# Trial Lenses in Vision Correction

Douglas A. Kerr

Issue 7
May 8, 2022

**ABSTRACT**

When prescribing corrective lenses (eyeglass lenses, contact lenses), the prescriber may determine the optimum parameters for the lenses with a technique involving calibrated "trial lenses" placed in an eyeglass-like "trial frame". The basic concept is straightforward, but there are myriad complexities involved in its actual use and working. In this article, this process and its apparatus are described in considerable detail. An introductory section gives background in some areas of lens theory and on the nature and correction of certain vision defects. Appendixes provide further detailed information on various of the topics.

## 1 CAVEAT

I am not an eye care professional, nor do I have any formal training in the practice in that field nor in its own unique branch of optical science. The information in this article is my own interpretation of the results of extensive research into the available literature, through the prism of my own scientific and engineering background and outlook.

## 2 TERMINOLOGY AND NOTATION

### 2.1 Refraction

In general optical theory, *refraction* refers to the "bending" of light rays by a lens or prism. But in the field of vision correction, *refraction* also refers to measuring the optical parameters of "refractive" flaws in a person's vision with the objective of properly specifying eyeglass lenses that will best correct those flaws.

Two techniques of refraction are commonplace, using the *refractor* and the *trial lens system*. This article discusses the latter.

### 2.2 Not that kind of trial lenses

Be sure not to confuse the term "trial lenses" (as used here), meaning a system of refracting a subject, with "trial lenses" meaning temporary contact lenses that are made from a proposed prescription and which the patient tries on to see if that if that prescription (and the "fit" of the lenses) really "works" for them.

### 2.3 Eyeglass lenses and contact lenses

While the general concepts described here apply to contact lenses as well as eyeglass lenses[1], many of the "wrinkles" are different in the case of contact lenses. The specific application of the principles to contact lenses is beyond the scope of this article.

### 2.4 Optometrists, ophthalmologists, and so on

I use the term *optometry* ("eye-measuring") to mean the science of measuring the refractive properties of the human eye, typically to the end of developing the "prescription" for corrective lenses (*e.g.*, eyeglasses or contact lenses).

But under the U.S medical licensure system, *optometry* refers to a specific health care profession, conducted by a certain type of highly-trained and licensed professional (an *optometrist*), who holds the degree of Doctor of Optometry (OD).

However, other licensed health care professionals are also authorized to conduct what I refer to as *optometry*, notably ophthalmologists, who hold the degree of Doctor of Medicine (MD) or Doctor of Osteopathy (DO) and who have specialized in the care of the eye (*ophthalmology*).

In addition, in many states, *certified ophthalmic technicians* and similar paraprofessionals are permitted to conduct refractions.

Ophthalmologists, though, might well be offended if I speak of their performing *optometry*, which they associate with the specific profession practiced by licensed *optometrists*, which ophthalmologists might look down upon as a "lesser profession".

So keep in mind that when I mention *optometry*, I mean the science, not the health care profession.

To avoid offense to any of these classes of professionals and the related paraprofessionals, in he interest of generality, in most parts of this article I will speak of a person performing optometry as an *refractionist*.

And because, perhaps in a research context, the person on whom the refraction is conducted may not be under "medical care", in the interest of generality I will usually not speak of such a person as the "patient" but rather as the *subject*.

---

[1] In technical writing in this field, the otherwise-archaic term "spectacle(s)" is often used rather than "eyeglasses".

## 3 THE REFRACTOR

A refractor[2] is the scary mask-like instrument that is placed in front of the subject's face while the refractionist turns various dials while asking. "Which is better, one [click] or two?". It is essentially an eyeglass simulator. It places lenses of different optical parameters in the path of the patent's vision. Simplistically, those parameters are varied by the refractionist, using the instrument's dials, until the subject experiences the best vision (in various contexts). The resulting "prescription", directly derived from the parameters of the lens in the final setting of the instrument, is a specification for the optical parameters of eyeglass lenses that should have the same effect on the subject's vision.

Figure 1 shows a typical refractor, made by Reichert Technologies, successor to the ophthalmic instrument business of American Optical Company (AO), once the most respected manufacturer of such in the U.S. It in fact closely follows an earlier design by AO.

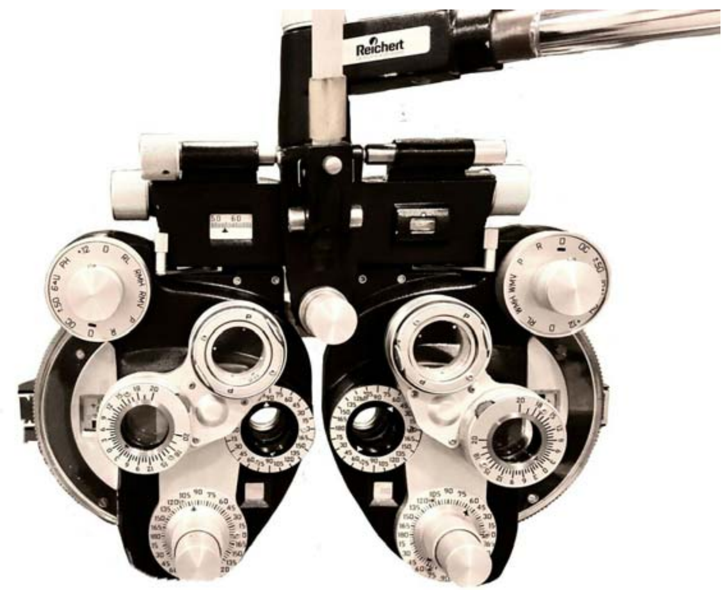


**Figure 1. Typical refractor**

## 4 THE TRIAL LENS SYSTEM

### 4.1 Description

The trial lens system also simulates the effect of potential eyeglasses in a more direct way. It uses a special (maybe a little scary) eyeglass

[2] Often called a "phoropter", although "Phoroptor" (note spelling difference) is a trademark of one manufacturer for their instruments.

frame (the *trial frame*) into which interchangeable, calibrated lenses can be readily placed (solo or in combination) until the best vision is obtained. The prescription reflects that final "setup", and thereby specifies the parameters of eyeglass lenses that should parallel the optical behavior of the final setup in the trial frame.

To help put this in context, in figure 2 we see a typical "contemporary" trial frame (here with no lenses yet in place). Its features and operation will be described in detail later.


**Figure 2. Typical contemporary trial frame**

"Contemporary" means that frames of this design are currently offered by many suppliers. But they are essentially "clones" of a frame series introduced by American Optical in 1938.

The entire "kit" of a trial frame and a large arsenal of different trial lenses (perhaps as many as over 250 of them) is often kept in a sloping-top cabinet with slots for each lens, usually with a tambour cover that can be closed to keep out the dust, mounted atop a wooden cabinet. This is often called a *trial case*, which name is also used for the set of lenses itself (and even for the refraction technique itself). Figure 3 shows a typical modern one.

**Figure 3. Cabinet-top trial lens case**

But there are also “portable cases”, about the size and shape of a briefcase, that can be used when the refractionist is visiting patients at a temporary clinic, a nursing care facility, or even their home. Figure 4 shows a typical modern portable trial lens case, with 228 lenses, sold by Reichert Technologies:

**Figure 4. Reichert 228-lens trial case**

We have a very similar (but smaller) set in our personal collection.

### 4.2 Pros and cons *vs*. use of a refractor

The refractor is, generally speaking, a more convenient tool for conducting refraction than a trial lens case.. It is always a little tricky to slip the various lenses into and out of their "slots" in the trial frame while the frame is in place on the subject's head.

On the other hand, often young children and some adults are intimidated by the refractor, and children in particular may enjoy the game with a trial frame as the refractionist works like a close-up magician in manipulating the trial lens system ("Is that a lens in your ear?").

From an objective standpoint, many experts feel that the indications given by skilled use of the trial lens system are more insightful than those gained with the refractor. Some clinics insist that, after the subject has been examined with a refractor, and a prescription drafted, the refractionist emulate that prescription with trial lenses and have the patent actually walk around, look at familiar documents, crochet, maybe swing a putter, and so forth, before confirming and issuing the prescription.

There are then logistic and economic considerations. A nice "portable" trial case is about the size of a large briefcase, and (as suggested earlier) can easily be taken to "storefront clinics", clinics in remote areas, nursing care homes, and the like. That is hardly practical for a refractor. A nice "professional grade" trial case with perhaps 266 lenses can be purchased today for less than $1000.00, while a first rate refractor with its necessary surrounds (stand, etc) can be many times as costly.

### 4.3 Accommodation

The term *accommodation* is used in the field of vision science to refer to the eye's ability to focus on objects at different distances. Ophthalmic lenses, as found in eyeglasses and contact lenses, are in part intended to overcome deficiencies in the eye that prevent the person from fully utilizing that capability.

In a "completely normal" eye, when the accommodation mechanism is relaxed, focus is (ideally) at infinity, or at least, at a great distance. As accommodation is exerted, the focus distance moves closer to the eye. The nearest focus distance usually increases with age. For a 30-year old, a near focus distance of 11 cm from the front of the eye is often considered "normal"

## 5 REFRACTIVE ERRORS OF VISION

The classical vision "defects" (often described as "refractive errors") are:

Hyperopia[3] ("far-sightedness") is the deficiency in which the total range of accommodation is "offset outward", such that distant objects (even at "infinity") can be focused, but the near limit is not nearly as close as is normal. From a theoretical standpoint, the far limit is "beyond infinity", although since there are no objects there that is not of any value to the person. But to the person, the significant effect is that near objects cannot be seen clearly.

Simply, we correct hyperopia with a converging (plus power) corrective lens.

Myopia ("near-sightedness") is the deficiency in which the total range of accommodation is "offset inward", such that close objects can be focused on but the far limit is short of infinity. To the person, the effect is that distant objects cannot be seen clearly.

Simply, we correct myopia with a diverging (minus power) corrective lens.

Note that in both these it is assumed that the person still has the normal "span" of accommodation; it has just been shifted from the desirable place (so one "end" is forfeit).

The basic cause of these two defects is that the focal length of the eye's lens system (which comprises two lens elements, the cornea and the "crystalline lens") is not appropriate for the distance from the lens system to the retina. The cornea is most often the principal villain in this.

Presbyopia ("old person's vision") is the deficiency in which the eye is able to make less than "normal" (perhaps no) change in the distance at which it focused".

It may be combined with *hyperopia*, which case the far limit of the range of vision is "beyond infinity", and the near limit may still be a large distance. Or it may be combined with *myopia*, in which case the far limit may be at a modest distance, and the near limit not much closer.

In "full blown" presbyopia, the eye cannot change its vision distance at all, so the near and far limits become the same (and what distance that is can be considered as a manifestation of myopia or hyperopia).

The basic cause of presbyopia is decline in the effectivity of the eye's mechanism for changing the focal length of the crystalline lens.

[3] Usually called in formal ophthalmological writing "hypermetropia".

We most commonly correct for presbyopia with bifocal lenses, which have one power to be used for vision at a distance and another (more plus) power (in a small "near vision" segment near the bottom of the lens) to be used for near vision.

Astigmatism is a refractive defect that is not a flaw in accommodation. It is basically caused by some part of the eye's lens system (most often the cornea) not having the same power in all directions (from its not having rotational symmetry).

In astigmatism, the eye cannot, in the same "state of accommodation", focus on a line at a certain distance running in one direction and a line at the same distance running in a different direction.

We correct astigmatism with a cylinder lens (or, more often, by having the effect of a cylinder lens combined with the effect of a sphere lens in the corrective lens). The axis of the cylinder lens must be appropriate to the orientation of the astigmatism being corrected.

# 6 LENSES

## 6.1 Introduction

This whole activity is about lenses—the lenses that will be used to correct a patent's vision, and the lenses used in the trial frame. Let us first review some pertinent things about lenses.

In figure 5, we will see some important properties of a biconvex, convergent lens.

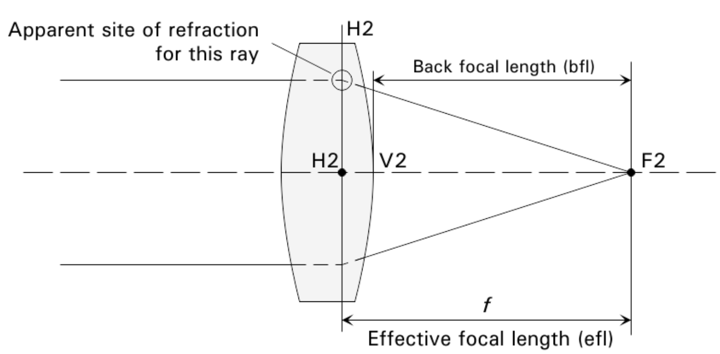


**Figure 5. Focal lengths of a lens**

We see the lens in an important situation. We consider rays of light, perhaps coming from the same point on an object at an infinite distance, that arrive parallel to each other and to the lens axis. In that situation, ideally, the lens will converge all such rays at a point on the axis called the *second* (or rear) *focal point* (F2).

We realize that in reality, the refraction of these rays (that is, the deflection in their direction of travel) occurs partly as they cross the front surface of the lens and further when they cross the rear surface.

