# The Vertex Power of Ophthalmic Lenses

Douglas A. Kerr

Issue 6
December 7, 2010

## ABSTRACT

The ability of a lens to converge or diverge rays of light that arrive on separate, parallel paths is quantified as the *refractive power* (or just *power*) of the lens. In most optical work, the power is defined as the reciprocal of the focal length of the lens. In the case of ophthalmic (vision correction) lenses, the "rated" power is the reciprocal of the *back focal length* of the lens, a different quantity. The power reckoned that way is called the **vertex power** of the lens. The rationale for the use of this convention has been written of *ad nauseam*, but rarely is the basic justification for it clearly revealed. This article seeks to do that.

The article also discusses the use of a *focimeter* to determine the vertex power of ophthalmic lenses, including some special conditions pertaining to bifocal lenses.

## CAVEAT

I am not an eye care professional, nor do I have any formal training in the practice in that field nor in its own unique branch of optical science. The information in this article is my own interpretation of the results of extensive (mostly quite recent) research into the available literature, through the prism of my own scientific and engineering background and outlook.

## ABOUT LENSES

The article is all about lenses, and how we specify and measure certain of their key optical properties. The terminology used can lead to confusion, so it is well to clarify these matters before proceeding.

### The thin lens conceit

When explaining a certain optical concept, or in making an approximate calculation in a system of lenses, we often employ a fictional creature called the *thin lens*. It is so-called because its overall thickness is considered to be essentially zero, which collapses into insignificance some of the complicating detail that goes on inside a lens. It is fictional because a lens gets its refracting ability by virtue of the curvature of its surfaces, and if there is curvature, then the lens cannot truly be of zero thickness.

### The effective focal length of a lens

The lens parameter *focal length*, which figures prominently into many optical equations, and in such fields as astronomy and photography predicts several important aspect of lens behavior, is called in formal writing the *effective focal length* of the lens. This makes it sound as if this quantity is not the real focal length of the lens, but rather only a focal length-like quantity that predicts the lens' behavior in a certain circumstance.

But in fact this **is** the "real" focal length of the lens. The somewhat-misleading moniker came about as a result of the historical development of the understanding of lens behavior.

In that regard, in figure 1 we see a lens (not the fictional thin lens) of the converging flavor (a result of both its surfaces being convex).

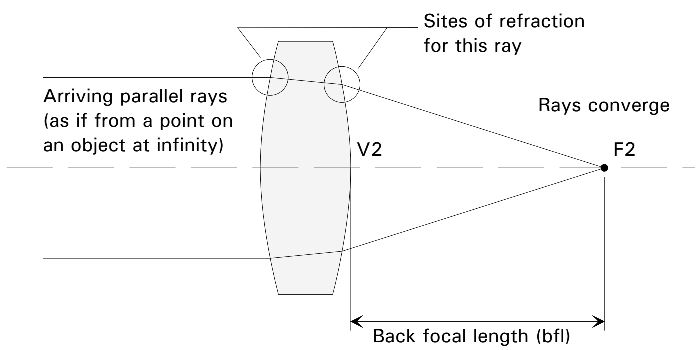


**Figure 1.**

We consider it to be receiving a number of rays or light, all traveling in parallel paths (and in fact, parallel to the lens axis). These are often thought of as rays emanating from the same point on an object being viewed which is at an infinite distance. (A distant star is a very close real approximation to this).

As a result of the refraction (deflection of the rays) that occurs at both the front and rear surfaces of the lens (we will for now speak of the side toward the object as the "front"), all these rays "converge" at a point, called the *second* (or rear) *focal point of the lens*, marked here F2.

Early workers recognized this as an important phenomenon, and realized that the location of this point, which affected the behavior of the lens, varied with the lens' physical nature (prominently with the curvatures of its two surfaces). Understandingly, they characterized this location in terms of the distance to the point from the *rear vertex* of the lens: the point on its rear surface that is on the lens axis. It is marked "V2" on the drawing. That distance is called the *back focal length* (or *back focal distance*).

However, as various equations for the behavior of a lens and of systems including several lenses emerged, it was soon realized that the back focal length rarely appeared as a parameter. What did show up frequently was another distance, generally slightly larger than the back focal length.

Because it was this distance, rather than the back focal length (bfl), that affected lens behavior, it was called the "effective focal length". Of course, that's a little bit like calling the weight of a brick its "effective weight", but that's the way the notation unfolded. And in fact, even today, this distance is called, in formal optical writing, the *effective focal length* of the lens. (The usual mathematical symbol for it is $f$.)

This distance is not measured from any point on the lens that can be easily physically identified (which is why it did not earlier come into play).

In figure 2, we get to see a little more about this distance.

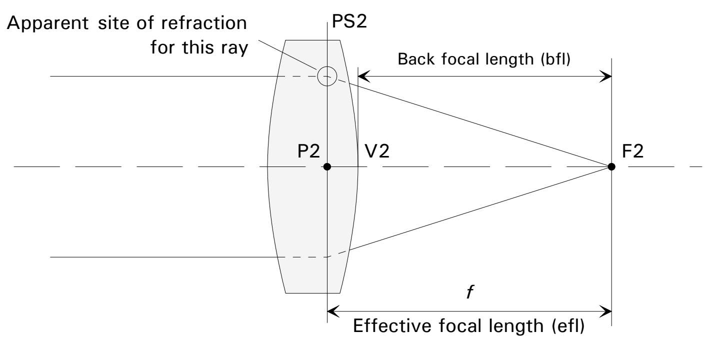


**Figure 2.**

Although it is not measured from any discernible physical feature of the lens, it is measured from an important conceptual place inside the lens.

It we look at one of the arriving rays, and then look at it as it exits the lens, it appears that the ray was deflected (refracted) as it crossed a certain plane, called the *second principal surface*[1] (labeled PS2).

We know of course that the ray actually was refracted once as it crossed the first surface, and again as it crossed the second surface (as seen in figure 1). But if we didn't know that, it would appear (from observation outside the lens) to have been refracted once, at PS2.

[1] Why **surface** rather than **plane**? Because in reality, it is often a curved surface. Treating it as a plane, as we do here, is another one of those conceits we use to simplify explanations.

(Warning—this is only true in the case of rays arriving parallel to the axis.)

