70th ***birthday series***

The Scheimpflug Principles

Douglas A. Kerr, P.E.

Issue 2
July 16, 2006

ABSTRACT

We can tilt the lens of a camera in order that the plane containing objects in perfect focus will not need to be parallel to the film plane, a desirable situation for many types of work, including architectural photography. It is often said that the required relationship between lens, film, and the desired plane of object focus is prescribed by "the Scheimpflug principle". It fact, two criteria must be satisfied, the second of which may be in terms of the classical Gaussian focus equation. However, we may instead use as the additional criterion a second principle also articulated by Scheimpflug. In this article we describe this whole situation along with the these two principles of Scheimpflug, and we show the equivalence of this additional principle of Scheimpflug to the Gaussian focus equation.

SUMMARY

In an ordinary camera setup, the points in space that are in focus ideally lie in a plane parallel to the film plane. There are situations in which this is not what we would prefer, such as in the photography of the façade of a building with the camera aimed diagonally upward, or the photography of a decorative pathway with the camera pointed diagonally downward.

By tilting the lens so that its axis is no longer perpendicular to the plane of the film, we can arrange for the plane of perfect object focus to be oblique. Various special cameras, or lenses, provide for this.

We often hear that for a certain oblique plane to be in focus the setup of the camera must obey "Scheimpflug's principle", which calls for three planes to intersect in a common line:

- The oblique plane containing the desired objects
- The plane of the film
- A plane through the center of the lens and perpendicular to its axis. (If the lens is not the famous fictional "thin lens", exactly how that plane is defined becomes complicated. Here we will avert that problem by assuming the use of the "thin lens".)

But a little thought reveals that this can't be the entire story. For any given position of the lens (in terms of both tilt and "focusing" setting), there are an infinite number of object planes that satisfy this condition. Clearly, all of them cannot be the plane of perfect object focus.

In fact, once we have tilted the lens to make the desired oblique object plane eligible to be in focus, we must still “focus on it” with the focus movement of the lens.

But when we move the lens to “focus on” the desired oblique plane, the arrangement described above is disrupted. Thus there is an interaction between lens tilt and lens focusing position that must be taken into account, complicating the actual practice.

If we go back to the geometric theory, it turns out that, for any given lens position (tilt and focusing setting), there are two conditions that jointly define the unique plane of perfect object focus. One is the Scheimpflug principle described above. The second condition can be an application to this situation of the classical Gaussian focus equation. Alternatively, it can be a second principle given by Scheimpflug, which states that these three planes must (also) intersect in a common line:

- The oblique plane containing the desired objects
- A plane through the center of the lens, parallel to the film plane
- A plane parallel to the plane of the lens (as described above) but forward of it by the focal length.

Thus, we find that two Scheimpflug principles can be involved in the geometry of photography of an oblique object plane.

INTRODUCTION

The problem

In an orthodox camera, the points in space at which an object may be placed and be in perfect focus (for any given setting of the camera's focusing mechanism) ideally form a plane surface parallel to the plane of the film. (In reality, the surface may not be exactly a plane.)

In many types of work, the objects we wish to be in best focus are often indeed essentially confined to a plane, but we cannot arrange for that plane to be parallel to the plane of the film and still meet other photographic objectives (such as having the camera in some place we can get to, or achieving the desired composition). Common examples include the photography of the façade of a building with the camera aimed upwards at an angle, or the photography of a decorative cobblestone pathway with the camera pointed downward at an angle.

It would thus be handy if we could rearrange things so that the plane of perfect object focus was not parallel to the plane of the film.

The solution

We can in fact do just that by arranging for the camera's lens to tilt so that its axis is no longer perpendicular to the film plane, as it would be in an orthodox camera.

"View cameras", such as are used in studio, technical, and architectural work, often are constructed so that camera "front" (on which the lens is mounted) may be tilted, and often the "back" (including the film holder) can be tilted as well, which can actually produce the same result.

In the case of single-lens reflex cameras, special lenses are available in which the lens axis can be tilted at an internal joint, and there are also adapters that can be interposed between conventional lenses and the camera body to enable the same movement.

Now, all we have to do is determine how the lens must be positioned and oriented so as to make the plane of perfect object focus be the one that contains our scene features of interest (such as the façade of a building, or the surface of a decorative cobblestone path extending forward from the photographer's location).