But if we consider the action from the standpoint of the lens as a "black box", then (in this case, with the arriving rays parallel), it appears that the rays are deflected just once, at a plane we call the *second principal surface*, labeled H2.[4] (The point on that surface at the optical axis of the lens is also labeled H2.)

It turns out that perhaps the most important property of the lens is its *focal length* (called, formally, its *effective focal length*, but don't let the "effective" throw you off—it is the "real" focal length). The common symbol for (effective) focal length is **f**. The SI unit is the meter (m).

For a reason we will see shortly, a distance that is critical in vision correction, in the scheme that has been adopted for that art, is the distance to the second focal point from the rear surface of the lens—to be precise, from the rear surface at the axis, a point called the *second* (or rear) *vertex* of the lens, V2. This distance is called the *back focal length*, bfl, (or *back focal distance*). The back focal length may be positive (for a converging lens) or minus (for a diverging lens); the power carries the corresponding sign.

The *refractive power* of a lens (often, just *power*) is the reciprocal of the focal length (that is, the *effective focal length*). There is no official symbol for this quantity.[5] For ease of recognition by the reader, I will use **P** here for the refractive power of a lens (and **S** for the related property *surface power*).

The modern scientific unit of refractive power is the inverse meter ($m^{-1}$), but the traditional unit (always used today in vision correction work) is the *diopter*, which has the same definition. A lens with a focal length of 1.00 m has a power of 1.00 diopter (1.00 D). A lens with a focal length of 4.00 m has a power of 0.25 D.

---

[4] Why "surface", not "plane"? Because in reality, this is usually a curved surface, not a plane. However, our theoretical examination of the behavior of rays is always conducted in an infinitesimal region near the axis (even though we don't draw it that way), and in that context it doesn't matter whether the surface is a plane of not. So we label it a surface (for rigor), and draw it as a plane (for convenience).

[5] In scientific writing the symbol $\phi$ is often used for refractive power, and, perhaps as a result, in technical ophthalmological papers, **F** is often used. I eschew that here owing to the possibility of confusion with **f**, focal length.

### 6.2 Two classes of lens

In the correction of vision, we will be concerned with the properties of two classes of lens.

<u>Spherical lenses</u>

The lens that is familiar in such areas as photography or astronomy is rotationally-symmetrical. That means that its cross section is the same shape in any plane containing the axis. A lens whose surfaces are portions of a sphere is the traditional and most familiar type of rotationally-symmetrical lens, and in the field of corrective lenses, rotationally-symmetrical lenses (whether their surfaces are actually portions of spheres or not) are referred to as *spherical lenses*.

The important performance consideration is that the refractive effect of such lenses is the same along any "meridian"—a transverse line passing through the axis at a specific orientation—whether the "6 o'clock to 12 o'clock" meridian or the "2 o'clock to 8 o'clock" meridian.

The power of these lenses may be of either the positive or negative sign, but it is most common in this field to speak of those as the *plus* and *minus* signs, and I will use that terminology here. It is the custom in this field to refer to spherical lenses for short as "sphere lenses" (or just "spheres"), and I will generally do that from here on.

<u>Cylindrical lenses</u>

The surfaces of a *cylinder lens* are portions of a cylinder (not necessarily circular cylinders, but typically such).

The important performance consideration is that their refractive effect is different in different directions (along different *meridians*). It is the greatest along a meridian that is a right angles to the cylinder axis (sometimes called the *power meridian*), zero along a meridian parallel to the cylinder axis (the *axis meridian*), and has intermediate values at meridians in between.

The refractive power of a cylinder lens is stated as along the power meridian and is denominated in diopters.

It is the custom in this field to refer to cylindrical lenses for short as "cylinder lenses" (or just "cylinders"), and I will generally do that from here on.

### 6.3 The use of vertex power

The effect of a lens on vision correction depends on the *power* of the lens (and here I mean in the conventional optical theory sense—the inverse of the *effective focal length*) and the distance from the first principal point of the eye's lens system to the second principal point of the corrective lens.

But using this value of power in this field is problematic, largely because the effective focal length is reckoned with respect to the second principal surface of the lens, whose location with respect to the physical lens may vary greatly. It may, like the center of gravity of a donut, not even be within the lens itself.

We can somewhat ameliorate this difficulty by using instead the back vertex power of the lens (the reciprocal of its back focal length, which is by definition reckoned with respect to the back vertex of the lens, the center of its rearmost surface).

Thus, in vision correction work, we (whenever possible) place the back vertex of the lens at a fixed distance from the eye (and if that is not practical in a specific case, we describe the alternate distance in those terms). Then, we specify lenses in terms of their back vertex power. And it all works out.

### 6.4 Astigmatic lenses

An astigmatic lens does not exhibit the same refractive power in all directions,. A cylindrical lens is the extreme case. It exhibits a certain refractive power in one direction, and none in the direction at right angles to that.

But we can have a lens in which the power is diffident along two directions at right angle, but not to the extent of a cylinder lens. In fact, the cornea of the eye may well be astigmatic, leading to the vision defect of astigmatism.

Appendix C discusses in some detail what an astigmatic lens does to the cone of light arriving from a point on some object.

## 7 BIFOCAL LENSES

For people with significant presbyopia, bifocal corrective lenses are often used. The "main lens" is made to provide the optimal correction for distant vision. In a small region at the bottom of the lens (the "near vision segment") the power is different so as to provide the optimal correction for near vision (at some specific distance, perhaps a distance suggested by the subject as relating to his most critical "near vision" usage).

## 8 THE PRESCRIPTION

The prescription that defines the parameters of the need corrective lens is typically written for each eye in a form such as this (there are several variants):

OD +3.50 +0.75 X 30 add 1.75

Add: 1.75

The meanings of the elements in this example are:

**OD** (*oculus dexter*): This part of the prescription is for the right eye. (For the left eye, the identifier would be OS, *oculus sinister*. If part of the prescription is to apply to both eyes, the identifier would be OU, *oculus uterque*).

+**3.50**: The lens overall has a sphere power component of +3.50 Diopters (+3.50 D).

+**0.75 X 30**: The lens overall has a cylinder power component of +0.75 D with its axis at 30°.

Note that the angle of the axis is reckoned as would be seen looking at the eyeglasses from the refractionist’s perspective, not that of the subject.

Because we are speaking of the orientation of a line (the axis of the lens), and not of the direction in which an arrow points, the range of possible values only spans 180°. In normal scientific terms, this range would usually be considered to run from zero (which, as in usual scientific or mathematical work, would be to the right) through just less than 180°

But because of the aversion, especially in past times, except in mathematical work, to the concept of “zero”, the orientation that we think should be designated “zero” is instead designated “180°” (which, given the cyclic nature of this scale, is actually quite apt).

And so an axis orientation of “zero” (a truly horizontal axis) is written in prescriptions as “X 180”

**add 1.75**: In the near-vision segment of the bifocal lens, the sphere component of the power is 1.75 D more plus than specified for the “base lens” (thus +4.25 D in this example). The sign of the add value is assumed to be plus, and often the sign is not marked (as in this example). The cylinder component of the power in the segment is taken to be the same as specified for the lens overall.

Most often the “add” is the same for both eyes, and then may be written only once on the prescription, possibly marked “OU”.

Sometimes (especially in technical articles) we will see this form:

OD +3.50 DS +0.75 DC X 30 add 1.75

where "DS" means "diopters, sphere" and "DC" means "diopters, cylinder".

## 9 THE TRIAL LENS "TOOLBOX"


**Figure 6. Typical modern trial frame**

### 9.1 The trial frame

Figure 6 shows again a typical modern trial frame, from our personal collection (seen earlier, here in a different view). This one was made in Japan (year not known), and closely follows the overall design of a series of American Optical Company trial frames designed in about 1938. It was acquired in February of 2022.

This frame, like its AO archetype, is made almost entirely of metal. Frames of a design almost identical to this. but with all major parts made of plastic, have been widely made and distributed.

The positions of the two lens-holding assemblies can be adjusted laterally by two small micrometer knobs in order to place the center of the respective lenses laterally in line with the patent's pupils. The positions can be read on scales. The sum of the two readings will be the *interpupillary distance* (more often, just "pupil distance") (PD).

Earlier AO models (*e.g.*, before 1938) used a single knob, with a double lead screw, to move both side "carriages" together (in opposite directions). But although this sounds very convenient, in reality we sometimes need to have the two lens-holding rings at different distances from the nosepiece (due to facial asymmetry), and so having separate controls for the two sides turns out to be valuable.

The nosepiece assembly, which actually provides for the location of the frame on the subject, can be adjusted in two ways. It can be moved fore-and-aft, so as to adjust the distance of the trial lenses from the eye to meet the standard distance or the special distance chosen for the subject. It can be adjusted up-and-down, so as to place the centers of the lenses vertically in line with the subject's pupils.

The temple pieces have adjusting knobs which adjust their vertical angle with respect to the frame proper, in turn adjusting the angular "tilt" of the lenses with respect to the line from the eye to the center of the aperture (the *pantoscopic angle*). The lengths of the temple pieces can also be adjusted to suit the location of the subject's ears.

On the "back side" of the frame, for each side, there are two features with notches into which a trial lens can be placed. A metal spring clip completes the location of the lens at a third point and holds it in pace.

On the "front side" of the frame, for the assembly on each side, there is a rotatable ring with two posts each having three notches to receive up to three trial lenses, plus a set of three metal spring clips to complete the location of the lenses and hold them in place. The front notches are spaced, axially, about 3.5 mm center-to-center. The rearmost of the three notches is about 7.5 mm center-to-center in front of the notches on the rear of the frame.

Each ring can be separately rotated with a small knurled knob. There is a friction brake on each control knob to prevent inadvertent shift in the ring orientation, and a locking screw to fix that even more securely.

Although all the trial lenses in the ring rotate together, this is only of importance to a cylinder lens, if present (and it is of no consequence for sphere lenses that might also be inhabiting these rings). A circumferential scale allows the orientation of the axis of a cylinder lens (indicated by a "tick mark" on the lens) to be ascertained. There is a curiosity in the origins of the scales that is discussed in Appendix B.

Note that, because the lenses are not "keyed" to the lens holder ring, even though the orientation of the ring may be "locked" with the locking screw, that is no guarantee that the orientation of the cylinder lens will not inadvertently be shifted by contact with its tab.

### 9.2 The trial lenses

The trial lens set typically includes as many as 266 lenses. They typically include:

- Sphere lenses, with a wide range of powers, both plus and minus, generally 0.12 D, 0.25 D, and then in steps of 0.25 D to some point, then by increasingly larger steps, perhaps to a maximum of 20.00 D. Usually two of each power are included, so that the same kind can be placed on both sides if needed.

- Cylinder lenses, again in a variety of powers, either plus and minus or perhaps both, usually in steps as described above, perhaps to a maximum of 6.00 D. Again, two of each power are included.
- Prism lenses, used to correct a defect in the ability of the subject to readily aim both eyes at the same point. (That aspect of vision correction is not discussed in this article.)
- Special lenses used for various special tests, "opaque" disks used to block one eye while the other is being measured, and so forth.

"Traditional design" trial lenses have thin metal rims (typically 1.5-1.7 mm thick), with a rounded edge )although the thickness of the glass just inside the rim may be 2.0 mm or so), and a nominal outside diameter of 38 mm (1.5").The clear diameter of the lens is about 36 mm.

In many sets of this style, all sphere lenses are symmetrical, having the same curvature (convex or concave) on both sides. In other sets (such as the one now in use here), sphere lenses of up to a certain substantial power, and many if not all cylinder lenses, have one flat ("plano") surface. Sphere lenses of greater powers (plus or minus) are generally symmetrical, with the same curve on both surfaces.

Here we see three lenses of this style from our personal collection (not from our current "set").

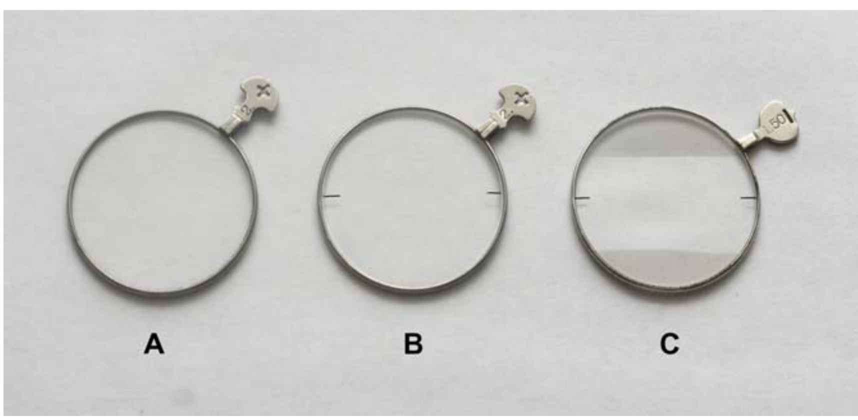


**Figure 7. Trial lenses—traditional**

Lens A is a sphere lens. Lenses B and C are cylinder lenses.

The cylinder lenses have a pair of tick marks on the glass near the rim. These indicate the orientation of the cylinder axis. They are often hard to see (I have artificially enhanced them in the illustration for clarity). They are not generally aligned with the location of the tab.