And it is fact the distance from the point on PS2 at the axis, called the *second principal point* (P2), to F2 that is the critical parameter in many lens and optical system equations—thus its labeling as the *effective focal length*. And it is in fact "**the** focal length" of the lens—the real one.

In a typical symmetrical positive power lens (such as the one shown), the second principal point is located about 1/3 of the distance from the rear vertex to the front vertex. For lenses of other shapes, it may be in a different place. It can even lie outside the physical lens.

The customary mathematical symbol for (effective) focal length is ***f***.

**The refractive power of a lens**

The *refractive power* (often, just *power*) of a lens quantifies the degree to which a lens, overall, converges (or diverges) rays of light that enter the lens along separate parallel paths.

It should not be confused with the following, which are different concepts:

- The "power" of a telescope, which is a synonym for the more precise property *angular magnification*.
- The "power" of a hand magnifier, eye loupe, or such.

In general optical work, the power of a lens is defined as the reciprocal of its focal length (meaning its effective focal length). The modern scientific unit is the inverse meter ($m^{-1}$), but the traditional unit (always used in ophthalmic lens work) is the diopter (D), which is identical. A lens with a focal length of one meter has a refractive power of 1.0 $m^{-1}$ or 1.0 D.

The usual mathematical symbol for power is $\Phi$ (upper-case Greek *phi*). But in ophthalmological writing, the symbol ***F*** is usually used. (Both $\Phi$ and *F* are evocative of **f**ocal power.)

The power carries an algebraic sign, as does focal length. For a converging lens, the power (and focal length) are positive. For a diverging lens, the power (and focal length) are negative.

**Spherical and cylindrical lens behavior**

in the discussions above, we have assumed that we are considering rotationally-symmetrical lenses. These have the same refractive power for refraction along any "meridian"; that is, for example, along a line between 6 o'clock and 12 o'clock, and along a line between 8 o'clock and 2 o'clock. In ophthalmology, these are called "spherical" lenses (although in fact we may use such lenses whose surfaces are not parts of spheres—aspheric "spherical" lenses).

The distinction is with "cylindrical lenses", whose refractive power varies with the direction of the meridian, reaching a maximum along a certain meridian and falling to zero at the meridian at right angles to that.

The refractive properties of cylindrical lenses are involved in the correction of the vision defect of *astigmatism*. The specification of their power follows the same concepts as for "spherical" lenses (here applying to the maximum power), but include as well the specification of the direction of cylinder axis—the meridian of zero power.

In this article, we will ignore the vision defect of astigmatism and thus the matter of cylindrical lenses.

## IMPERFECTIONS IN THE HUMAN EYE

The theme of this article is in particular about lenses used to "correct" various imperfections in the behavior of a human eye. We will review this area to set the stage for our main topic.

The term *accommodation* is used in the field of vision science to refer to the eye's ability to focus on objects at different distances. Ophthalmic lenses, as found in eyeglasses and contact lenses, are principally intended to overcome deficiencies in the eye that prevent the person from fully utilizing that capability.

The three basic "defects" in accommodation (often described as "refractive defects") are:

Hyperopia[2] ("far-sightedness") is the deficiency in which the total range of accommodation is "offset outward", such that distant objects (even at "infinity") can be focused, but the near limit is not nearly as close as is normal. From a theoretical standpoint, the far limit is "beyond infinity", although since there are no objects there that is not of any value to the person.

Myopia ("near-sightedness") is the deficiency in which the total range of accommodation is "offset inward", such that close objects can be focused on but the far limit is short of infinity.

Note that in both these it is assumed that the person still has the normal "span" of accommodation; it has just been shifted from the desirable place (so one "end" is forfeit).

---

[2] Usually called in formal ophthalmological writing "hypermetropia".

The basic cause of these defects is that the focal length of the eye's lens system (which comprises two lens elements, the cornea and the "crystalline lens) is not appropriate for the distance from the lens to the retina.

Presbyopia ("old person's vision") is the deficiency in which the eye is able to make less change in the distance at which it focused than is "normal". It may be combined with *hyperopia*, which case the far limit of the range of vision is "beyond infinity", and the near limit may still be a large distance.

Or it may be combined with *myopia*, in which case the far limit may be at a modest distance, and the near limit not much closer.

In "full blown" presbyopia, the eye cannot change its vision distance at all, so the near and far limits become the same (and what distance that is can be affected by myopia or hyperopia).

The basic cause of presbyopia is decline in the effectivity of the eye's mechanism for changing the focal length of the crystalline lens.

## CORRECTION OF REFRACTIVE DEFICIENCIES

### Ophthalmic lenses

As we mentioned above, the principle role of ophthalmic lenses is to "correct" refractive deficiencies, allowing the patient to effectively attain good vision for objects over a range of distances. It is the administration of this process that is the specific thrust of this article.

### One outlook

One outlook on the role of ophthalmic lenses is that the corrective lens, together with the eye's lens system, form a new "compound lens", whose focal length is appropriate to the location of the retina. A converging lens (with a positive power) can correct for hyperopia; a diverging lens (with a negative power) can correct for myopia.

A second outlook is that, with the corrective lens in place, the eye sees the object being observed as if it is at a distance at which the eye can focus.

And there is a third outlook, which we will actually use here.

### The spectacle plane

Under either of those two outlooks, we will find that the effect of a corrective lens on overcoming the particular defect depends on both eh power of the lens (in the usually optical sense) and on the distance of the lens from the eye (in particular, the distance between the second principal point of the lens and the first principal point of the eye).

The whole matter of measuring the patient's vision defect, prescribing a lens to correct that, and constructing a lens to fulfill the prescription would be made much simpler is we generally chose to place the lens at a fixed distance in front of the eye (as defined above).

But there is a fly in the ointment. The location of the second principle point of a lens "on the lens" varies significantly with the shape of the lens. Suppose we were to adopt for general use a standard distance between the first principal point of the eye (which lies in a fairly predictable location with regard to the front of the eye) and the second principal point of the lens. We would find that for some lens shapes the lens would be placed so far from the face as to look silly, while for other lens shapes it would have to be pressed hard against the face (or worse).