Enter Scheimpflug

In this regard, we often hear that the necessary relationship between the plane of object focus, the lens, and the film is defined by "the

Scheimpflug[1] principle", named in honor of Theodor Scheimpflug. That principle is often described this way:

> In order to have the objects in a certain oblique plane all in perfect focus, we must tilt the lens so that this object plane, the plane of the lens [we will assume here a "thin" lens], and the plane of the film all intersect in a common line.

Alas, the story is not that simple. The necessary relationship between the object plane, the lens, and the film must follow two imperatives, only one of which is that stated just above. In this article, we will investigate this whole situation at length.

The image medium

The principles discussed here are equally applicable to film or digital cameras. For conciseness, I will consistently refer to the image acceptance medium as "the film".

Acknowledgement

Much of what I know about this I learned from an excellent article by Harold M. Merklinger, *Scheimpflug's Patent*, which appeared in *Photo Techniques*, Nov./Dec. 1996. (An Internet source given at the end of this article.) Merklinger is also the author of the book, *Focusing the View Camera*.

Merklinger's article establishes some terminology which I will generally follow here. He points out that the concept generally described as "the Scheimpflug principle" (as stated above) is actually the third principle pertinent to this area articulated by Scheimpflug in his landmark patent (1904), and that the additional criterion due to Scheimpflug we will discuss here is one of two parts of what Merklinger describes as "Scheimpflug's fourth principle", also articulated in that patent. I'll use these terms here for consistency with Merklinger's work. [2]

LENS MOVEMENTS

The matter with which we are concerned here implicitly involves two lens movements, *focusing* and *tilt*. The mechanical arrangements for executing those movements vary between camera designs. To avoid

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[1] Pronounced (approximately) "**Shime**-floog".

[2] Scheimpflug did not discover the "third principle", which he evidently learned of from a 1901 patent by Jules Carpentier; in any case it is considered to be based on a theorem of projective geometry by Girard Desargues (1591–1661). Scheimpflug greatly enlarged the understanding of the application of the principle to photography, so he gets the credit!

any misunderstanding, let's take a quick look into the matter of these lens movements.

Tilt refers to changing the axis of the lens from its orthodox orientation, perpendicular to the film plane. In fact, this is itself a "two degrees of freedom" movement. The lens may have a component of tilt about a horizontal axis and/or a component of tilt about a vertical axis. This can also be looked at as the lens having a certain angle of tilt about some axis whose orientation itself may vary (perhaps vertical, perhaps horizontal, perhaps something in between).

In a full-featured view camera, the actual mechanism of tilt usually matches the first outlook, and in fact there the two components of movement are usually separately described as "tilt" and "swing". With a "tilt" lens, the actual mechanism matches the second outlook, in which the entire lens is rotated to place the axis about which the lens will tilt in the proper orientation.

In the figures in this articles, we finesse the matter of the two degrees of freedom of the tilt movement by always taking our view of the system along the axis of tilt, so that to us it seems as if the only tilt is about an axis perpendicular to the paper.

Focusing refers to what we change (on other than a view camera) by operating the focusing ring on the lens. We can think of it as changing the distance from the lens to the film.

But in many lenses, focusing is done by merely moving certain groups of elements within the lens. This serves to complicate our tidy view of the focusing movement (although it does not disrupt the principles of either basic camera focusing nor of the specialized topic of this article).

Now, on a camera that provides for a tilt movement, there are two basic ways the focusing movement can be implemented:

- The lens (or its movable elements) may move along a path that is always perpendicular to the film. This is typical of the arrangement in a view camera (if it is the camera front that is tilted, not the back).

- The elements may move along the "tilted" axis of the lens. This is typical of the arrangement for a single-lens reflex camera lens with integral tilt movement, or mounted on a tilt adapter. It also pertains to a view camera if the "tilt" is done by tilting the back and not the lens board.

This distinction has no impact on the basic geometric principles and concepts that are involved, but they do influence how we must manipulate the camera to apply the concepts.

Finally, note that in almost all lenses of interest to us, there is a substantial distance between the two sets of nodal/principal points of the lens (the lens is not "thin"). And in fact, for lenses in which focusing is not done by moving all elements together, this distance may vary with the focusing movement. Here, we avert concern with this by assuming a "thin" lens.