In some cases, the cylinder lenses also have “frosted” portions outside a central roughly-rectangular clear portion (see lens C). The cylinder axis of the lens runs along the center of the clear portion. This “masking” has some optical advantage in the operation of the measurement process. An additional purpose of the frosting is to allow the refractionist to quickly grasp the general orientation of the cylinder lens without having to refer to the tick marks, as well as to readily distinguish the cylinder lenses from the sphere lenses. But not all cylinder lenses have this feature.

The metal tabs serve to hold the lenses for manipulation, and also carry a designation of the lens power.

In some cases, the difference between plus and minus power is conveyed by the color of the tab, in some case by the shape of the tab, in some cases with a plus or minus sign pierced through the tab, and in some cases with two or more of those. (Those shown are from a set in which the sign was conveyed both by a pierced mark and by the shape of the tab.)

In some sets, the class of lens (sphere *vs*. cylinder) is indicated by the shape or color of the tab (a very subtle color difference being commonly used—maybe a feeble gold wash on the tabs of the cylinder lenses).

For non-symmetrical lenses (with one plano surface) there is no mark as to which is the plano side, nor is there any generally-given recommendation as to in which orientation to use when placing the lenses during refraction. However, that matter can make a subtle difference to various issues we will discuss later.

Lenses of this style are offered by almost all trial lens suppliers.

For a reason we will see shortly, lenses of this type are sometimes called “full-diameter” lenses.

In figure 8, we see the trial frame earlier seen, now with three lenses of this style in the “stack” on each side.

The lenses in position 1 (on the back of the frame) are the sphere lenses for basic vision correction (we see the frame resting on their tabs). On the front, in position 2, are cylinder lenses for the correction of astigmatism (they have the red tabs). In position 3 (frontmost) are plus sphere lenses used in the final stage of the process for determining the increment of power in the “near vision segment” of the final eyeglass lens (called the “add” in the prescription).


**Figure 8. Trial frame with lenses in positions 1, 2, and 3**

## 9.3 A matter of "art"

It does not matter how the sphere lenses are oriented, so (from an optical standpoint) we can arrange for their tabs to be located anywhere we wish, When a cylinder lens is to have a certain axis orientation, since it is symmetrical, we can have the tab at either of two positions (180° degrees apart), but only at those positions.

So part of the art of the refractionist is to prevent any of the tabs from colliding with the subject's nose, while also (for those in positions 2 and onward) preventing the tab shanks from interfering with any of the supporting posts and clips on the ring. (Recall that the ring may have any arbitrary orientation.)

## 9.4 Reduced-diameter lenses

For various reasons, there is an advantage to having the clear diameter of the lenses smaller than the 36 mm of the "traditional" type of lens (perhaps 25 mm or even as little as 18 mm).

This is generally implemented with lens "mounts" of anodized aluminum (or, for lower cost lenses, molded plastic).


**Figure 9. Reduced diameter trial lenses**

Figure 9 shows two trial lenses of this type (from Marco Ophthalmic, Inc.) from our personal collection. They are of very nice construction, with anodized aluminum mounts.

In these, the mount has a nominal outside diameter of 38 mm and an edge thickness of 1.7 mm (so as to fit nicely in trial frames designed for “traditional” trial lenses). These two have a clear lens diameter of nominally 25 mm. In the full Marco repertoire, above a power of 8.00 D the clear diameter is 18 mm.

The left lens is a sphere, the right one a cylinder (see discussion below on identification).

Typically, in such sets, the sphere lenses are of the meniscus form (curved overall, convex on the front surface and concave on the back surface). This can be advantageous from the standpoint of consistency of vision through different parts of the lens. The reduced diameter configuration aids in the design of such lenses; in a full diameter context, their “bulge” might be so much that adjacent lenses in a trial frame would interfere with each other.

Often the lens sets of this style are said (or at least suggested) to be *additive* (a topic that is discussed at length in section 13).

Normally the sign of the power is included in the marking, and as well, the mounts are usually black for plus power and red for minus (as in fact is prescribed by the international standard for the properties of trial lenses).

The distinction between sphere and cylinder lenses is often only shown by virtue of the fairly prominent tick marks on the mount showing the axis of cylinder lenses.

In the Marco lenses, there is also the distinction that for cylinder lenses, the power is only marked once on the mount (generally aligned with the "power" meridian direction of the lens), while for sphere lenses, it is marked twice, the two marks 90° apart (perhaps to be evocative of the fact that for a cylinder lens, the refractive power is exerted in more than one direction, all directions, actually).


**Figure 10. Trial frame with reduced-diameter lenses**

In figure 10, we see the trial frame earlier seen, now with two lenses of this style in the "stack" on each side (one on the back of the frame, and one in the rotatable rings on the front).

Note that, for this particular set of lenses, the lenses are intended to be mounted with the marked side away from the subject. When sphere lenses are put in position 1, at the "back" of the frame (as for the basic sphere lens used to correct distant vision), this results in the markings being against the frame where they are hard to see.

Since the cylinder lenses are typically put on the front of the frame (in position 2), that works out nicely for them—until we get to the stage of determining the "add" for near vision, when another sphere lens is put in front of the cylinder lens.

So when everything is working ideally, and we want to write down what lenses we have in the frame (perhaps directly on a prescription blank), we cannot easily see the designations (nor, in fact, can we then see the tick marks on the cylinder lens so as to see its axis angle).

### 9.5 The corneal vertex distance

A lens of a certain back vertex power will only have a consistent effect on vision if the lens is located at a predictable distance from the eye. So we seek to, whenever possible, have the eyeglasses designed to have a standard distance (often considered to be 12 or 14 mm) from the rear vertex of the lens to the front of the eye (the corneal vertex).

And, since the trial lens setup is intended to be a model of the eyeglass lens to be prescribed, we seek to, whenever possible, have that distance from the rear vertex of the lens stack to the corneal vertex.

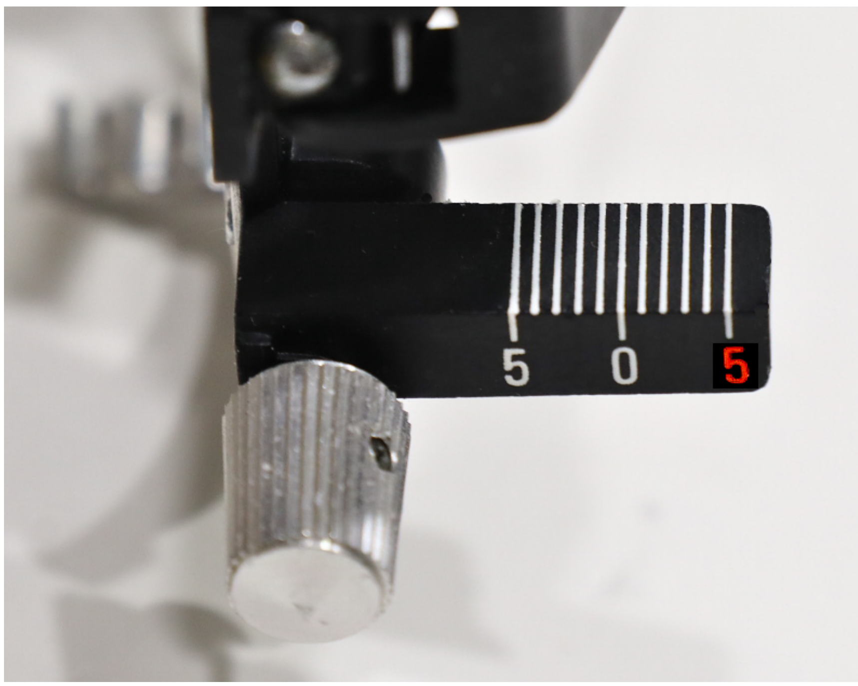


**Figure 11. Typical corneal distance sight**

In fact, many trial frames have a gunsight-like arrangement that allows the refractionist to ascertain that distance, and to adjust it by adjusting the axial location of the trial frame (perhaps by adjusting the axial position of the nosepiece). We see that “sight”, on the frame shown earlier, in figure 11.

In this frame, if the corneal vertex is seen to be at the position of the “0” mark on the scale, the distance from the corneal vertex to the center of the thickness of the mount of a lens in the “back of the frame” position is about 14 mm.

Note that this is not the distance to the corneal vertex from the back vertex of the lens, which would depend on where that back vertex was located with respect to the center of the lens mount. But for typical trial lenses it would be close.

## 10 TRIAL LENS EXAMINATION

There are various procedures recommended for the conduct of a refraction using a trial lens system. They all have various subtle advantages.

Here, I will discuss the setup for one procedure, one that perhaps best illustrates the technical principles involved.

In this procedure, the sphere trial lens that represents the basic power of the final lens overall is placed in the rearmost position (or "cell"), "position 1"—closest to the eye. If a cylinder component is involved, the trial cylinder lens is placed in "position 2"—farther from the eye.

Without describing the actual step-by-step procedure (which is very "crafty"), the objective is to end up with a combination of the power of the sphere lens (in position 1) and the power and axis orientation of the cylinder lens (in position 2) that produce the best distant vision with the subject observing a "distant" target.

Then, to determine the optimum lens properties for near vision (as will be implemented in the near vision segment of the final bifocal lens), we place an additional plus sphere lens in a further position in the frame ("position 3"). One advantage of this, rather than replacing the main sphere lens with ones of successively more-plus power, is that the result comes out directly in "add" notation.

Imagine that we end up with this "stack" in the frame for the right eye when best distant vision has been attained:

Cell 1 (nearest face): Sphere, +3.50 D

Cell 2: Cylinder, +0.75 D, axis 30°

After we have added the third lens, and best near vision has been attained, we have this stack:

Cell 1 (nearest face): Sphere, +3.50 D

Cell 2: Cylinder, +0.75 D, axis 30°

Cell 3 (outermost): Sphere, +1.75 D

Then we write this as the prescription for the actual lens to be made:

OD +3.50 +0.75 X 30 add 1.75

## 11 THE JACKSON CROSS CYLINDER

As discussed above, when determining the optimum parameters for the cylinder component of the lens (for correction of astigmatism), the refractionist puts in place cylinder lenses of various powers, and adjusts the lens axis to various angles, until the best near vision is attained. The subject's identification of when the vision is best may be a bit "vague".

There is a clever technique, using a tool called the *Jackson cross cylinder*, that can confirm that the power, and axis, of the trial cylinder lens are indeed optimum, or if not, will help us to refine those values.

Appendix A describes this in detail.

## 12 THE DUALITY OF THE CYLINDRICAL LENS

The action of the cylindrical lens, when present, is to increase, or maybe decrease, the power of the entire lens system along a certain meridian.

We do this because the power of the eye's own lens system, due to astigmatism, is not uniform for all meridians. So a broader view of the cylinder lens is that it overcomes this lack of symmetry in the eye's lens system.

Now in the actual corrective lens (and thus in the trial lens stack that emulates it), if we needed to have a power of power of +4.00 D "vertically", and +3.00 D "horizontally", we could think of, conceptually, doing it in at least two ways:

- Have a "base" power of our hypothetical lens system of +3.00 D in all directions (created by a sphere lens component, with power +3.00 D) plus a power of 1.00 D in the vertical direction (created by a cylinder lens component, with power +1.00 D, and a horizontal axis, thus having its power in the vertical direction), or

- Have a "base" power of +4.00 D in all directions (created by a sphere lens component, with power +4.00 D) less a power of 1.00 D in the horizontal direction (created by a cylinder lens component, with power -1.00 D, and a vertical axis, thus having its power in the horizontal direction)

Note that these two views (predicated on different hypothetical lens assemblies) would lead to different lens prescriptions:

- +3.00 +1.00 X 180

- +4.00 -1.00 X 90

Yet both of thee would describe the same lens (by describing the same behavior of the lens). The former description is said to use "plus cylinder" notation, and the latter one, "minus cylinder" notation.

This duality of the cylinder lens has been both a blessing and a curse in optometry.

A detailed demonstration and discussion of this, plus a discussion of its principal effect on actual optometric practice, is given in Appendix D.

# 13 THE "ADDITIVITY" ISSUE

## 13.1 The ideal

Suppose that for the eye in question best vision is attained with this "stack" in the trial frame::

- Sphere lens, power +2.00 D (in position 1 in the test frame, on its back, nearest the eye)
- Cylinder lens, power +1.00 D, axis 180 (in position 2 in the test frame, on its front)

So, for that eye, we write the prescription as:

+2.00 +1.00 X 180

This implies a corrective lens with the following refractive properties:

- In the vertical direction, +3.00 D (the refractive effect of the cylinder component adding to that of the sphere component in this direction).
- In the horizontal direction, +2.00 D (the cylinder component does not add to the effect of the sphere component in this direction).

## 13.2 The fly in the ointment

But in fact that prescription does not (quite) describe the refractive properties of the trial frame "stack". In the vertical direction, the stack has a refractive power greater than +3.00 D.

It would if the two lenses involved were the hypothetical "thin" lenses so beloved in optical theory lectures, and if they were placed in intimate contact. Then the power of the cylinder lens (in the direction of its power meridian) would add to the power of the sphere lens to give the power of the stack in the power meridian direction of the cylinder lens.