Thus, a more pragmatic norm is adopted. Here, we ordinarily seek to place the rear surface of the lens (precisely, its rear vertex, which is the point on its rear surface on its axis) at a standard distance from the front of the eye (and thus a standard distance from the eye's first principal point). That intended location of the rear vertex of the lens is called the *spectacle plane*.

A result of the adoption of this practice is that the effect of the lens does not follow consistently from its power; for a certain corrective effect, the needed power of the lens will depend on its shape.

We will see shortly that, fortunately, in a context where the rear vertex of the lens is located at a standard distance from the eye, it will be the vertex power of the lens (the reciprocal of the back focal length) that tells us its corrective effect.

In the next section, we will demonstrate (we might even say "prove") that convenient fact.

## DEVELOPMENT OF THE VERTEX POWER CONVENTION

### The far point and near point outlook

For our interests, it is useful to look at the vision defects, and their correction, in a way involving the concept of the *far point* and *near point* of the eye. We will start with the far point, which will first come into the story.

### The far point of the eye

The far point of an eye is the point at which it is focused when the focusing muscles are fully relaxed—the greatest distance at which the eye can be focused. In an "ideal" eye, this distance is very nearly infinite.

We can look at the mechanism of hyperopia as being that the eye's far point is significantly "beyond infinity", a physically-meaningless notion but one that is quite tractable mathematically. We can represent this graphically, to guide our analysis, as the far point being **behind** the eye (equally physically-impossible, of course, but much easier to draw).

**Correction of hyperopia**

A convergent lens (positive power) placed in front of the eye will effectively "relocate" the far point to a desirable location in front of the eye, near infinity.[3]

This is reminiscent of the situation in photography when the lens takes an *object* at a certain distance and from it creates, on the other side of the lens, at a certain distance, an *image* of the object. Here, the lens takes the near point (just a location, not a real object, and in an "impossible place"), and nicely creates its alter ego on the far side of the lens—in this case, at or near infinity.

So there is much voodoo to all this. We just have to have faith in the mathematics of things unseen, or unseeable.

The following figures do not follow a consistent scale. In general, the region including the eye and lens is presented in one scale. The distances to points behind the eye are in a different scale, and the distances to points in front of the eye are in a different scale yet, all in the interest clarity of the principles.

In our first scenario, in figure 3, we see this outlook on the use of a convergent lens to correct for hyperopia.

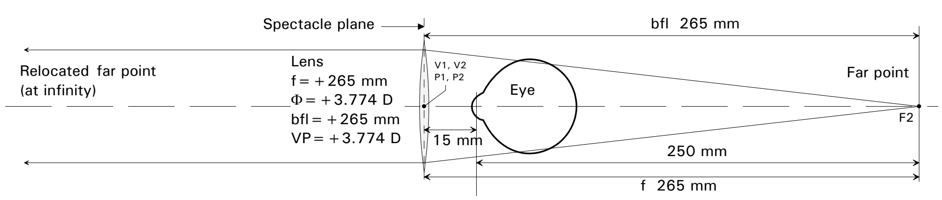


**Figure 3. Correction of hyperopia (thin lens)**

The far point of this eye is assumed to be located 250 mm behind the first principal point of the eye. In accordance with conventional vision correction practice, we place the *rear vertex* of the lens, V2, (the point on its rear surface at the axis) at a standard distance, usually 15 mm

---

[3] In actual practice, we may wish to relocate the far point only to a distance of 20 feet or so, but we will assume infinity as it makes the story more tidy.

as seen here, from the eye's first principal point.[4] This location is referred to as the *spectacle plane*.

Of course in a thin lens, both front and rear vertexes, and both the first and second principal points, are all in the same place, so we might just say, "we place the lens 15 mm in front of the first principal point of the eye." But we must remain aware of the separate roles of the various points, which will later become distinct. (We draw the thin lens with a finite thickness just so that we can recognize that it is a lens.)

Thus the actual far point of this eye is 265 mm behind our lens. If we want the "conjugate" of that point (the point where the lens will create an alter ego of the point) to be at infinity, then the near point must lie at the second focal point, F2, of the lens. That is behind the second principal point of the lens (P2) by the focal length of the lens (f).[5]

Recall that rays from any point on an object at infinity arrive at the lens parallel (as we can see in the figure). This is why they (by definition) converge at the second focal point.

In our fanciful "thin lens", the second principal point (P2) is at the same place as the rear vertex (V2).

Thus we can see that the focal length of this lens must be +265 mm (the positive sign is known since the lens is converging). By definition, the power of the lens is the reciprocal of its focal length, or +3.774 diopters (+3.774 D).[6]

The *back focal length* of a lens is the distance from the rear vertex to its second focal point. But in our thin lens that will be the same as the focal length: +265 mm. In ophthalmic lens work, the *vertex power* of a lens is defined as the reciprocal of the back focal length. In our thin lens that will be the same as the power of the lens, +3.774 D.

Now, wasn't that easy?

---

[4] In actual standard practice, we place it 13.75 mm from the front of the eye (the front vertex of its cornea), which we assume is 1.25 mm in front of the eye's first principal point.

[5] That is, the *effective focal length*, to use its formal name.

[6] This is the *power* of the lens as used in most optical work, the reciprocal of the *effective focal length*.

But now we must consider a more nearly-real lens, a "thick" lens. In figure 4, we see one of those used to deal with the same eye's hyperopia.

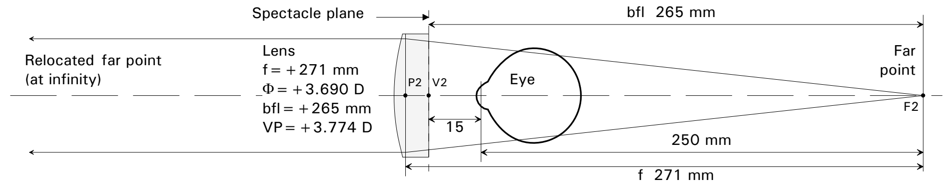


**Figure 4. Correction of hyperopia (thick lens)**

Since it's the same eye, the far point is still 250 mm behind the first principal point of the eye. Again following normal ophthalmic lens practice, we place the lens so its rear vertex, V2, is 15 mm in front of the first principal point of the eye.