BRINGING OUR OBJECT PLANE INTO FOCUS

Two degrees of freedom

In practice, the plane of object focus is established by the layout of our "subject array" (perhaps it is the façade of a building), and our task is to manipulate the camera settings to bring that plane into focus. The plane itself is defined in "two degrees of freedom" (if we limit ourselves to planes that are perpendicular to our "paper", a constraint that simplifies our work). We can think of those two degrees of freedom as being:

- The inclination of the plane (perhaps measured as a departure its parallelism with the film plane, the situation we enjoyed before we started fooling around with lens tilt)

- The "location" of the plane, as for example the distance to it along the lens axis.

Thus is it not surprising that to bring a particular plane into focus, we must manipulate the camera lens with "two degrees of freedom"— the lens tilt and its "focus setting". (This assumes that we limit look at the system along the axis of the tilt, an outlook consistent with the assumption above about the orientation of the plane.)

Then, taking the next step in scientific logic, as we prepare to examine "on paper" the matter of bringing our object plane into focus, it will not be surprising that there will be two technical conditions that will have to be met for the plane to come into focus.

The first step

We often hear that to bring some arbitrary oblique object plane into focus it is (merely) necessary that we arrange to satisfy "Scheimpflug's principle". This is typically stated as:

> In order for an oblique object plane to be in focus, these three planes must intersect in a common line:

- The oblique plane containing the desired objects
- The plane of the film
- A plane through the center of the lens and perpendicular to its axis.[3]

Now just a moment ago we saw that there would have to be two conditions satisfied if a particular object plane were to be brought into focus, so already we can tell that this isn't going to be the whole story. But we'll proceed as if it were.

Because it makes later parts of the geometric analysis proceed more tidily, I will assume that the have a camera in which the "tilt" function is done by tilting the camera back (with the film), and in which focusing is also done by moving the back (as is possible on many view cameras).

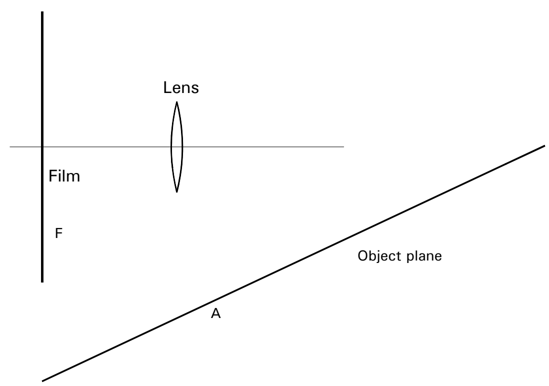

Figure 1.

In figure 1, we start with the lens and back in some arbitrary initial position (we label the film plane F), and show the oblique object plane on which we want to focus, plane A.

In figure 2, we apply "the Scheimpflug principle" by tilting the back until planes F, L, and A all intersect in a common line, which we label m. (We of course see it only as a dot since the line is perpendicular to our paper).

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[3] If the lens is not the famous fictional "thin lens", exactly how that plane is defined becomes complicated. Here we will avert that problem by assuming the use of the "thin lens". We will examine the situation for a "thick lens" in Appendix B.

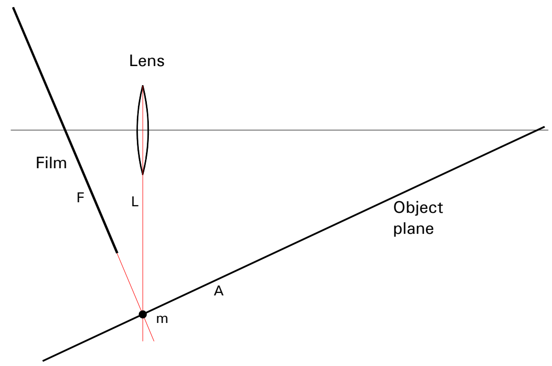

Figure 2.

Have we brought plane A "into focus" by doing this? Probably not.

If we look at figure 3, we see that planes A′ and A′′ also satisfy Scheimpflug's principle, since they pass through line m, through which planes F and L also pass.

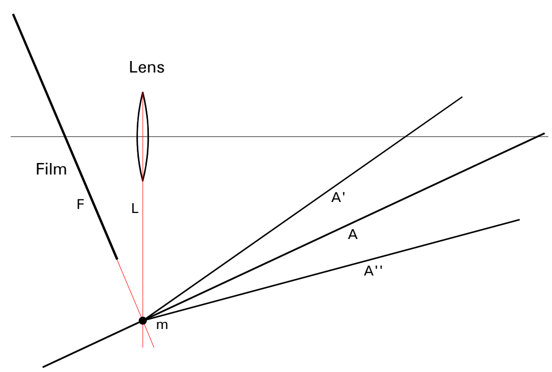

Figure 3.