But these are real, "thick" lenses, and mounted so that they are about 7.5 mm apart, center-to-center. as a result, the cylinder lens and its distance from the sphere lens constitute a sort-of telescope, which "amplifies" the power of the cylinder lens. And so, considering the vertical direction in this case, the vertex refractive power of the combination (in the direction of the power meridian of the cylinder lens) differs from the sum of the vertex powers of the two lenses.

This discrepancy would make the actual corrective lens, made to exhibit the refractive behavior described by the prescription, not give the "best vision" attained in the trial frame refraction.

### 13.3 How much discrepancy?

Broadly, the magnitude of this discrepancy increases with the powers of the two lenses involved. A digital simulation of four combinations of a sphere lens and a cylinder lens from a typical set, based on physical parameters measured here, show the errors indicated in this table (all values in diopters):

| Row | Cylinder power | Sphere power | Total of powers | Joint power | Error |
|---|---|---|---|---|---|
| 1 | +2.00 | +4.00 | +6.00 | +6.093 | +0.093 |
| 2 | +3.00 | +4.00 | +7.00 | +7.153 | +0.153 |
| 3 | +2.00 | +8.00 | +10.00 | +10.097 | +0.097 |
| 4 | +3.00 | +8.00 | +11.00 | +11.158 | +0.158 |

The columns ""Cylinder power" and Sphere power" are the vertex power of the respective lens, as marked on the lens. [6]

The column "Total of powers" gives the totals of the powers of the two lenses in the train.

The column "Joint power" gives the actual back vertex power of the stack, calculated from the physical properties of the lenses, with the spacing between them they would have if placed in the trial frame seen earlier.

The column marked "Error" shows the error due to "non-additivity". The sign convention here for the errors is that a plus sign means that the joint power is greater than the sum of the powers of the two lenses.

An error of ±0.25 D is usually considered "just bothersome" in this field. We note that, for the four lens combinations shown, the "non-additivity" error itself never reaches that level.

Nevertheless, this error situation deserved attention.

---

[6] There is actually a complication here. These lenses are not marked with their actual (calculated) vertex powers, but rather with the sum of their surface curvatures. But that is not part of the issue of interest here. Accordingly, I have transformed the data to what it would be if the actual powers were the "handy" values seen in the table and were the marked powers as well.

An error of ±0.25 D is usually considered "just bothersome" in this field. We note that, for the four lens combinations shown, the "non-additivity" error itself never even approaches that level. Even when the errors due to inaccurate marking of the individual lens vertex powers are included, the error does not reach that level. For greater cylinder powers, the error would reach that level.

I note that in this model, for lenses that are plano on one side, that side was placed "toward the eye". That does not result in the smallest errors for the combinations of these lenses, but is typically the usage that is recommended.

### 13.4 "Effective power" of the cylinder lens

Given the situation described above, we can justifiably speak of, for the cylinder lens, **in a particular setting with a certain spherical lens**, its *effective power*. This is the power it contributes (in the direction of its power meridian) to the power of the stack of two lenses.

For example, in row 2 of the table above, the power of the sphere lens is +4.00 D, and by itself, that would be the power of "the whole stack". Se we can say that the *effective power* of the sphere lens is just the same as its actual vertex power.

But if we then place a cylinder lens with a power (in its power median direction) of +3.00 D in front of the sphere lens (at a certain spacing), the power of the whole stack is now +7.153 D. Thus we can think of the cylinder lens as having contributed +3.153 D to that total power. So we can say that the *effective power* of that cylinder lens, **in this setup**, (in the direction of its power meridian) is +3.153 D.

### 13.5 The Tillyer additive vertex power scheme

In about 1918, Edgar Tillyer, the "lens wizard" at American Optical Company, devised a clever scheme to avert this discrepancy, often called the "additive vertex power" system. It calls for us to follow certain requirements when designing the lenses of our sphere and cylinder trial lens sets:

Firstly, all the sphere lenses (of various powers):

- Have the same front surface curvature.

- Have the same center thickness.

  Given these two requirements, to set the power of the lens we must work only with the curvature of its rear surface, but that is doable.

- Are mounted so the front vertex is in a consistent axial location (n the "standard" trial frame).

Then, all cylinder lenses:

- Are mounted so the back vertex is in a consistent axial location (n the “standard” trial frame).

The result of the last two imperatives is that there will be a constant distance between the adjacent vertexes of the cylinder lens and sphere lens in any “setup”.

We mark all the sphere lenses with their actual (back) vertex power.

For the cylinder lenses, we mark each one with a value calculated by a complicated formula involving:

- The vertex power of the cylinder lens.
- The front curvature of the lenses in the sphere lens set (fixed).
- The axial center thickness of the lenses in the sphere lens set (fixed).
- The standard spacing between the adjacent vertexes of the front and rear lenses when in the trial frame (fixed).[7]

That value is sometimes called the “effective additive vertex power” of the lens.

Note that we are free to have each of the cylinder lenses have any shape we wish, as might be chosen to meet further design objectives. There is no requirement for consistency in the curvature of either surface or in the axial thickness.[8]

Having done all that, the (back) vertex power of the stack of any sphere lens and any cylinder lens, each in the proper position in the trial frame, will be the sum of the markings on the two lenses.

If we want “zero” power in the rear position (perhaps the subject needs no correction for hyperopia/myopia, but only for astigmatism)), we must nevertheless put in the rear position a “lens” that fulfills the norms regarding front surface curvature and center thickness but has zero back vertex power. Otherwise, the effect of the cylinder lens on the overall cylinder vertex power of the stack would not be that

[7] I say “standard” as if this distance were standardized, or even specified by an industry standard, but that is just wishful thinking.

[8] There is a very small implication of the thickness of the anterior lens with regard to near vision measurement, but this is typically negligible.

deduced from its marked power. "Additive" trial lens sets generally include one or two such "zero" lenses.[9]

Readers interested in the mathematics behind the Tillyer system might wish to read "Tillyer's Additive Trial Lenses System", by the same author, probably available where you got this.

### 13.6 A further benefit

A subtle advantage of trial lens sets made additive via the Tillyer plan, not often mentioned, is that by definition the distance from the rear vertex of the sphere lens to the subject's corneal vertex will be constant for all powers of the sphere lens. This obtains because of the requirement that all sphere lenses have the same center thickness and that all sphere lenses be mounted so that the front vertex is in a consistent position with respect to the lens mount and thus with respect to the trial frame.

## 14 CURRENTLY AVAILABLE "ADDITIVE" TRIAL LENS SETS

### 14.1 INTRODUCTION

Many modern trial lens sets of a certain style (different from the "traditional" style referred to above), amenable to being designed to follow the Tillyer additive plan, are described by the provider's literature in terms that suggest that they are in fact additive. Whether or not this nicety is actually fulfilled is another matter altogether. Here is what I know about three of those sets

### 14.2 The Marco "Custom" trial lens sets

Marco Ophthalmic offers trial lens with two styles of lens. The "Custom" sets have reduced diameter lenses, generally of meniscus design, in very nice aluminum mounts. The literature for the Custom sets seems to indicate that the lenses of those sets attain additivity. Here is one statement of that:

- Fully additive effective power.

which is a quite suitable definition of "additivity" as I have discussed it.

However, a review of lens properties in the company's data sheet, along with measurements taken here of a few samples of the Custom

---

[9] Often called *plano* lenses, since a lens with a "plano" (planar) surface on each side has zero power, and these lenses (even though there is nothing "plano" about their form) have that same power (zero). In fact, when the spherical lens required for the prescription has zero power, sometimes "plano" is written rather than "0.00D".

lenses, does not show any evidence that these lenses are intended to conform to the Tillyer plan for achieving additivity.

Thinking that there might be some scheme (of which I was unaware) other than Tillyer's for achieving additivity, I made calculations of the overall back vertex power of various hypothetical pairs of cylinder and sphere lenses from the Marco set (in what I considered to be a reasonable setting insofar as the spacing between the two lenses was concerned, namely that which would be produced in a representative trial frame). This did not show any evidence that additivity was achieved in this lens set.

Both according to the data sheet and the measurements taken here, both the sphere and cylinder lenses in this set are marked with their nominal power (meaning the sum of the surface powers) rather than with the actual vertex power (which of course is influenced both by the surface curvatures and the center thickness).

This table shows the error for all four possible combinations of the two sphere lenses and two cylinder lenses from this set that we have and thus for which we could verify and determine **all** the pertinent dimensions. The inter-lens spacing was based on the dimensions of the lenses and the inter-mount spacing on our reference trial frame. All values are in diopters

| Marked power | | Marking error | Additivity error | Total error |
|---|---|---|---|---|
| Cylinder | Sphere | | | |
| 2 | 4 | 0.208 | 0.084 | 0.291 |
| 4 | 4 | 0.143 | 0.234 | 0.377 |
| 2 | 8 | 0.275 | 0.110 | 0.385 |
| 4 | 8 | 0.211 | 0.275 | 0.486 |

All the dimensions given in the data sheet were found to be accurate for all these specimens.

"Marking error" is the error due to the actual lens power differing from the marked power. "Error 2" is the error due to non-additivity. "Additivity error" is the difference in the joint power of the two lenses against the total of their actual powers. The total error is typically on the order of 4.5% of the total marked powers of the two lenses involved,

It seems that the suggestion that these trial lens sets are "additive" is very questionable.

As to the fact that the powers marked on the lenses are not even their actual vertex powers, I personally found that disappointing. Yet it seems that this is a common practice for trial lenses.

### 14.3 The Topcon "Deluxe" trial lens sets

Topcon Healthcare (part of a large conglomerate once known as "Tokyo Optical Company") offers trial lens sets with two styles of lens. The literature for the "Deluxe" sets (the "reduced diameter" style seems to indicate that the lenses of those sets provide additivity.

Several statements in the instruction sheet for the latter series of trial lens sets, including a table giving what seems to be the actual vertex power *vs*. the "marked" power for the cylinder lenses in the series, suggests that these lenses are in fact intended to follow the Tillyer system of providing additivity. However, I do not have enough information on the detailed properties of the lenses in the series to confirm this possibility.

Among other things, that document seems to say:

- The sphere lenses are marked with their actual vertex power (this being a good idea generally as well as one of the conditions of the Tillyer additivity plan).

- The cylinder lenses are labeled by what is described as a "corrected" power, which well might be the effective vertex power of the lens as paired with a sphere lens of the set in a Topcon trial frame.

It may well be that these trial lens sets are deservedly described as "additive".

### 14.4 The Magnon TLX trial lens sets

These sets are made by H. Ogino & Co. Ltd. of Yokohama, Japan. They are reduced diameter lenses of meniscus design, in aluminum mounts similar to those of the Marco reduce diameter sets.

The catalog sheet for these sets uses language strongly suggesting that they are additive. But we have no further data to look into that. (I have requested data from the manufacturer.)

## 15 "CORRECTED CURVE"

We often see. especially for "reduced diameter" sets, that a trial lens set uses "corrected curve" design. This can mean one of two things (and in fact. possibly both):

a. The lenses are of the "meniscus" type ("corrected" meaning that the refractive effect is "more consistent" for off-axis lines of sight).

b. The lens design produces "additivity" ("corrected" meaning to avoid the error in the joint power of a two-lens train compared to the sum of their "marked" powers).

Often, when it seems that meaning (b) is intended, the description explains the meaning with terminology perhaps like this:

> The corrected curve additive design compensates for thickness and space between combined lenses, which allows the exact correction to be read directly from the total lens power in the trial frame.

Sounds like “additivity” to me. Maybe.

## 16 NEAR VISION EFFECTIVITY ERROR

### 16.1 Scope

This topic here is presented in the context of the “emulation” of an eyeglass lens with trial lenses, but it apples equally well to eyeglass lenses themselves.

### 16.2 Introduction

In the field of ophthalmics, the refractive power or lenses is stated in terms of their back vertex power. Recall that this is defined as the reciprocal of the back focal length (bfl), which is the distance from the back vertex of the lens to the point where the rays originating at a point on an object “at infinity” converge to a point image.

But if we analyze a lens system with an object at a “near” distance (as when the subject is regarding a near object through the lens), The behavior of the lens (in terms of the relationship between object and image distances is not (necessarily) that suggested by the “rated” back vertex power. This discrepancy is described as *near vision effective error* (NVEE).

### 16.3 About “vergence”

In the discussion to follow, I will use for the first time in this article the term *vergence*, a term not that often heard, so I thought I would take a moment to discuss it.

*Vergence* is a general term in optics that embraces both of the more-familiar terms *convergence* and *divergence*, and is used to quantify either in the analysis of a lens system. It is generally applied to the rays originating at a single object point as they are observed at some later place in a lens system. It can be thought of as describing the rate at which the rays converge or diverge at that place.

The sign of the vergence is plus for convergence and minus for divergence. If we consider convergence, the value of the vergence is the inverse of the distance from the point of reference to the subsequent place at which the rays from such a single point on an object will converge to an image point. If we consider divergence, the value of the vergence is the inverse of the distance from the point of reference to the point, if we were to project the diverging rays backward, at which the projected rays would converge to a point.