We assume the use of a plano-convex lens, with its convex surface forward. In such a lens, the second principal point (P2) is exactly one-third the thickness of the lens back from the front vertex. We assume the thickness of the lens to be 9 mm (that's pretty fat, but makes the important mathematical relationships show up clearly). Thus P2 will be 271 mm from the far point of the eye.

As in the first scenario, to have the lens relocate the far point to infinity, it must be at the second focal point of the lens, F2. So the *focal length* of this lens must be +271 mm, and thus its *power* will be +3.690 D.

The *back focal length* of the lens, which is the distance from its rear vertex to its second focal point, will then turn out to be 265 mm, and its *vertex power*, the reciprocal of the back focal length, will be +3.774 D.

Recall that the two scenarios each correct an identical situation of hyperopia, while honoring the standard ophthalmic practice of having the back vertex of the lens 15 mm in front of the first principal point of the eye.

We note the following interesting facts:

- The power (in the usual optical sense) needed with the thick lens is less than with the thin lens. (That's because its second principal point is farther from the eye.) The amount of this difference depends on the particular "shape" of the thick lens.

- The vertex power needed in the case of the thick lens is the same as with the thin lens.

And this is why, in the field of vision correction, a lens is "rated" in terms of its vertex power rather than its (ordinary) power.

> If we assume that the lens is always mounted with its rear vertex at a certain distance from the eye, then its effect on vision correction is consistently indicated by its vertex power, regardless of the shape of the lens.

When the ophthalmologist or optometrist determines, by testing with an instrument, the "power" of a lens needed to produce the best corrected distant vision, he states that in the prescription in terms of vertex power.

He doesn't need to be concerned with what shape of lens will be used (which is of course influenced by many decisions made at the time the glasses are purchased, as well as by practical manufacturing considerations).

Of course, for this to work out, the finished glasses must place the rear vertex of whatever lens is used at the distance from the eye presumed by the ophthalmologist (13.75 mm from the front vertex of the cornea if no special instructions are noted on the prescription). This is hopefully taken care of by the design of the eyeglass frame.

In reality, at the optical shop, the thousands of lenses they have in stock (ready to just be cut to the perimeter shape need to fit the chosen frames) [7] are all marked on their little envelopes with their *vertex power*. A lens of the proper shape, type of material, coating, impact resistance, etc. is chosen that is marked with the vertex power specified in the prescription.

**Myopia**

In myopia (nearsightedness), the person is unable to focus on distant objects. We can look at the mechanism of myopia as being that the eye's far point is located not near infinity but rather at a relatively-short distance.

To mitigate myopia, we use a diverging lens. We can understand its action in terms of its relocating the eye's far point from a relatively small distance to a place "near infinity". The process is conceptually identical to what we saw for hyperopia, and various implications of it are just as seen in that case. However, the model of lens action now involves the equivalent of a "virtual image", which makes it a bit

[7] You didn't think they grind and polish those puppies in the back room of the optical shop "in about an hour", did you?.

trickier to illustrate. We'll leave it up to the reader to imagine that whole process.

**The far point of the eye**

Before we look into the correction of presbyopia, we note that the *near point* of an eye is the closest point at which it can be focused. In an ideal eye, this might be at a distance of about 10 inches from the eye.

**Presbyopia**

In *presbyopia*, the eye is not able to change very much (or at all) the distance at which it is focused. In "full-blown" presbyopia, the eye cannot change focus distance at all. In such a case, the near point is at the same place as the far point.

But for near vision we do not wish that the near point is at infinity (as for the far point in the case of distant vision). Rather, we wish it to be "relocated" to a point that represents the distance at which we wish to be able to focus on "near objects". Clearly, this requires a lens of different power than we use to correct the basic "distant vision" refractive error of the patient.

**The bifocal lens**

We deal with this dichotomy through the use of bifocal lenses. In these, there is a region (normally at the bottom of the lens) in which the power (in either way we can describe that) is more positive than in the rest of the lens. This region is called the "near vision segment" of the lens (or just "segment" for short). The user looks at near objects through that part of the lens (but at distant objects through the "main" part).

As we might guess by comparison with the earlier part of this discussion, the task of the near vision segment (which we can think of as a "second lens in the same frame") is to relocate the eye's near point to a more desirable location. The location chosen may depend on the person's visual habits: what kind of "near" object is the most important, and at what distance from the eye is it normally found.

In figure 5, we see this process at work.

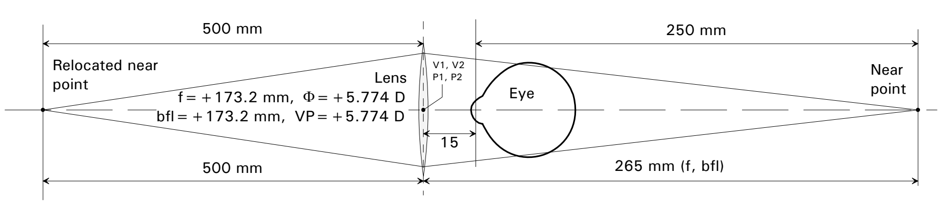


**Figure 5. Correction of severe hyperopic presbyopia (thin lens)**

The eye assumed here exhibits “full-blown” hyperopia, with the near point at the same place as the far point (250 mm behind the first principal point of the eye) seen in the earlier example. We have chosen the ideal near point location as 500 mm (about 19.7 inches) in front of the lens.

We will start again with our fanciful thin lens. As before, we assume that it will be mounted so that its rear vertex is 15 mm in front of the first principal point of the eye. Since this is a thin lens, that means “the lens” is at that point.

Thus, the eye’s near point will be 265 mm behind the lens. We look to the lens to create its alter ego at a distance of 500 mm in front of the lens.

From those distances, we can calculate that the focal length (*f*) of a lens that will do that must be +173.2 mm; its power ($\Phi$) will thus be +5.774 D. Since this is a thin lens, the back focal length (bfl) is the same as the focal length, +173.2 mm, and the vertex power (VP), the reciprocal of the back focal length, will be +5.774 D (the same as the power of the lens—this is always so for a thin lens).

Before we proceed, note that the difference between the vertex power needed to correct distant vision (+3.774 D) and the vertex power needed to correct near vision at a distance of 500 mm (+5.774 D) is exactly +2.000 D.