Yet they can't all be the plane that is in focus. And that should not be surprising, since so far we have attended to only one of the two critical cameras movements, satisfying only one geometric condition.

To bring plane A, our object plane, into focus, we must also the camera's using movement"), which we have said we will do by adjusting the "position" of the back. We in fact will determine when plane A is really in focus by applying the Gaussian focus equation, which tells us the relationship between object and image distances for a properly focused image:

$$\frac{1}{P}+\frac{1}{Q}=\frac{1}{f} \qquad (1)$$

where *P* is the distance from the object to the first principal point of the lens, *Q* is the distance to the image from the 2nd principal point of the lens, and *f* is the focal length of the lens.

We see the result of the Gaussian focus equation applied to our current settings situation on figure 3 (for some assumed focal length of the lens). We've applied it to points on the lens axis, the only place it will strictly apply if we have a lens corrected for flatness of field, which we have tacitly assumed.

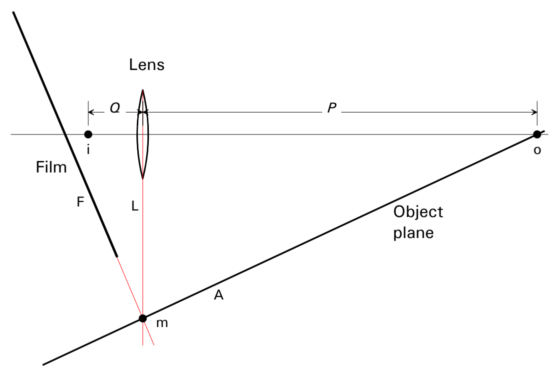

Figure 4.

Well, guess what? For object point **o**, the image point falls at **i**, which is not on the film. So, as we have suspected (since we have not yet attended to the focusing movement of the camera) we have not yet brought plane A into focus.

So we need to pivot the film plane, F, about line m (so we continue to observe the Scheimpflug principle already satisfied) until it passes through point **i**. Now object plane A is in focus.

A graphical solution

Now in this part of the exercise we presumably had calculated the location of point **i** (by way of the distance Q) numerically. But there is a graphical solution ("by construction") for the proper tilt and position of the film plane. It is essentially a graphical solution of the Gaussian focus equation as it applies in this situation.

In doing this, we will refer to geometric objects as we see their projections on our paper—that is, for line m, we will speak of "point m", for plane K we will refer to "line K", and so forth.

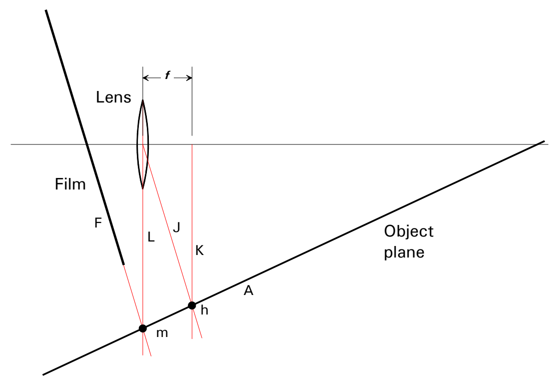

Figure 5.

We begin by drawing line L (representing the lens plane) until it intersects line A (representing the desired object plane). This gives us point m. Then we mark off, to the front of the lens from its center, the focal length of the lens (*f*). From that point, we draw a line (K) perpendicular to the lens axis to intersect A at point h. We next draw a line (J) from the center of the lens through point h.

Now we draw a line though point m and parallel to line J. This is the line representing plane F, which is where we must position the film.

Re-enter Scheimpflug

In fact the relationship between planes J, K, and A—that they must all intersect in a common line (h)—was articulated by Scheimpflug as a second condition that must be satisfied to bring plane A into focus. It can be stated as:

> In order for an oblique object plane to be in focus, these three planes must intersect in a common line:

- The oblique plane containing the desired objects
- A plane through the center of the lens, parallel to the film plane
- A plane parallel to the plane of the lens but forward of it by the focal length.

In Scheimpflug's landmark patent, he identifies the principle we often hear of (the one we applied in figure 2) as his "third principle" (the first two relate to other aspects, not of concern here). Building on this, Merklinger calls the other Scheimpflug principle "Scheimpflug's fourth principle". We will follow that notation here.