Vergence is denominated in the unit *diopter* (D), just as for the power of lenses or lens surfaces (and the two are closely related [10]).

### 16.4 Quantifying the near vision effectivity error

To quantify the near vision effectivity error (NVEE), we first consider the lens of interest, with an object at the “chosen” near distance being observed. We calculate where the image of a point on that object would be formed, and take the reciprocal of its distance from the back vertex of the lens as the vergence of the light rays (at that back vertex) in this first situation.

Then we consider a thin lens, that fictional creature so beloved to optics lecturers, with power the same as the rated rear vertex power of our actual lens, and located at the location of the rear vertex of that lens. We calculate where the image of the point on the imagined object would be formed, and take the reciprocal of its distance from the back vertex of the lens as the vergence of the light rays (at that back vertex) in this second situation.

The difference between these two vergence values is the *near vision effectivity error*. The sign convention is that the error is negative if the vergence with the lens of interest is less than the vergence with the hypothetical thin lens. We can think of this situation as the actual lens having a lesser effective power for the viewing of a near object than its “rated” back vertex power.

How this value is used in lens planning is beyond my ken.

## 17 ANOTHER WRINKLE FOR NEAR VISION MEASUREMENT

As I mentioned earlier, in the case of a bifocal lens, the prescription describes the power in the near vision segment in terms of an “add” (an increment to the power of the “main lens”).

When refracting with trial lenses, we could determine the optimal power in the near vision segment by changing the power of the sphere lens in position 1 of the trial frame until best near vision was obtained. We would then take the difference between that power and the sphere lens power earlier determine for distant vision correction, and report that difference as the “add”. And in fact, if we were doing the refraction with a refractor (phoropter), that is just what we would do.

---

[10] For a lens system regarding an object at infinity, the vergence of the rays as they leave the back vertex of the lens system is the same as the back vertex power of the lens system.

But when refracting with trial lenses for the near vision correction, the normal procedure is to leave the sphere lens (optimized for distant vision) in position 1 in place and put positive sphere lenses in the front of the stack (in position 2, position 3 if there is a cylinder lens in place) until best near vision is attained. The power of this added lens is then reported as the “add” value on the prescription. Very tidy.

Except that, owing to the same issue we heard of in connection with the “additivity” issue, the sphere power of the lens stack is not exactly the sum of the powers of the main sphere lens (in position 1) and the added sphere lens (in position 2 or 3). Thus the “add” value determined this way is not necessarily the needed increment of the corrective lens sphere power in the near vision segment.

Even if our trial lens set is “additive”, that feature (for several reasons) does not come into play to correct this error.

## 18 CONTACT LENSES

Many people needing corrective lenses opt for contact lenses rather than eyeglasses. The basic principles of measuring the needed correction, by way of the trial lens system, are the same as described here. But there are a number of special wrinkles, the most prominent of which is the need to convert the prescription as suggested by the trial lens refraction (probably predicated on a corneal vertex distance of 12 or 14 mm) to a vertex distance of zero (since the lens will lie against the cornea). This matter is beyond the scope of this article.

## 19 INDEX OF APPENDIXES

Appendix A—The Jackson cross cylinder

Appendix B—A curiosity in the axis scales on trial frames

Appendix C—Astigmatic lenses

Appendix D—Plus *vs*. minus cylinder practice

## 20 ACKNOWLEDGEMENTS

Special thanks to Wayne Starling of Marco Ophthalmic, Inc., a major supplier of ophthalmic and optometric instruments, for (in 2010, when this project began!) his patient assistance in helping me as I tried to understand how the additive vertex power system is implemented and utilized in modern trial lens sets.

Thanks to Steven Falco for calling to my attention several typographical errors in an earlier issue.

#

## Appendix A

### The Jackson cross cylinder

In the use of trial lenses, the optimal optical parameters are determined empirically. To ascertain an initial value for the sphere power, lenses of different sphere power are put in place with the until the subject reports that the most clear vision is obtained. The indication will typically be refined by "bracketing": if the subject reports that +1.50 is better than +1.75, and that +1.50 is better than +1.25, then +1.50 is chosen.[11] It is here that we first encounter that classical refractionist's patter, "Which is better, one or two".

A similar approach is used to get an initial choice for the cylinder power and cylinder axis for astigmatism correction. But in this regime, the perceptual impact of less-than-ideal correction is a bit more subtle than in basic sphere correction. And the trial frame-trial lens setup does not lend itself to simple back-and-forth change between two "bracketing" values.

To improve the "bracketing" process with regard to the cylinder correction, we use an ingenious system based on a tool called the Jackson cross cylinder (JCC) [12]. We see a typical one in figure 12.


**Figure 12. Jackson cross cylinder**

Its lens is a Stokes lens. It can be thought of as a composite lens comprising two cylinder lenses, with the same magnitude of power (typically in the range 0.25 to 0.50 D) but one plus and one minus, with their axes at right angles. A pair of dots or tick marks of one

---

[11] If two adjacent trial values are reported by the subject as giving the same result, the more plus is usually chosen, as this seems to be the best in term of practical eye accommodation (this is called the "push plus" approach).

[12] Less frequently, "crossed cylinder", better grammar actually.

color (usually white or black) shows the direction of the axis of the plus cylinder component and a pair of dots or tick marks of another color (usually red) shows the direction of the axis of the minus cylinder component. The power may be marked on one of the tick marks (as seen in figure 12) or it may be marked on the handle (as suggested in 13, below(.

Figure 13 shows the orientation of the axes of these two lens components with respect to the handle.

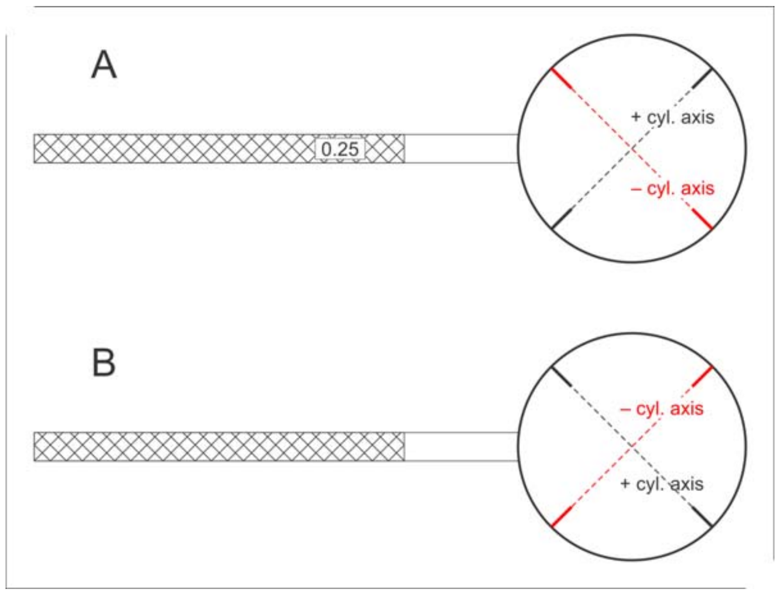


**Figure 13. Jackson cross cylinder— both orientations**

We see this with one face up in panel A of the figure. In panel B, we see it the other way up, as if we had just twirled the handle by 180° ("flipped" it). Note that this has in effect interchanged the axes of the plus and minus components.

In figure 14, we see the overall refractive effect of the JCC compound lens in a polar plot:

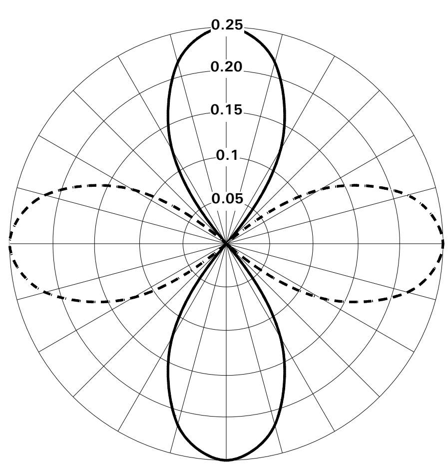


**Figure 14. Cross cylinder power polar plot— ±0.25 D**

It shows the refractive power of the lens as a function of the angular direction in which we observe that power. The dotted line represents minus values of the refractive power in the respective direction.

Now, let's see this wondrous gadget at work. Our first task will be to refine the interim choice of cylinder power. We have already put in place a cylinder lens with what we think is about the correct power, and have set its axis to what seems to be a good first position.

We then hold the JCC by its handle (in the orientation shown by panel A of figure 13) and place it in front of the trial lens "stack" for the eye being worked on, holding it so that the red tick marks (or dots) are aligned with the axis direction of the cylinder trial lens.

We will follow the optical action on figure 15.

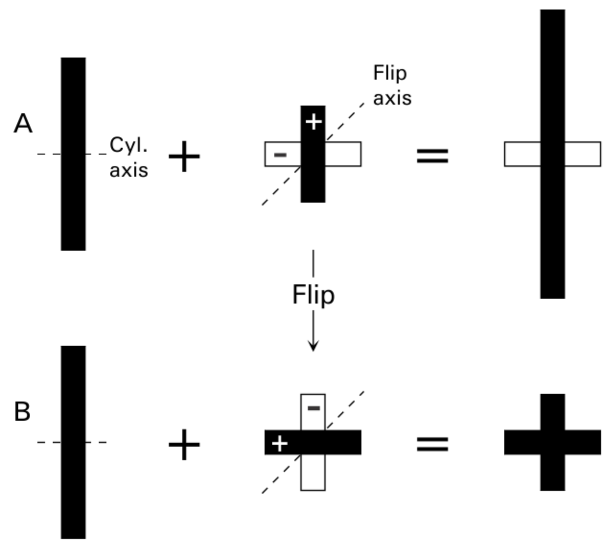


**Figure 15. Cross cylinder—refinement of cylinder power**

In panel A, we see the initial setting of the cross cylinder unit as described just above. On the left, we see a bar representing the refractive power of the basic cylinder lens that is in place (we assume its axis to be horizontal, thus its power meridian is vertical).

In the center we see the cross cylinder lens. Its magnitude, compared to that of the cylinder lens proper, is probably a bit unrealistic. I made it that way to most easily allow its effect to be seen. The black bar represents the power meridian of its plus component; the white bar shows the power meridian of its minus component.

On the right, we see the joint effect of the two. Note that the presence of the cross cylinder lens has:

- **Increased** the effective (plus) cylinder power over the power of the cylinder lens proper by the magnitude of the cross cylinder powers (for example, 0.25 D).

- Added a minus cylinder aspect at right angles to the plus power (essentially inconsequential to what the subject sees).

The refractionist says, "which is better, one . . ."

With that, the refractionist flips the cross cylinder lens by merely twirling its handle by 180° (the unit now has the "way up" as seen in panel B of figure 13). we move to panel B of the figure, and the refractionist says, ". . . or two?"

Now note that now the presence of the cross cylinder lens has now:

- **Decreased** the effective (plus) cylinder power from the power of the cylinder lens proper by the magnitude of the cross cylinder powers (for example, 0.25 D).
- Added a plus cylinder aspect at right angles to the plus power (inconsequential).

If the subject reports that there is no difference between the vision with these two different magnitudes of the net plus component , we can conclude that the current power of the cylinder lens proper (which is midway between those two powers) is the optimum cylinder lens power—situations "one" and "two" have "bracketed" the optimum value.

If the subject prefers "one" (the situation in panel A), then more cylinder power than that of the cylinder lens itself is optimum. The refractionist puts in a higher cylinder power and repeats the process until an optimum has been reached. The converse is true if the subject prefers "two" (the situation in panel B).

It can be shown that that neither situation "A" nor "B" represent any significant change in the overall sphere power of the optical path. Thus that aspect of the "correction" remains appropriate throughout this process.

Next, we will use the cross cylinder unit to refine our choice of cylinder axis. We rotate the JCC so that the handle axis is aligned with the axis direction of the cylinder trial lens. Again it will be the "way up" seen in panel A of figure 13.

We will follow the optical action on figure 16.

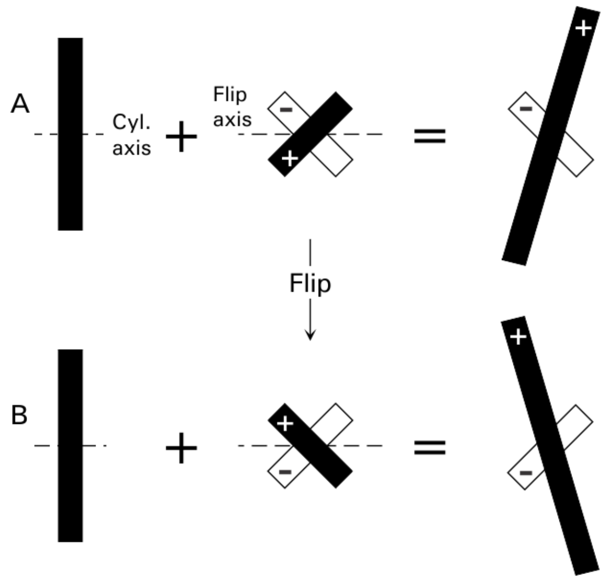


**Figure 16. Cross cylinder—refinement of cylinder axis**

Note that, initially (panel A) the presence of the cross cylinder lens has:

- Rotated the axis (and the power direction) of the cylinder correction slightly clockwise.
- Added a minus cylinder aspect at an angle to the plus power (essentially inconsequential).