That difference in vertex power is exactly the reciprocal of the assumed near vision distance, 500 mm. This is no accident. The math makes this inevitable, given that we have assumed that the eye’s far point and near point are the same and that we will relocate the far point to infinity. Simply, to move this “fixed-focus” eye’s focusing distance from infinity to 500 mm requires an additional +2.000 D of correcting lens vertex power. To move its focusing distance from infinity to 333 mm would require instead an additional +3.000 D of correcting lens vertex power. (All this assumes we are using a thin lens.)

Let’s now do the near vision correction with a “thick” lens, as seen on figure 6.

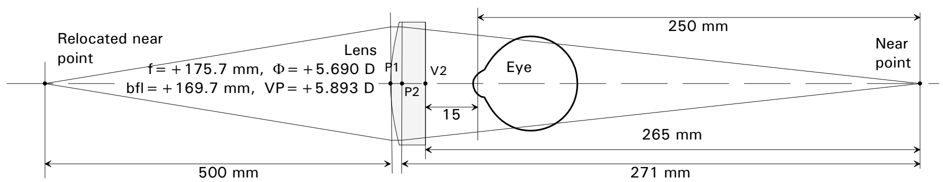


**Figure 6. Correction of severe hyperopic presbyopia (thick lens)**

The story goes pretty much as in the various previous scenarios. Our desired "relocated" near point is now 500 mm from the first principal point of the lens, P1 (yes, we have to work with that now; in a lens of this shape it falls at the front vertex), but the actual near point is 271 mm from the second principal point of the lens. We go through all the math and find out that for the lens to do its job it must have a focal length ($f$) of +175.7 mm, and thus a power ($\Phi$) of +5.690 D.

The second focal point of the lens (F2) will be a distance equal to the focal length, 175.7 mm, behind the second principal point (P2). Thus it will be 169.7 mm behind the rear vertex of the lens. (We don't show F2 on the drawing since it is not an optical point of interest in this scenario, other than in the mathematical reckoning we are doing just now.)

So the back focal length (bfl) of the lens will be 169.7 mm, and the vertex power (VP) will be its reciprocal, +5.893 D.

Now, if we compare the two lenses, which we "designed" to have the same effect on the eye's near vision (for our specific choice of a desired near vision point[8]), we find the following:

- The power (in the usual optical sense) needed with the thick lens is **less** than with the thin lens.

- The vertex power needed in the case of the thick lens is **greater** than with the thin lens.

This latter is different than what we found when we played the entire game with regard to distant vision correction. In fact, here we seem to have lost the wonderful convenience of being able to think only in terms of a lens' vertex power when considering how it will affect vision correction, regardless of the lens shape.

We can quantify the impact of this situation by noting that, for the particular near vision distance we have assumed, our thick lens has exactly the same effect on the near point as the thin lens we saw in figure 5, whose vertex power is +5.774 D. Thus we can say that, for the particular near vision distance we have assumed, the *near vision effective vertex power* of our thick lens is +5.774 D.

Often, this situation is described as a *near vision effectivity error* (NVEE) of -0.119 D—the difference between the near vision effective

[8] You will find me using this qualifier, "for our specific choice of a desired near vision point", or its equivalent, over and over again. It is pivotal to many of the issues to come, but often overlooked, and I want to make sure that we remain conscious of it.

vertex power of the lens, +5.774 D and its actual vertex power, +5.893 D.

If we thoughtfully reflect on figures 5 and 6, we will realize that this “error” emerges as a result of the second principal point of the lens not being coincident with the rear vertex, as it is in a thin lens, and from other implications of the lens shape.

More thoroughly, it turns out that the value of the NVEE depends on the actual vertex power of the lens and its “shape”.

## PRESCRIBING THE LENS

### Near vision effective vertex power

We can see at this point that a prescription for bifocal lenses ideally should state the needed *near vision effective vertex power* of the near vision segment. (We’ll see later that this value is actually written in an indirect way.)

### Refracting the patient

Commonly, the needed lens properties for vision correction are determined by the examiner (a process called “refracting the patient”) using an instrument called a refractor. This is the scary mask-like instrument behind which one sits while the examiner says, “Which is better, one (click) or two (click)”.

In this instrument, the patient looks through a “trial” corrective lens, whose vertex power can be varied over a large range in steps of 0.25 D by moving a wheel, marked in terms of vertex power. The examiner varies this power, with the patient regarding a “distant” target, until the patient reports the best vision.

The vertex power of the trial lens in the refractor at this point (indicated on the wheel) is taken to be the desired vertex power of the lens (of the main part of the lens, if a bifocal lens is involved).

If a bifocal lens is involved, we then have the patient regard a near target, and change the vertex power of the trial lens until best vision is again reported. The vertex power of the trial lens in the refractor at this point is taken to be the desired vertex power of the near vision segment.

What about near vision effectivity error (NVEE) in the trial lenses? Well, for our purposes, we can think of the trial lenses as having the properties of the imaginary “thin lens”. Thus, even when the trial lenses are working in a “near vision” context (the patient regarding a near target), the indicated vertex power (on the wheel) is in fact the effective vertex power.

So we can regard the specified power of the near vision segment implied by the prescription as the needed *near vision effective vertex power* in the segment.

**The "add"**

I have so far intentionally avoided the introduction of an important convention of vision correction practice. It would have had no meaning on what we have done so far, and obsession with it can misdirect our thoughts. But it will raise its head in what is to come, so I now need to introduce it. It is a convention that goes back to an earlier era of "refracting" a patient.

When a prescriber has determined, in the case of a bifocal lens, the desired vertex power for the "distant vision" (main) part of the lens, and then for the "near vision" (segment) part of the lens, he does not write both directly in the prescription. Rather he writes the desired power for the main part of the lens, and then how much more positive than that is the desired power in the segment. This latter number is called the "add" (from the way it is written on the prescription).

So for this specification for the lens we saw before, rounded to increments of 0.25 D (as is the practice):

Vertex power in the main part of the lens: +3.75 D

Vertex power in the segment: +5.75 D

the prescriber writes:

+3.75 add +2.00

It is important to remember that the "add" value is not an optical property of the near vision segment. It would not inherently appear in any optical equation. It is only part of a historical system of notation whose sole purpose is to describe the vertex power of the near vision segment.