In appendix C, we prove that in fact that Scheimpflug's fourth principle (in the presence of Scheimpflug's third principle) turns out to be a graphical solution of the Gaussian focus equation.

THE ANGLE OF LENS TILT

We can actually determine the required angle of tilt with a little trigonometry. We see how on figure 6. Here, we think in terms of a camera in which the lens itself is tilted (the more common practical case).

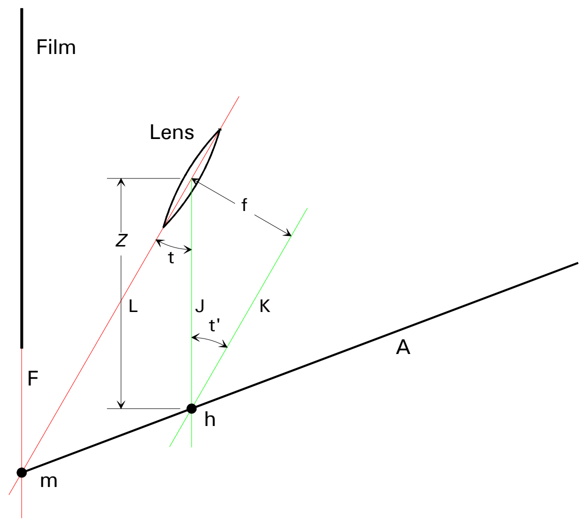

Figure 6. Angle of lens tilt

On this figure, we assume that the lens has already somehow been properly adjusted (in both tilt and "focusing movement") to bring object plane A into focus, and we will look, after-the-fact, into its angle of tilt, *t*.

Because planes L and K are, by definition, parallel, angle *t*' will be equal to angle *t*. If we designate as *Z* the distance from the center of the lens to the desired plane of object focus, (plane A), measured along a plane parallel to the film (plane J), we find that this angle is given by:

$$t = t' = \arcsin\frac{f}{Z} \qquad (2)$$

where *f* is the focal length of the lens and **arcsin** denotes the trigonometric *arc sine* (or *inverse sine*) function: the angle whose sine is the ratio that follows.

But in practice, if we haven't yet put the lens into its final position (which is why we are interested in the angle, *t*), we don't know the geometry in which to determine the distance *Z*.

However, in most situations of actual interest, the range of lens movement we might encounter in making the "focusing movement" is really very small, and thus we can assume some arbitrary lens position and have little error in determining the distance Z.[4]

An example of this determination, for two "non-closeup" situations, is given in Appendix A.

ACTUAL PRACTICE—DISCLAIMER

As the reader can well imagine from the preceding discussion, in practice there is art and craft as well as science and geometry involved in setting up a camera for a desired oblique plane of perfect object focus. I have no experience in this arena, and so I will leave advice about how to actually play the instrument up to genuine virtuosi.

REFERENCES

An excellent article by Jeff Conrad on the topic of the utilization of camera movements is available (as of this writing) on the Internet, here:

`http://eosdoc.com/manuals/?q=tilt-shift_desc`

This is an excellent paper by Robert Wheeler which gives an extensive mathematical treatment of the topic and related issues:

`http://www.bobwheeler.com/photo/ViewCam.pdf`

Merklinger's paper on Scheimpflug's patent is available (as of this writing) here:

`http://www.trenholm.org/hmmerk/SHSPAT.pdf`

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[4] This will not necessarily be so in closeup work.

APPENDIX A

Estimation of the distance *Z*

To bring our object plane into focus, we must attend both to the angle of lens tilt and to the "focusing" setting. If we can predetermine the required angle of tilt and set the lens accordingly (tilt lenses, for example, often have a scale showing the tilt angle in degrees), it is then often practical to bring the whole object plane into focus with the focusing movement by observation in the viewfinder or on the ground glass.

We saw in the body of the paper that the required angle of tilt of the lens is given by:

$$t = \arcsin\frac{f}{Z} \qquad (3)$$

where *t* is the angle of tilt, *f* is the focal length of the lens, and *J* is the distance from the center of the lens to the desired plane of object focus, measured along a plane parallel to the film plane. So of course, to determine the needed tilt, we need to know the value of *Z*. If our project is, say, photographing a decorative sidewalk, we can just measure the distance *J* with a tape measure. We need to measure from the lens to the sidewalk, holding the tape measure about parallel to the film plane (and thus it will hit the sidewalk behind us, since the entire camera is tilted downward).[5]

But in an architectural photographic task, we probably can't do that, since the line H (to which we must measure to determine *Z*) is likely far underground. Fortunately, we can generally determine *Z* by simple calculation based on some things we can readily measure (or at least estimate).