The refractionist says, "which is better, one . . ."

With that, the refractionist flips the cross cylinder lens by twirling the handle by 180° (we move to panel B of the figure, and to the "way up" seen in panel B of figure 13), and says ". . . or two?"

Now the presence of the cross cylinder lens has:

- Rotated the axis (and the power direction) of the cylinder correction slightly counterclockwise.
- Added a minus cylinder aspect at angle to the plus power (essentially inconsequential).

If the subject reports that there is no difference, we can conclude that the axis of the cylinder lens proper is the optimum cylinder lens axis—situations "one" and "two" have "bracketed" the optimum value, which is the current value of the cylinder lens itself..

If the subject prefers "one" (the situation in panel A), then clearly a cylinder axis angle less than that now in place (that is, farther clockwise) is optimum. The refractionist decreases the cylinder lens axis angle and repeats the process until an optimum has been reached.

The converse is true if the subject prefers “two” (the situation in panel B).

It can be shown that that neither situation “A” nor “B” represent any significant change in the overall sphere power of the optical path. Thus that aspect of the “correction” remains appropriate throughout this process.

**Historical background**

Sir George Gabriel Stokes developed a variable-power cylinder lens, which he described in an 1849 paper, suggesting its use to determine the optimal cylinder power to correct astigmatism. It comprises two cylinder lenses, one with a plus power, and the other with a minus cylinder power of the same magnitude.

The two lenses are mounted together so that the relative orientations of their axes could be varied. When the two axes are aligned, the net power (cylinder and sphere) is zero. When the two axes are at right angles, the situation is that illustrated in figure 14: along one meridian there is a plus cylinder power of a certain magnitude (the magnitude of the power of either component lens), and along the other meridian, at right angles, there is a minus cylinder power of the same magnitude.

With some other angle between the two components, the composite lens presents an intermediate plus cylinder power on one meridian, and the same minus cylinder power on the opposite meridian.

Note that in this situation, the overall result can be considered equivalent to a lens with both either:

- A plus sphere component and a minus cylinder component, or
- A minus sphere component and a plus cylinder component

The use of a Stokes lens to determine the optimum cylinder power in eyeglasses was done in a context where the optimal sphere power was found by placing sphere trial lenses with varying power, from a set, into a trial frame. A Stokes lens was then added to the mix, with its “setting” (angle between the axes of its components) adjusted to vary the cylinder effect, and its overall angle adjusted to vary the cylinder axis. And of course, a sphere component was also added by this, which had to be taken into account in recording the overall indicated prescription.

Edward Jackson, in 1897, published a paper in which a Stokes lens, fixed at the “cylinder axes at right angles” state, could be used (by “flipping”) to increment and then decrement the power of a “trial” cylinder lens, allowing for convenient “bracketing” to ascertain the optimal cylinder power (just as we saw above in figure 15). Thus a

Stokes lens, with the axes of the two elements fixed at right angles, became the “Jackson cross cylinder” lens.

Twenty years later, in 1907, Jackson published another paper in which he showed that this same “cross cylinder” setup could be also used to make incremental changes in the effective axis of a cylinder lens, allowing for convenient “bracketing” to ascertain the optimal cylinder axis (just as we saw above in figure 16).

It is worth noting that, from about 1930, a Jackson cross cylinder feature was available on manual phoropters.

-#-

## Appendix B

### A curiosity in the axis scales on trial frames

Because the axis of a cylinder lens runs "in both directions", we only need to describe its orientation with a scale 180° in length. In geometry and related scientific disciplines, we would probably consider that scale to run from zero to "just less than 180°". If implemented as the cylinder lens axis scale on a trial frame, we might expect that to look like panel A in figure 17:

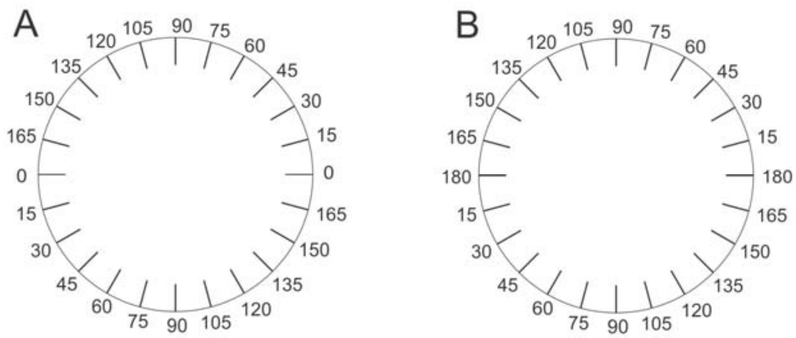


**Figure 17.**

But, as I mentioned in section 8, there has often been an aversion to the use of the value "zero", and thus it is the practice on optometry to refer to what we would otherwise regard as "zero" as "180°" (which, owing to the cyclic nature of this scale, is really perfectly apt).

So them we might expect the scales on a trial frame to be as we see in panel B above.

But in fact, in the American Optical Company trial frames, from early on (*ca*. 1890), the scale used was based on what we see on figure 18.

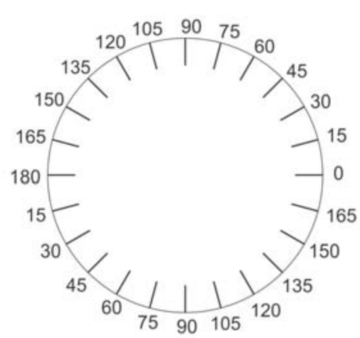


**Figure 18.**

Suppose we actually have a scale like that. The cylinder lenses have a tick mark at both ends of the axis, and so for an "exactly horizontal" orientation, one tick mark would fall opposite the “0” mark on the scale and the other one opposite the “180” mark on the scale. In that case, which number would the optometrist write? Why, “180”, of course.

This gets even more odd when we consider the actual scales found on typical trial frames. We can see this in figure 19


**Figure 19.**

and schematically in figure 20.

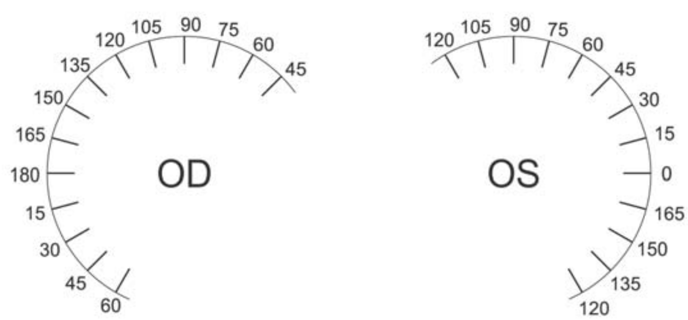


**Figure 20.**

The scales are typically (as seen in the figures) not complete, in part to allow greater clearance around the subject’s nose..

Now consider a cylinder lenses having the axis “exactly horizontal” on both sides. In the right eye section (OD–on our left in the figure), one tick mark would fall opposite the “180” mark, and the other one would fall where there was no scale. In the left eye section (OS–on our right in the figure), one tick mark would fall opposite the "0" mark, and the other one would fall where there was no scale.

So what would the optometrist write in the OD section of the prescription? Why, "180", of course. So what would she write in the OS section of the prescription? Why,"180", of course.

Most currently-available trial frames that generally follow the classical AO design have the scales marked this way. Trial frames of other designs may have the scales marked this way, or may or in all places have the "zero" marked "180", or in all places have the "zero" marked "0".

-#-

## Appendix C

## Astigmatic lenses

### C.1 Lens free of astigmatism

Imagine a spherical lens, perhaps for a simple camera. At the moment it regards a "point source object" at a great distance (theoretically, at infinity). The lens is free from astigmatism, meaning that its refractive power is the same in all directions.

The following figure shows what happens "downstream" from this lens.

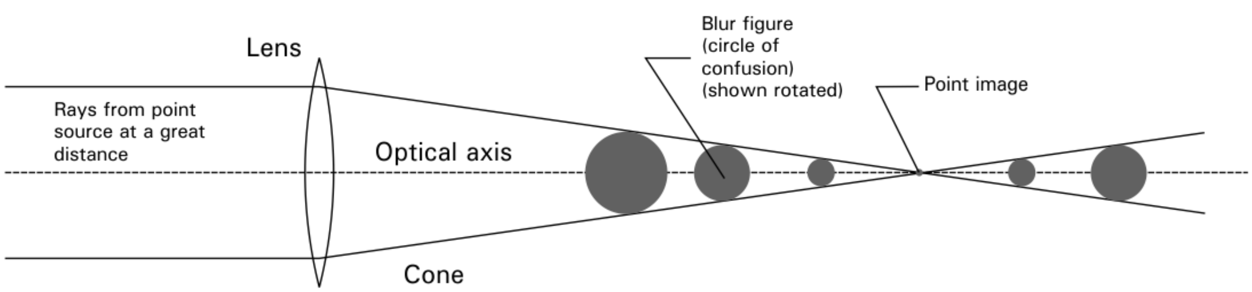


**Figure 21.**

The rays from the object are converged by the lens, and their envelope is a cone. The figure shows the cross-section of this cone at various distances from the lens; the cross-section figures have been rotated by 90° so we see their size and shape. We assume that their is no film or other opaque surface that blocks the rays; thus they, in theory, continue (to our right) to infinity.

Assuming that the lens has a circular aperture, the envelope of the rays is a circular cone.

At a certain distance from the lens the rays in the cone converge to a point image. Of course, if this lens were in a camera, we would place the film at that location to get a "perfectly focused" image (for objects at a great distance).

If we were to place the film at a point closer to the lens, there the rays of the cone are incompletely converged, forming a circular "blur figure" (also called a "circle of confusion"). The image of an actual object, composed of such a blur figure from every point in the object, would, of course, be a "blurred" image.

If we were to place the film at a distance further from the lens, at that point the rays from a point on the object would have converged and then diverged, also forming a blur figure. So the image of an actual object on the film in that case would also be blurred.

These of course are situations of "imperfect focus"

### C.2 Lens with astigmatism

Next we will consider a lens with astigmatism. That means that its refractive power is not the same in all directions. In the figure that follows, the lens is assumed to have greater power in the vertical direction than in the horizontal direction. The two shapes of the lens, one solid and one dashed, suggest how the lens might get that behavior.

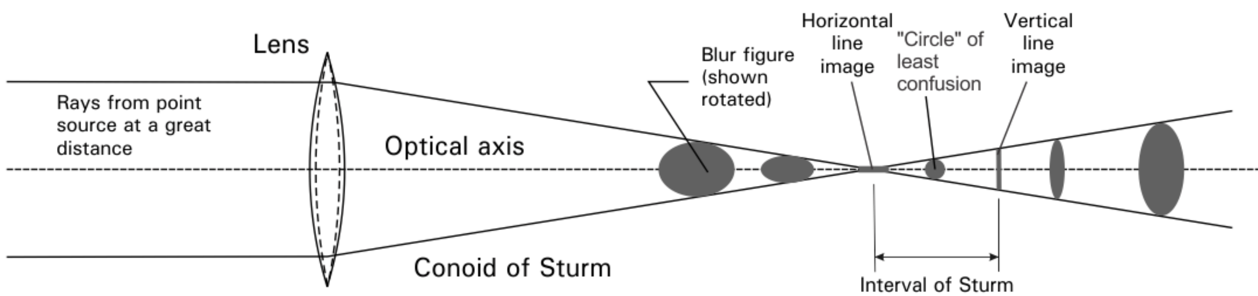


**Figure 22.**

In this case the envelope of the rays is not a cone, but rather what we can think of as a deformed cone (known as the Conoid of Sturm)[13]. Again, we see the cross sections of this conoid at different locations, as before rotated so we can see them.

At moderate distances from the lens, this cross-section (the shape of the "blur figure" that would be created on the film were it put at that place) is an ellipse with its greater dimension in the horizontal direction. As we move farther from the lens, the ellipse becomes smaller but also relatively "narrower", its vertical dimension decreasing faster than its horizontal dimension.

At a certain place, the "ellipse" gets a zero dimension in the vertical direction; it has collapsed into a vertical line. If we put film there, the image of this point source object would be a line. (In the figure, this line is shown with substantial thickness to improve its visibility.)

As we continue away from the lens, the cross section becomes an actual ellipse, but this time with its longer dimension vertical. As we

[13] Named in honor of French mathematician Jacques-Charles-François Sturm (1803-1855), who developed this model of astigmatism.

proceed, the ellipse gets larger in its vertical dimension, but smaller in its horizontal dimension. At a certain point, its horizontal dimension becomes zero; it has again collapsed into a line, this time a vertical line (again shown thick for visibility).

The axial distance between those two line images is known as the “interval of Sturm”.

Note that there is no distance from the lens where the rays converge to a point image (to produce, from the collection of all points on an actual object, a perfectly focused image). The cross section that is the “smallest” (and there is some disagreement as to how that should be defined) is called the *circle of least confusion*, where “circle” is perhaps a bit optimistic. Note that if the astigmatic lens (in a camera) is focused so that this figure falls on the film, this does not necessarily create the “visually sharpest” image.