**DISPENSING**

When the patient takes the prescription to an optician to have the eyeglasses made (that is, to have the lenses "dispensed"), the lenses that are provided (that is, chosen from thousands of factory-made lenses in the back room, all needing only to be ground to the right outline shape to fit in the chosen frame) must have, in the segment, the near vision effective vertex power implied by the prescription.

Their **actual** vertex power must differ from that by the negative of the NVEE that is involved for the lens power and the near vision distance that is intended. The latter distance may not be mentioned on the prescription, so perhaps a "normal" value is assumed. (However, we can make a fairly good guess of the intended near vision distance from the "add" power in the prescription.)

## TESTING THE LENS

We may wish to test a completed pair of eyeglasses to be certain that the prescription has been accurately implemented. Testing of corrective lens vertex power is commonly done with an instrument called a *focimeter*. It essentially determines the back focal length of the lens, and then reports the reciprocal of that as the vertex power. We can have it do that for the main part of a bifocal lens and then for the near vision segment.

### Principle of the focimeter

Before we see some implications of that process, we need to understand how a focimeter works. Figure 7 is a conceptual presentation of the scheme it uses.

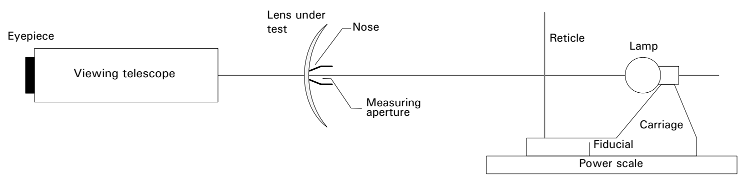


**Figure 7. The focimeter (conceptual)**

The lens to be tested is normally placed with its rear vertex against a fixed open conical tip of the measurement aperture colloquially called the "nose". To the right of the nose is a movable carriage carrying a target in the form of a pattern of crossed lines on a reticle, illuminated from behind.

At the left is a viewing telescope, adjusted so as to be precisely focused at infinity.

If the target happens to fall at the second focal point of the lens, then a "real" image of the target is formed, to the left, at infinity, which can also be considered a "virtual" image to the right, also at infinity. If the target is not at the second focal point, the image formed is not at infinity.

The operator regards this virtual image (generated by the lens under test) though the telescope. Since the telescope is focused at infinity, the virtual image will only be seen in sharp focus if it is also at infinity. The operator moves the carriage with a handwheel until that occurs. At that time, the reticle must lie exactly at the second focal point of the lens.

Because the back vertex of the lens is at a known position in the instrument (against the nose), the position of the reticle is relative to the back vertex, so the distance noted at this time is the back focal length of the lens.

The focimeter reports the reciprocal of that distance on a dial. This is the *vertex power* of the lens.

Note for future reference that when we say that the lens under test creates an image of the target "at infinity", this means that the rays from any point of the target exit the lens (to the left) parallel.

**Measuring bifocal lenses**

In measuring a bifocal lens with the "basic" technique, we first measure the vertex power in the main part of the lens by placing the lens so that its optical center is in the center of the measurement aperture nose. (We can tell where the optical center is from things we can see through the instrument.) We record that on our report.

Then we raise the lens (the glasses sit on a little "elevator") so that the appropriate point in the segment (not its optical center, for various reasons) is centered on the nose, and measure the power there.

We subtract the value for the main part of the lens, determined earlier, from this value for the segment, and record that as the "add" for the segment.

Remember, the "add" value is not any optical property of the lens. It is just the numerical difference between two optical properties, the vertex power in the main lens and the vertex power in the segment.

**Which power is that for the segment?**

Unfortunately, this doesn't really do what we want. This process will (via the usual "add" convention for writing it) report the "actual" vertex power of the segment. But the "add" of the prescription specifies, by addition to the "basic" power of the lens, the desired *near vision effective vertex power* of the segment. And that is what we should determine by measurement to find out if the lens actually fulfills the intent of the prescription.

Can we measure the effective vertex power of the segment with our focimeter? Yes. We can measure it precisely by the "auxiliary lens" technique, and more conveniently, but a little less precisely, by the "reversed", or "front vertex", technique. We will look briefly at each of these; they will get further treatment in two appendixes.

**The auxiliary lens technique**

Here the task is to determine the *near vision effective vertex power* of the near vision segment.

We begin by placing immediately in front of the lens being tested an auxiliary lens, whose vertex power is the negative reciprocal of the near vision distance for which we want the effective power of the lens. For our examples in figures 5 and 6, with a near vision distance of 500 mm, that would be a –2.000 D lens. We may not be told that assumed distance by the prescription. We can, however, reasonably assume that it is the reciprocal of the “add” power specified for the segment.

After we do that, the focimeter tells us the vertex power of the combination of the two lenses—a composite lens. That turns out to be dependent on the *near vision effective vertex power* of the segment (for the near vision distance of interest), not its actual vertex power.

Specifically, if we subtract from the vertex power reading the power of the auxiliary lens (observing the algebraic signs), we will have the *near vision effective vertex power* of the lens under test (for the near vision distance of interest).

The technical basis of this technique is covered in appendix A.

**The “reversed” measuring technique**

In this technique, the lens is placed in the focimeter “reversed”: with its front vertex against the nose of the focimeter. The indicated vertex power is measured in the main part of the lens and in the near vision segment. The difference between those readings is taken as the “add” of the segment.

This technique does not necessarily yield an exact value for the “add” that indicates the near vision effective vertex power, but the error is typically far smaller than the precision to which we normally report vertex power.

A detained discussion of this technique will be found in Appendix B.

**SUMMARY**

We have seen how “rating” corrective lenses in terms of their (back) *vertex power*, rather than their *power* (in the usual optical theory sense) supports a practical, consistent regimen of vision correction management. We also have seen that direct measurement of the vertex power of the “near vision segment” of a bifocal lens does not tell us the parameter we actually need to know to precisely grasp the effect of that segment on near vision correction. Finally, we have seen how two special measurement techniques can provide us with that critical parameter.

#

# APPENDIX A

## The auxiliary lens measurement technique

The object of this technique is to determine the *near vision effective vertex power* of the near vision segment of a bifocal lens.