Figure 7 shows schematically a typical "architectural photo" application. Here, to include the entire building façade tidily in the camera's field of view, we have tilted the camera's axis upwards by the elevation angle *e*.

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[5] Recall that, since we haven't yet made the focusing movement, we don't actually know precisely the final location of the lens, but in cases such as we describe the final position of the lens—which can only vary by only a small amount compared to the various distances involved—does not significantly disturb the reckoning we are discussing here.

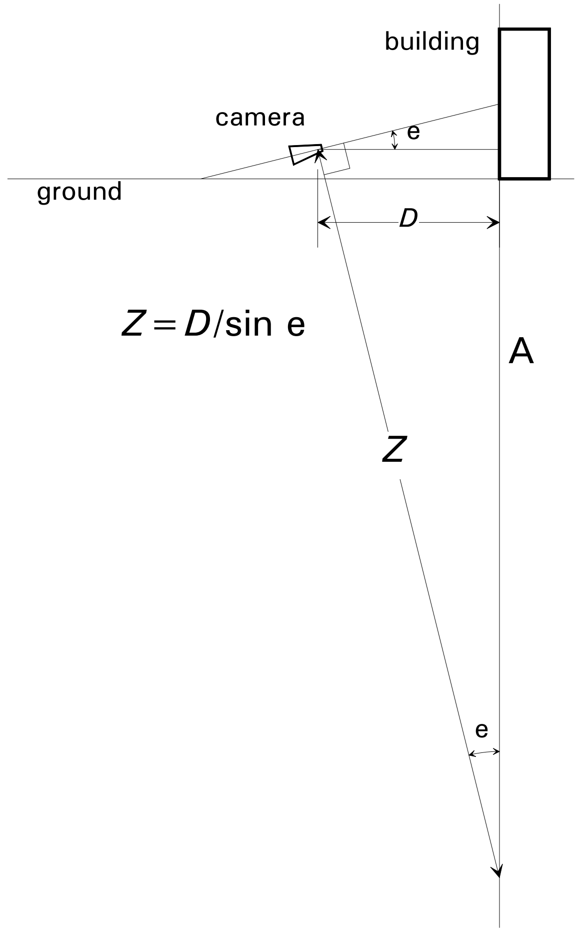

Figure 7. Determination of the distance *Z* in an architectural photo situation

The arrangement allows for the use of simple trigonometry, after application of the principle of similar triangles, to determine the value of *Z*. As inputs, we need:

- The horizontal distance, *D*, from the camera to the building face, and
- The angle of elevation of the camera axis, *e*.

Then the distance *Z* is given by:

$$Z = \frac{D}{\sin e} \qquad (4)$$

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APPENDIX B

With a thick lens

The presentations in the body of the article, for convenience, shows the fictional “thin lens”, in which both nodal points and both principal points all lie at the very center of the lens.

Of course, most lenses with which we are concerned depart substantially from that model. Figure 8 shows how the two Scheimpflug principles apply to a practical “thick” lens.

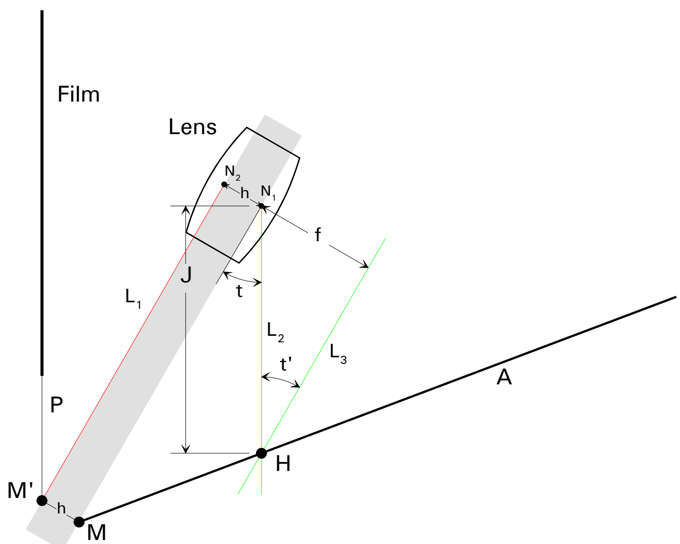

Figure 8. Scheimpflug and the thick lens

Analysis of a system involving a thick lens is facilitated if we recognize that it can be made by taking the same system with a thin lens, cutting it through the very center of the lens, and separating the two halves of the system by a distance equal to the distance between the two nodal points of the actual thick lens. That “no man’s land” we have inserted between the two halves of the world (I show it shaded in the figure) is sometimes called the *hiatus* of the lens, and thus I use the symbol *h* for its thickness—the distance between the nodal points.