For clarity, I have spoken of the astigmatic lens as in a camera. But of course when the lens system of the eye is astigmatic, this same model applies.

### C.3 Our “avatar” for lens behavior

As we consider the effect of cylinder lenses during refraction, we will use this pair of line images as an avatar for the entire conoid. An example of that would be as we see in figure 23.

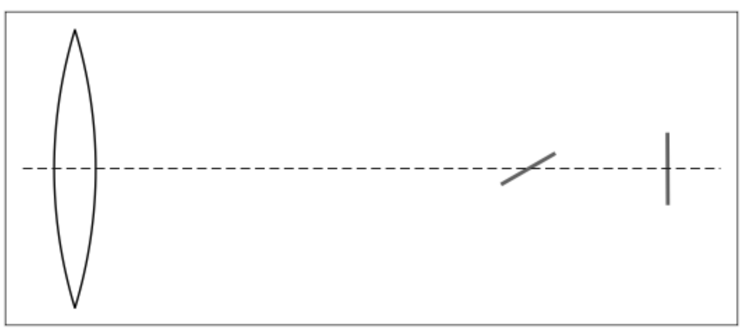

**Figure 23.**

The line images are shown in a “cavalier oblique projection”; the horizontal line image is shown at an angle, a convention for such a drawing projection, but at its actual length. (If this were an orthographic view, that line image, which is “end on”, would “appear” as an infinitesimal dot, no help to us.)

To get a little ahead of the telling of that story, I note that when we use this avatar, as we apply correction of the astigmatic component of the eye’s lens system, the two line images move more closely together (which we typically show in our illustrations), and get shorter (which we typically do not show).

When the stigmatism has been completely neutralized, the situation becomes that shown in figure 22, with a point image being generated. Still, in our illustrations, we continue to show the two line images, now at exactly the same location, with arbitrary sizes, as a reminder of "how we got there", as we see in figure 24.

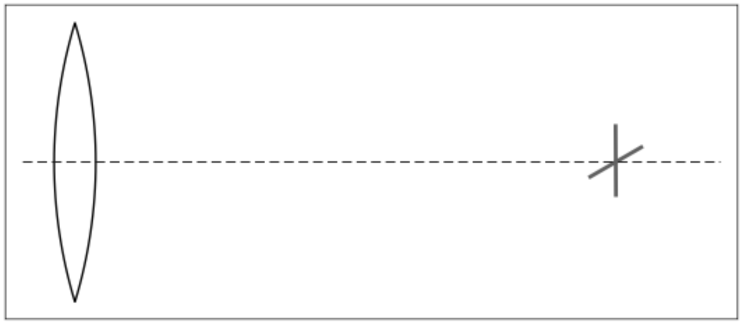

**Figure 24.**

We will use this "avatar" in Appendix D.

-#-

# Appendix D

## Plus *vs*. minus cylinder practice

### D.1 Introduction

Whether the refraction is done with a refractor (phoropter) or with trial lenses, in the prescription:

- Ophthalmologists almost always describe the astigmatism correction on the prescription in terms of a plus cylinder lens power (plus an axis angle).
- Optometrists almost always describe the astigmatism correction on the prescription in terms of a minus cylinder lens power (plus an axis angle).

It turns out that, for all practical purposes, we can describe the needed overall behavior of the eyeglass lens using either form of description. I will give a demonstrative proof of that later.

And if we have a prescription in "minus cylinder" form, it can easily be converted to "plus cylinder" form, or *vice-versa*. I'll give the equations a little later.

We might well wonder not only why do both forms exist, and of course, why does one profession consistently use one and the other profession the other?

### D.2 The tools

So how do the "tools" differ for the two practices?

<u>Refractor (phoropter)</u>. Any given make and model of refractor is made in two different versions. One has all plus cylinder lenses in its cylinder stage, and the other all minus cylinder lenses.

How could we tell, if we came across a refractor, which flavor it was? There will likely not be a nameplate that tells.

But in almost all refractors, the currently-set cylinder power is shown with a small number that appears in a small window. It is black for a plus power, and red for a minus power. A "plus cylinder" refractor has only plus cylinder lenses, so the number seen in that little window will always be black. A "minus cylinder" refractor has only minus cylinder lenses, so the number seen in that little window will always be red.

<u>Trial lens system</u>. If a "custom" set of trial lenses is ordered, normally it will be specified with only plus cylinder lenses or with only minus cylinder lenses, depending on the practice of the user (depending in turn on which profession is involved).

If a "package" set is bought, it will generally have both plus and minus cylinder lenses, and the user will just ignore one or the other.

### D.3 Demonstration of equivalence

Figure 25 gives an intuitive demonstration of why we can compose any desired overall behavior with a sphere lens and a cylinder lens of either sign of its power.

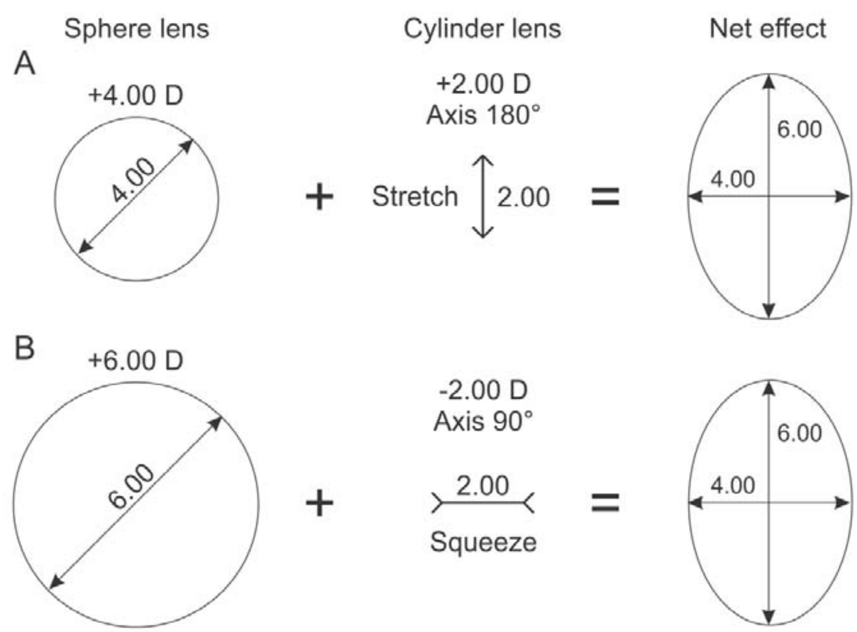


**Figure 25.**

In panel A we have a sphere lens of power +4.0 D and a cylinder lens of power +2.0 D with axis at 180° (so its refractive power is exerted in the vertical direction). On the right, we see that the effect of the cylinder lens is to "stretch" the overall pattern (since the sign of its power is plus) in the vertical direction.[14]

In panel A we have a sphere lens of power +6.0 D and a cylinder lens of power -2.0 D with axis at 90° (so its refractive power is exerted in the horizontal direction). On the right, we see that the effect of the cylinder lens is to "squeeze" the overall pattern (since the sign of its power is minus) in the horizontal direction.

We see that the net result is the same in either case. Thus either of these prescriptions leads to the same eyeglass lens:

- +4.00 +2.00 X 180

[14] For convenience I have drawn the plot of net power with meridian angle, on the right, as an ellipse. It is actually not exactly an ellipse.

- +6.00 -2.00 X 90

Now, which way is the lens actually made? Well, of course the lens is not made by taking two lenses and cementing them together. Rather, the surfaces of the lens are ground so at to produce the overall refractive syndrome suggested at the right on the figure.

Note that, if for some reason (such as described in section D.6) we wanted to only specify the cylinder component with a plus power, or maybe with a minus power, we can always do that.

Note that the same concept applies to the actual eyeglass lenses themselves. For any given prescription (which we have seen unambiguously specifies the overall optical performance of the eyeglass lens), regardless of which convention the prescription uses, that optical performance can be composed in the actual lens with either a plus or minus cylinder component.

However, there are many ramifications this choice when actually realizing the lens, and these may lead to whether, for a given overall lens prescription, the actual cylinder component of the lens performance it achieved with a plus or minus cylinder component.

## D.4 Conversion between conventions

These equations will allow conversion of a prescription written in the "minus cylinder" convention to the "plus cylinder" convention and *vice versa*.

Here, $S$ is the sphere power, $C$ is the cylinder power, and $A$ is the axis angle of the cylinder component. Subscript $P$ indicates that the factor is as would appear in the "plus cylinder" convention; subscript $N$ indicates that the factor is as would appear in the "minus cylinder" convention.

Have minus cylinder form, convert to plus cylinder form

$S_P = S_N + C_N$ (Be sure to observe the sign of C, here and below)

$C_P = -C_N$

$A_P = A_N + 90$

unless that would give a result greater than 180, in which case:

$A_P = A_N - 90$

Have plus cylinder form, convert to minus cylinder form

$S_N = S_P + C_P$

$C_N = -C_P$

$A_N = A_P + 90$

unless that would give a result greater than 180, in which case:

$A_N = A_P - 90$

It is fascinating that these look identical regardless of the direction of conversion!

## D.5 Two refraction techniques

### D.5.1 Accommodation by the eye

The human visual system, faced with an out-of-focus image, tries to attain good focus by *accommodation*; that is, by changing the shape of the crystalline lens of the eye, done by way of the ciliary muscle, which surrounds the lens capsule.

When refraction is done (either with a refractor or trial lenses), the eye's attempt to accommodate, by shifting its focus state, can interfere with some of the maneuvers of the refraction procedure. Thus we must in some way disable, or frustrate, the eye's attempt at accommodation.

### D.5.2 Clycloplegic refraction

One approach is to, when refraction is to be done, is to instill into the eyes a *cycloplegic medication*[15], which paralyzes the ciliary muscle (induces *cycloplegia*), and thus keeps the eye's focus state fixed. That state is with the ciliary muscle relaxed, which puts the lens into its greatest focal length (as for focus on an object at a great distance, ideally and theoretically at an infinite distance).

It turns out that, with cycloplegia in effect, the cylinder aspect of the refraction can be essentially equally-well conducted with either plus or minus cylinder lenses.

But, at the time the practice of refraction was being "normalized", another consideration led to a preference for using plus cylinder lenses in cycloplegic refraction.

I will put off explain that "other consideration" just now so as not to slow down the real story. I will discuss it later, in section D.7.

### D.5.3 Manifest refraction

In this technique, clycloplegia is not used. The name suggests that this is the refraction "as seen" (manifest), meaning without interference with the eye's accommodation action by cycloplegia.

Here, we still need to avert interference with the measurement process by the eye's action of accommodation, as it tries to maintain good focus. This is done by a clever ploy.

---

[15] This medication is usually also a *mydriatic*—it causes the eye's pupil to dilate (open wide). This has its own advantage in the refraction process.

Simplistically, we start by putting in front of the eye a sphere lens with a large plus power. This will cause the eye to be focused, regardless of the "setting" of its own lens, at a very close distance. The result is that the image of the test chart (typically 20 feet way) will be gravely blurred. This process is spoken of as "fogging" the eye.

The rest of the process is complicated to explain, and so again I will defer its discussion until later (in section D.8).

But a key point is that, for a rather subtle reason, the process works more handily if negative cylinder lenses are used in the refraction.

So, in fact, when conducting manifest refraction, it is seen as desirable to use minus cylinder lenses.

What about the notion that if we use minus cylinder lenses in the refraction, the prescription will be written in "minus cylinder" form, and in turn, that will suggest that (if the lens is made in the once-common practice, will suggest the use of a minus cylinder surface on the front of the lens, which is not desirable?

Well, that is all just folklore. Even in "the day", lens makers were well able to covert a prescription in "minus cylinder" form to "plus cylinder" form, so it could be used as a recipe for making the lens.

### D.6 Why the difference by profession?

Above we saw the basic differences between clycloplegic and manifest refraction. And overall, clycloplegic refraction was the easiest to actually do—the "maneuvers" we less tricky than for manifest refraction (even when the latter was done with negative cylinder lenses).

However, under the medical practice laws of the various states (which differed greatly), there were (still are) are certain things that a licensed ophthalmologist (who had to start off with "MD" training) can do that a licensed optometrist (whose training, while very extensive, is not that of an MD) may not do.

And in many states, at an earlier time, one of those prohibited things was to prescribe or administer "medication". The definition of "medication" varied from state to state, but in many cases included that which was used to induce cycloplegia.

So, in those states, licensed optometrists, unable to induce cycloplegia, were forced to us the less-convenient manifest form of refraction.

> The licensed optometrist associations in the various states were typically supportive of relieving those restrictions. By now, we should be able to imagine that the licensed ophthalmologist associations in the various states were not in general supportive of relieving those restrictions.

Of course, in fact, today, in all ( think) states, licensed optometrists are permitted to prescribe, and administer, a range of medications, including the medication used for cycloplegia.

But the die was cast as to the use of plus and minus cylinders by the two professions.

