The concepts of the *time constant* and *half life* of certain physical processes

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INTRODUCTION

In an important class of physical processes, at any instant the property of interest declines at a rate proportional to its value at the instant. As a consequence, over any time period of a certain duration in the life of the process, the value of the property of interest will decline to the same fraction of its value at the beginning of the period. This is often described as the process following a *negative exponential function* of time.

In some fields of work, such a function is characterized by the length of the period over which the property declines to 1/e of its value at the beginning of the period. Often this parameter is called the *time constant* of the process.

In other fields of work, such a function is characterized by the length of the period over which the property declines to 1/2 of its value at the beginning of the period. This parameter is called the *half life* of the process.

This article explores this type of time function, some of its applications, and those two values used to characterize it.

1 THE PROCESSES OF INTEREST

1.1 Introduction

In an important class of physical processes, at any instant the property of interest declines at a rate proportional to its value at the instant. As a consequence, over any time period of a certain duration in the life of the process, the value of the property of interest will decline to the same fraction of its value at the beginning of the period.

1.2 Examples

Examples of processes of this type include:

- In electrical engineering, the decline in the voltage of a capacitor that is being discharged through a resistor.

- In medicine, the decline in the concentration of a medication in the body as it is metabolized and the products excreted.

- In nuclear physics, the decline in the amount of a radioisotope as it "decays" into other (typically non-radioactive) isotopes.

2 EQUATIONS

2.1 The classical form

The classical form of the equation describing a generalized process of this type is:

$$x_t = x_0 e^{-\frac{t}{\tau}} \quad (1)$$

where X_t is the value of the variable of interest, x, at time t; x_0 is the value of the variable x at time t_0; e is the Naperian base; t is the time at which we are interested in the value of x; and τ (lower-case Greek *tau*) is the parameter of the equation.

We can see why such a process is said to follow a *negative exponential function of time*.

Often, especially in electrical engineering, the parameter τ is called the *time constant* of the process. From the equation we can see that this is the time at which the value of x has declined to 1/e of its value at t_0 (approximately 0.368 x_0). This will be true regardless of the instant we adopt as t_0; over **any** period of duration τ, the value of x declines to 1/e of its value at the beginning of the period.

This is the form most often found in electrical engineering.

2.2 Another form

This same mathematical function can also be written this way:

$$x_t = x_0 2^{-\frac{t}{h}} \quad (2)$$

There, the parameter of the equation, h, is called the *half life* of the process. It is the duration of the time period over which the value of x will decline to 1/2 of its value at the beginning of the period.

This is the form often found (or implied) in many cases in medicine and nuclear physics. The name "half life" seems especially apt in those usages: after any period of that duration, only half of the "stuff" of interest at the start of the period is still "alive".

2.3 Equivalence

The two forms are in fact precisely equivalent, if we take proper care with their parameters. A little bush-league algebra tells us that:

$$h = (\ln 2)\,\tau \qquad (3)$$

or, approximately:

$$h = 0.693\tau \qquad (4)$$

Thus, a process with a *time constant* of 1.00 second has a *half life* of (approximately) 0.693 seconds. A process with a half life of 1.00 second has a time constant of (approximately) 1.44 seconds.

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