We begin by placing immediately in front of the lens being tested an auxiliary lens whose vertex power is the negative reciprocal of the near vision distance for which we want the effective power of the lens. For our examples in figures 5 and 6, with a near vision distance of 500 mm, that would be a −2.000 D lens. We may not be told that assumed distance by the prescription. We can, however, reasonably assume that it is the reciprocal of the "add" power specified for the segment.

We show this arrangement in figure 8—almost. We actually draw the auxiliary lens a substantial distance in front of the lens under test. This allows us to better show what happens in the region between the two lenses (which in the real technique is of negligible size).[9]

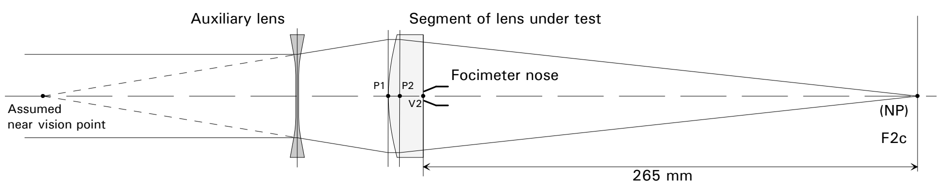


**Figure 8. Auxiliary lens technique—conceptual**

Remember, this is the near vision segment we are talking about—we have just drawn it so it looks like a regular lens for graphic symmetry.

We see that the auxiliary lens would take rays, imagined as starting at point NP (the location of the near point in our actual vision correction layout), which have been converged by the lens under test so they would meet at the assumed near vision point, and, before they could get there, intercepts them and diverges them so they are parallel. Thus, they are now rays heading to make an image at infinity. Then, by definition, point NP must be the second focal point of the composite lens (F2c).

Now we'll go to the actual arrangement (figure 9.)

[9] Purists may note that if we actually did this, the auxiliary lens would have to have a higher power than the one that will appropriate for the actual technique.

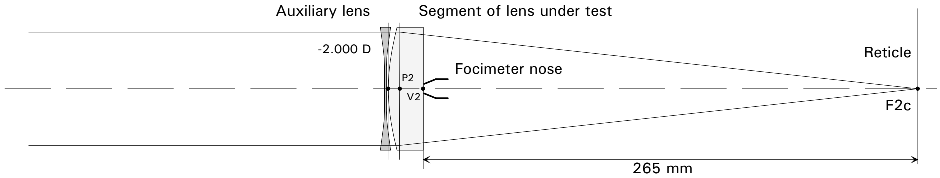


**Figure 9. Auxiliary lens technique—actual setup**

Here we actually put the auxiliary lens snug up against the front vertex of the lens under test. It is concave to the rear, so this is probably doable.

In this situation, the region in which the rays lie along lines converging toward the relocated near point has shrunk to zero size, so we cannot see it as we could in figure 8. But the concept is unchanged.

We assume the actual center thickness of the auxiliary lens is very small.[10] As mentioned a little while ago, we chose its power to match the distance from the front vertex of the lens under test to the assumed near vision point (500 mm): a power of -2.000 D (negative because this is a diverging lens). [11]

Having done that, as in the earlier figure, the second focal point of the composite lens (F2c) will fall 265 mm behind its rear vertex (the rear vertex of the lens under test, V2).

With V2 against the measuring nose, we move the carriage until the image of the target is in perfect focus. This will happen when the reticle is at F2c, The focimeter will conclude that the back focal length of whatever is between its nose and its telescope is 265 mm. The focimeter then reports the reciprocal of that as the vertex power of whatever it is measuring: +3.774 D.

With the two elements of this “composite lens” essentially in intimate contact, the power of the combination will be very nearly the sum of the powers of the two elements. Its leftmost element (the auxiliary lens) has a power of −2.000 D (we made it have that). We subtract that value from the reported vertex power for the combination, +3.774 D (properly observing the algebraic signs), and get what we will take to be the effective vertex power of the other element (our lens under test): +5.774 D.

---

[10] This is not the imaginary thin lens assumption; a real biconvex lens can be made as thin at its center as we think is safe!

[11] Because of the shape of the lens, it really doesn’t matter whether we think of this as its (conventional optical) power or its vertex power from one side or the other.

The difference between that power and the power of the main portion of the lens (**measured in the usual way**) can be thought of as the “near vision effective add” of the segment. Again, it is not any intrinsic optical property of the lens. It is just a numerical way to state the *near vision effective vertex power* of the segment.

We determined earlier, by reference to the thin lens situation, that the effective vertex power of this lens in near vision correction, with the intended near vision point 500 mm in front of the lens, is +5.774 D. Thus we see that the auxiliary lens technique has accurately reported the *near vision effective vertex power* of our segment.

#

## APPENDIX B

### "Reversed" measurment of the near vision segment

The object of this technique is to determine the near vision effective vertex power of a the near vision segment in a bifocal lens.

In this technique, the lens is placed in the focimeter "reversed": with its front vertex against the nose of the focimeter. The indicated vertex power is measured in the main part of the lens and in the newer vision segment. The difference between those readings is taken as the "add" of the segment.

In this appendix, we look into the principle of this technique.

**Background**

In a bifocal lens, the vertex power in the near vision segment is more positive than the vertex power in the main, or "distant vision", part of the lens. In most bifocal lenses, this is implemented by giving the front surface of the lens, in the segment region, a greater curvature than in the remainder of the lens' front surface.

If we wish, we can visualize this as our having appliquéd a small convex "mound" of lens material on the front of the basic lens. This mound is most often described as the "near vision addition" of the lens. The concept can be seen in figure 10.

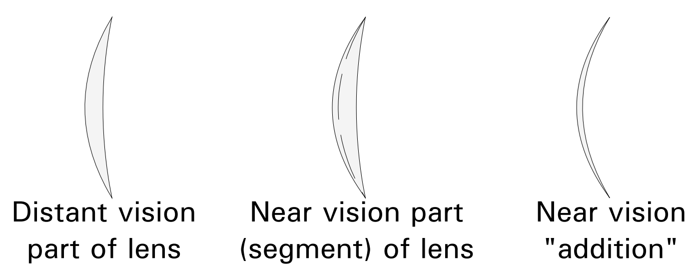


**Figure 10. Composition of near vision segment**

The left panel represents the distant vision (main) part of the lens (drawn as a full-size lens). The center panel represents the near vision segment (again drawn as a full-size lens). The dashed line shows, for comparison, the front surface profile of the distance vision portion. The part beyond that is considered to be the near-vision "addition". The right panel shows the "addition" in isolation, as if it had been excised from the lens.