In the figure, we first recognize a bogus line M (called M’) defined by the intersection of the film plane, P, and the plane passing through the second nodal point, N2, perpendicular to the lens axis. We then transport that line by the distance *h* along a line parallel to the lens axis, in effect moving it to “the land beyond the hiatus”, which is where the plane of perfect object focus lives.

We then locate the hinge line, H, in the same way as before. The plane of perfect object focus, A, is the plane that passes through lines M and H, just as before.

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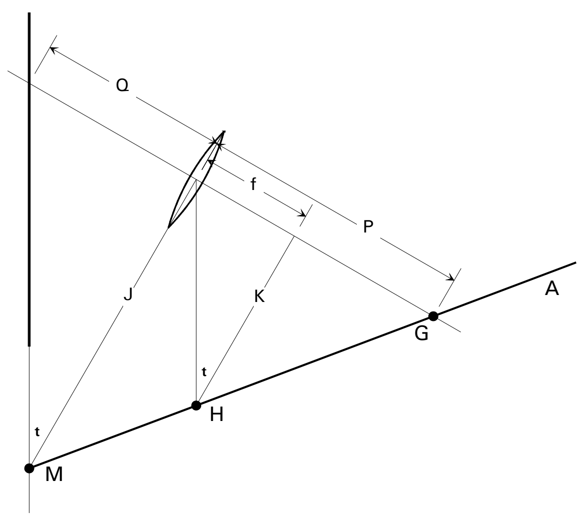
APPENDIX C

Scheimpflug's fourth principle and the Gaussian focus equation

In the body of the article we learned that the an oblique object plane would be the plane of perfect focus if the setup satisfied Scheimpflug's third principle and also satisfied either the Gaussian focus equation or Scheimpflug's fourth principle.

Here we will demonstrate that, in a system following Scheimpflug's third principle, the Gaussian focus equation and Scheimpflug's fourth principle are equivalent.



First, we review the Gaussian focus equation:

$$\frac{1}{P} + \frac{1}{Q} = \frac{1}{f} \qquad (5)$$

where P is the distance from the object to the first principal point of the lens, Q is the distance to the image from the 2nd principal point of the lens, and f is the focal length of the lens.

We can safely apply this to the oblique object plane situation if we work along the axis of the lens.[6]

Note that the distance f appears in the geometry as prescribed by the definition of Scheimpflug's fourth principle.

[6] It applies at any angle if the lens is not corrected for flatness of field, but if it isn't, the surface of perfect object focus is not a plane, so we'd better not go there!

First we will solve equation 5 for P to most readily allow later comparison with the rewriting of Scheimpflug's fourth principle:

$$\frac{1}{P} = \frac{1}{f} - \frac{1}{Q} \tag{6}$$

$$\frac{1}{P} = \frac{Q - f}{fQ} \tag{7}$$

$$P = \frac{Qf}{Q - f} \tag{8}$$

On the figure, because of the parallelism of the various planes under the definitions of the two Scheimpflug principles, the two angles marked *t* are equal.

Then from basic trigonometry, we can write

$$J = \frac{Q}{\tan t} \quad \text{and} \quad K = \frac{f}{\tan t} \tag{9}$$

Now, by similar triangles:

$$\frac{J}{P} = \frac{K}{P - f} \tag{10}$$

Solving for P, we get:

$$P = \frac{fJ}{J - K} \tag{11}$$

Substituting from Equation 9, we get:

$$P = \frac{f\left(\frac{Q}{\tan t}\right)}{\left(\frac{Q}{\tan t}\right) - \left(\frac{f}{\tan t}\right)} \tag{12}$$

Multiplying numerator and denominator by tan *t*, we get:

$$P = \frac{Qf}{Q - f} \tag{13}$$

Quod erat demonstrandum.

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