### D.7 Why the preference for plus cylinders in cycloplegic refraction?

"In the day", it was generally the view that the prescription was not just a specification for the overall optical performance of the eyeglass lens but in fact a recipe for making it. And in that era, when the eyeglass lens was made, it was common to have the sphere surface (which determined the sphere power) ground into the back surface of the lens, and the cylinder surface (which determined the cylinder power), if applicable, ground into the front of the lens.

And at the time, it was at the time considered desirable for this cylinder surface to be convex (a plus cylinder) rather than concave (a minus cylinder). The reason is that a spherical surface (either convex or concave), or a cylindrical surface (either convex of concave), can be ground onto a glass blank in the most straightforward way.

As we saw above, a given overall optical result of an eyeglass lens can be attained with either a plus or minus cylinder component (in part by properly choosing the axis of the cylindrical component), so the lensmaker was free to use either a plus or minus cylinder form for the front surface of the lens, regardless of the form of the prescription.[16]

So, given the view of the time that the prescription was a "recipe" for making the lens, working backward from that it became desirable for the prescriptions to be written in "plus cylinder" form.

---

[16] It was later realized that better optical performance would be attained if both the sphere and cylinder components of the lens power were done by the rear surface of the lens. This required a surface that was in fact a section of a torus, said to be a *toric* surface. Grinding that was more challenging than for a spherical or cylindrical surface, but schemes for doing that were invented and made it practical to use this more desirable design. Both spherical and cylindrical aspects of that rear surface curve are minus. The front surface is spherical, and is made to produce the proper sphere power of the lens.

And, again working backward, since the prescription notation was most handily derived directly from the refraction procedure, it became most common to use plus cylinders in that process.

## D.8 Why the preference for minus cylinders in manifest refraction?

### D.8.1 Introduction

Again, here a pivotal need is to disable, or at least frustrate, the eye's attempt to accommodate as different trial lenses are put in place, in this case, trial cylinder lenses.

Let's first consider an eye whose lens system has a greater power in one meridian than the opposite one (that is, which has astigmatism), and which in addition has a "relaxed" focal length that is too great for the dimensions of the eye (as thus the eye suffers as well from hyperopia.

We see a fanciful quasi-oblique presentation of an important result of that in this figure[17] [18], using our "avatar" for the effect of the Conoid of Sturm, the pair of line images

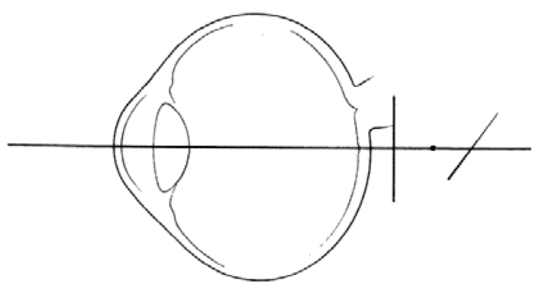

**Figure 26.**

This is predicated on the refractive power of the eye's lens system being greatest in the horizontal meridian and thus being the least in the vertical meridian.

Imagine that this eye is regarding a point source of light at a great distance. Because of the inconsistency of refractive power in the different meridians, rather that the light rays from the point source being converged into a point image at some point, here (theoretically) two line images are formed. one (horizontal) at a point behind the retina, and the other (vertical) at a point farther behind the retina.

---

[17] The principles of this representation are discussed in Appendix C/

[18] The figures in this section were adapted from those in a paper by Jay H. Kaufman, and may be original with him. They are used here under the doctrine of fair use.

Of course, no actual images can be formed behind the retina (which is opaque), but just imagine that this problem has been magically relieved.

Now before I proceed, I must note that, in actual refraction, the object is not a point source but rather a test chart of some sort (perhaps a Snellen chart or its equivalent). Thus the actual image situation (theoretically, behind the retina) is much more complex that a pair of two line images. But we will magically still follow the adventures of the images creates from a single point on that test chart, with the two line images the lens creates as their avatar..

**D.8.2** "Fogging" the eye

The first step in the refraction process is to put a sphere lens of substantial power in front of the eye. The result is that our two heroes, the two line images, are now created farther forward, perhaps even, theoretically, in front of the eye, like so:

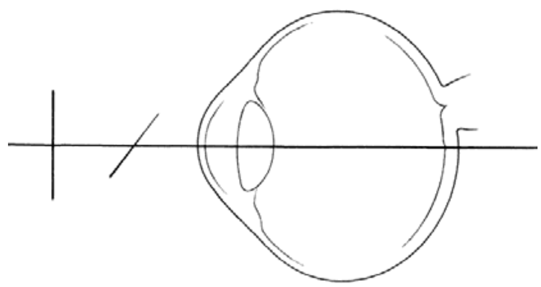

**Figure 27.**

Of course, these hypothetical images, being located in front of the lens, are "virtual", but that doesn't make them of any less value to us at this point. The important thing is that they are so far removed from the retina that the image on the retina is gravely blurred, probably beyond recognition, as if the scene were being viewed through a thick fog. And in fact, this action is spoken of as "fogging" the subject's view.

The accommodation system, unsatisfied with this image not being on the retina, tries to move them back by an increase in the eye lens's focal length. but it runs out of range with the situation still almost as seen just above, maybe like so:

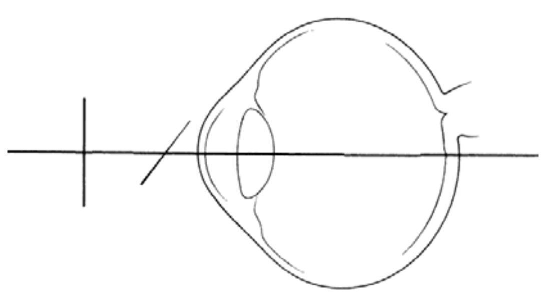

**Figure 28.**

The subject's view of the test chart is still severely "foggy".

We next reduce the power of the spherical lens, at one point leading to a situation like this:

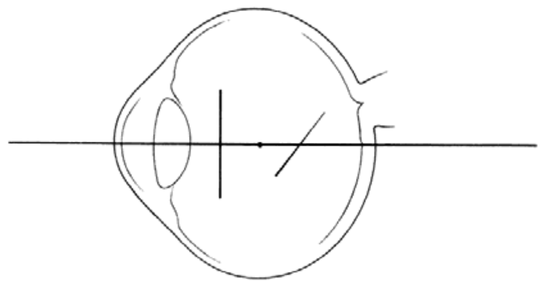

**Figure 29.**

Perhaps now the subject can begin to see the test chart, although very badly out of focus. The accommodation system tries to move these images (on the average) onto the retina, but again that would require an increase in the focal length of the eye lens, and it is already at its greatest focal length. The accommodation system is "against the maximum focal length stop". And we want to keep it there.

We continue to decrease the power of the sphere lens until we get a situation like this:

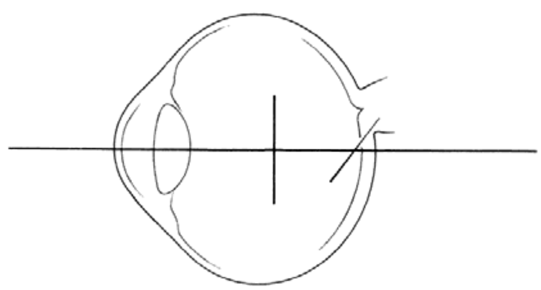

**Figure 30.**

I note that in reality the two line images now are smaller than in the prior figure. But our convention is to use the graphic presentation of the line images at constant size as our avatar for the entire situation downstream from the lens.

Now the rearmost (horizontal) line image lies almost on the retina (so the horizontal line image on the retina will be only slightly out of focus), but the frontmost line image (vertical) is still quite a way from the retina, so the vertical line image on the retina is substantially out of focus.

And again, the diligent accommodation system attempts to, "on the average", put the two line images on the retina. But again, that would require an increase in the focal length of the eye lens, and it is already at its greatest focal length.

So the accommodation system remains frustrated, and thus cannot change the focal length of the eye's lens as the remaining steps of the refraction take place.

Interestingly enough, although at this point both line images can be seen, the horizontal one a bit blurred, but neither as if seen through a fog, this situation is still described in optometric jargon as the eye "still being fogged".

**D.8.3** Neutralizing the astigmatism—minus cylinder lenses

Now we begin to neutralize the astigmatism. We put in place a minus cylinder lens of small power with its axis vertical (and thus its power is in the horizontal direction). Its effect is the reduce the discrepancy between the overall power of the lens system between the (in this case) horizontal and vertical meridians. The result is that the axial separation between the two line images is reduced (the vertical line image being moved toward the retina.

Then we perhaps have this:

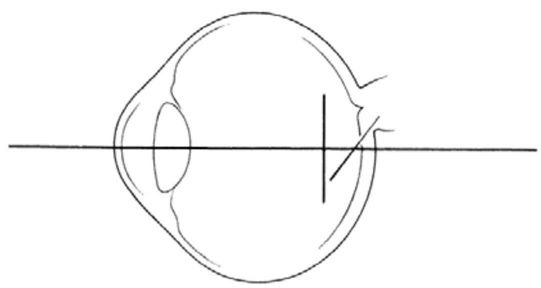

**Figure 31.**

W continue increasing the magnitude of the minus cylinder power until we have this:

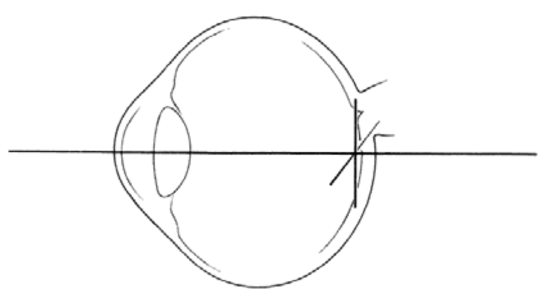

**Figure 32.**

In reality, the two line images have now shrunk to infinitesimal size, so it is a point image that is created. Still, for consistency, I continue to show the two line images, at constant size, as the avatar for what is happening.

Here, the astigmatism is completely neutralized, but the image is not quite on the retina, and thus is still a little out of focus.

So we decrease the sphere power a little until we have this:

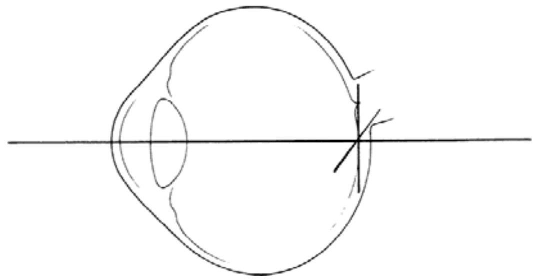

**Figure 33.**

Again, in reality, it is a point image that now lies on the retina, even though I show our familiar characters, the two line images, at constant size.

We have neutralized all the refractive errors in the eye, and write down the lens combination we have in the trial frame as the (distant vision) prescription for this eye.

**D.8.4** Neutralizing the astigmatism—plus cylinder lenses

Now suppose we did this using plus cylinder lenses. We will start with this situation (achieved as described earlier):

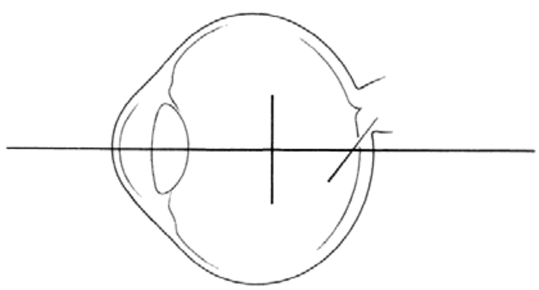

**Figure 34.**

Now we start to neutralize the astigmatism by putting a plus cylinder lens in front of the eye, with its axis horizontal (and thus its power is in the vertical direction). As before, its effect is the reduce the discrepancy between the overall power of the lens system between the (in this case) horizontal and vertical meridians. In particular, it increases the overall power in the vertical direction. The result is that the axial separation between the two line images is reduced (the horizontal line image being moved away from the retina).

We then might have this:

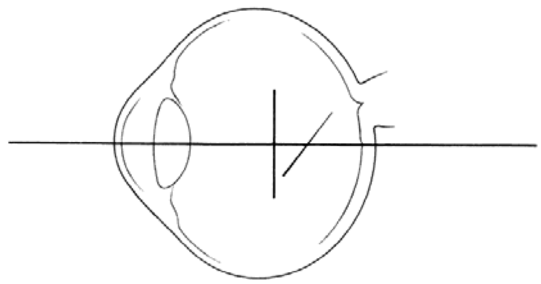

**Figure 35.**

But now neither of the line images are in very good focus, and it will be hard for the subject to report on changes seen as we neutralize the astigmatism. So we must now decrease the power of the sphere lens to again put the rearmost (horizontal) line image near the retina.

Very possibly, as we continue to increase the plus cylinder power toward the value that will completely neutralize the astigmatism, we will again need to decrease the plus sphere power to keep the entire image (here represented by the two line images) in a reasonable state of focus.

**Punch line alert:** This need to perhaps continually change the sphere power as we change the cylinder power is an inconvenience not present in the minus cylinder scenario.

**D.8.5** Conclusion

And that, my patient readers, is the reason that, when doing manifest refraction (that is, without benefit of cycloplegia), it is considered that the use of minus cylinder lenses is preferred over the use of plus cylinder lenses.

-#-