When an eyeglass prescription is implemented, we can imagine that first a main lens is designed with the vertex power specified in the prescription. Then an "addition" is incorporated, by way of a more positive curvature of the front surface. It will serve to increase the power of the lens in the segment region.

Let's look at just how that works—it will be critical to what follows. As before, we will assume a patient with "full-blown" myopic presbyopia, as a consequence of which the eye's near point is at the same place as the far point. (This simplifies the story.)

In figure 11, we see the lens we had earlier designed to correct that eye's hyperopia, but now with the near point labeled. Since we assume that to be in the same place as the far point, this figure is identical to figure 6 (except that we have taken off a lot of the clutter). The lens "designed" in figure 6 had a vertex power of +3.774 D.

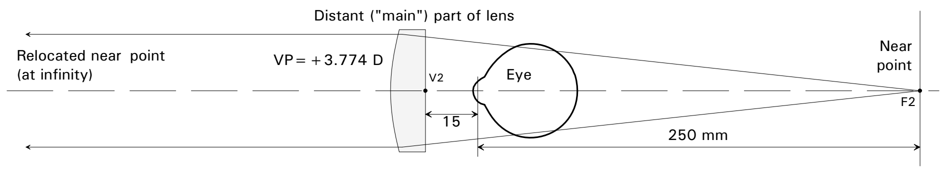


**Figure 11. Relocation of near point by distant part of lens**

Now, we will again develop our near vision segment. Unlike in figure 6, where we "designed" the segment in terms of its optical parameters, in figure 12 we will use a "construction" technique, actually sticking on a salient "addition" to form the segment.

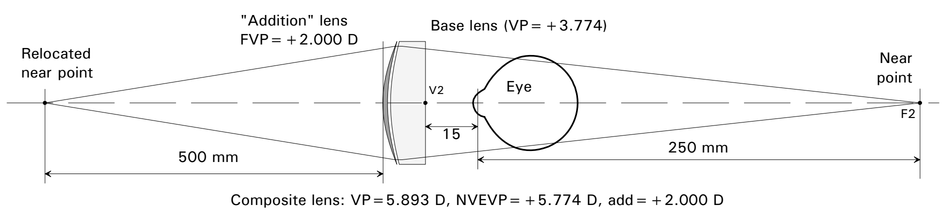


**Figure 12. "Construction" of near vision segment**

The addition is shown as a meniscus lens (curved front and back), so that it can be "plastered" onto the front surface of the base lens. We have shown a small gap between it and the base lens to emphasize its "independent" nature at this point in its life.

We see the place to which we wish to relocate the eye's near point, again 500 mm in front of the (entire) lens.

Since the rays to the front of the distant part of the main lens in figure 11 are parallel, then the power required of our "addition", which will converge those parallel rays to the relocated near point, must be +2.000 D (the reciprocal of 500 mm).

Now when we "stick the addition onto" the face of the base lens, nothing changes. And thus this built-up lens must have the same optical properties as the integrated segment earlier seen in figure 6 (since it has the identical effect on near vision: relocating the near point to 500 mm in front of the lens).

But we note that the back vertex power of that segment is not 2.000 D greater than the vertex power of the base lens (which is the distant vision correcting lens seen in figure 4). It is 2.119 D greater.

And the same will be true of the segment we just "assembled"—its vertex power will be 2.119 D greater than the base lens. That is because when we make a composite lens by combining two lens "elements", the power of the composite lens will be in general be greater than the sum of the powers of the two elements.

But as we saw earlier, it is not the vertex power of the near vision segment that we expect to be a certain amount greater than the vertex power of the base lens—it is the *near vision effective vertex power*, which is less than the vertex power proper.

In the case of this composite lens, the near vision effective vertex power turns out to be exactly 2.000 D diopters greater than the vertex power of the base lens—just what we want.

Thus, by pasting on the front of the base lens an "addition" with a vertex power (that's its *front vertex power*, to be precise) of +2.000D, we have made the *near vision effective vertex power* of the segment 2.000 D greater than the power of the "base" (distant) part of the lens.

Let me restate this pivotal fact in a more general way:[12]

| |
|---|
| The amount by which the near vision effective vertex power of a segment is greater than the vertex power of the "base" lens (an amount reflected in the "add" of the prescription) is exactly the front vertex power of the "addition" that creates the segment (as if excised and measured on its own). |

Now, back to inspecting our bifocal lens to determine its conformity with the prescription. If we could measure the front vertex power of the addition, that should match the "add" of the prescription (which is based on the near vision effective vertex power of the segment).

Clearly, in practical work, excising the addition for this purpose is out of the question.

[12] The failure to clearly articulate this key fact is the "missing link" in many technical papers that attempt to explain and justify—often, to me, unconvincingly—the "reversed" measurement technique for a bifocal segment.

However, it turns out that, if we examine the lens "reversed"—with its front toward the nose of a focimeter, making vertex power measurements in the distant portion of the lens and in the near vision segment, and take the difference between those readings, this will be very nearly the front vertex power of the addition.

And so that result will be very near the prescription "add" that describes the near vision effective vertex power of the segment.

This technique is theoretically less precise than the auxiliary lens technique, but still should give results well within the precision to which we normally state vertex powers (0.25D—or at best, 0.125 D). And it is more convenient that the auxiliary lens technique.

This table summarizes the results of a numerical simulation of two detailed lens designs (a meniscus overall design in both cases, with the addition on the front surface). The "add" values of the prescriptions are intended to indicate the near vision effective vertex power of the segment.

| | Lens design 1 | Lens design 2 |
|---|---|---|
| Prescription | +4.000 add +2.000 | +3.000 add +1.500 |
| Front vertex power of the addition | +2.000 | +1.500 |
| Estimated front vertex power of the addition from "reversed" measurement | +1.954 | +1.469 |
| Error | –0.046 | –0.031 |

#