

Describing the Potency of Light

Douglas A. Kerr, P.E. (Ret.)

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INTRODUCTION

In many types of technical work it is necessary to quantify the “potency”¹ of light. The matter is complicated by the fact that there are many distinct circumstances in which the potency of light is a consideration, each having its own physical concept, dimensionality, and units of measure.

In this article we describe these circumstances and the way in which the potency of light is quantified for each.

BACKGROUND

In the discussions of quantifying the “potency” of light, several crucial concepts are encountered with which the reader may not be familiar. In this section, we will give some insight into these crucial concepts.

Dimensionality

Dimensionality² is the property of a physical quantity that shows how it relates to the seven fundamental physical quantities: length, mass, time, electric current, thermodynamic temperature, amount of substance,³ and luminous intensity. Any physical quantity can be described in terms of one or more of these seven quantities.

¹ I use the rather vague term “potency” here rather than, for example, “intensity” or “brightness”, since all those terms have specific meanings applicable to only one of the various “circumstances” in which we may encounter the need for light measurement and description, as are in fact described in this article.

² Often called by mathematicians “dimension”. Since that word has a quite different common meaning, we will use the admittedly-more-clumsy term *dimensionality* here to avoid any misunderstanding.

³ This is the property that relates to the notion, in chemistry, of “one mole” of a substance. It is related to the matter of molecular weight.

For example:

- the width of an object is a quantity with dimensionality *length*
- the area of a two-dimensional geometric figure is a quantity with dimensionality *length squared*
- the velocity of a moving object is a quantity with dimensionality *length per (unit⁴) time*.

We can write the dimensionality of a quantity in an algebraic form. For example:

- We can write the dimensionality of velocity, “length per (unit) time”, this way: l/t , where l represents length, t represents time, and the word “per” in the verbal description is represented by the division sign (“/”).
- We can write the dimensionality of area, “length squared”, as l^2 . We can write the dimensionality of electric charge, “current-time”⁵, this way: It , where I represents current and t represents time. However, in the symbolic form we often use a dot to represent multiplication, as for example $I \cdot t$, in order to avoid any impression that the adjacent symbols form a word.⁶

Now consider the quantity “number of eggs in this box”. It does not work in terms of any of the seven fundamental physical quantities. It just works in terms of a number (sometimes called a “counting number” to emphasize this). Such a quantity is said to be *dimensionless*.

Units

The unit in which a quantity is described must have a dimensionality consistent with the dimensionality of the quantity. There are of course many different units used for any given quantity. Length may be reckoned in units of inch, foot, yard, meter, fathom, furlong, or many others.

⁴ The word “unit” is often included after “per” for clarity, but is not needed for the technical definition. When we get into more complicated expressions of dimensionality, we often eliminate the word “unit”.

⁵ Meaning “current times time”.

⁶ This is in fact the preferred convention in scientific writing.

In engineering and scientific work, it is preferred to use only the units of the International System of Units (SI, from the first two initials of its name in French), the “modern metric system”. Other units, however, are frequently encountered, and we will introduce the principal non-SI unit(s) for the various quantities as we go.

Specifics of conversion between the SI units and common non-SI units for photometric quantities are given in Appendix B.

Units for derived quantities are formed from one or more of the basic units. These compound units can be written with the symbols for the units in the same algebraic form as that in which we represented dimensionality. In the case of units, we do not commonly use the dot to indicate multiplication, but rather a hyphen, in both verbal and symbolic forms: electric charge is measured in the SI unit *coulomb*, which corresponds to the *ampere-second*, or A-s.

The quantity *angle* is an interesting case. Angle does not correspond to any of the seven fundamental physical quantities. In fact, angle can be thought of as working by counting “revolutions”. Thus angle is a dimensionless quantity.

Angle is nevertheless not unitless. Common units for measuring angle are the *revolution*, the *degree* (1/360 of a revolution) and the *radian* ($\frac{1}{2\pi}$ revolution). All are “dimensionless units”⁷.

The unit *radian* is defined thus: if we construct two lines outward from the center of a circle of radius r , and the length of the circumference which they embrace is equal to r , then the angle between them is one radian.

Solid angle

If we look through the viewfinder of a camera, we are able to see a certain amount of “space”. We could describe that in terms of two angles: perhaps the amount of space we can see is 30 degrees horizontally and 20 degrees vertically. But how do we describe the entire amount of space we can see (independent of its shape)? The quantity we use is *solid angle*.

⁷ *Dozen* is in fact a dimensionless unit.

More generally, if we have a cone or pyramid of infinite “extent”, how do we describe “how fat it is inside at the apex”? We do that in terms of the included *solid angle* of its apex.

Solid angle is measured in terms of the unit *steradian*.⁸ If we construct a cone or pyramid with its apex at the center of a sphere of radius r , and the region of the sphere surface embraced by the cone or pyramid has an area r^2 , then the included solid angle of the apex of the cone or pyramid is one steradian.

A solid angle is often represented by the symbol ω (lower-case Greek letter *omega*).

RADIOMETRY AND PHOTOMETRY

Measurement and description of the “potency” of electromagnetic radiation (in several situations) constitutes the field of *radiometry*. Light is in fact electromagnetic radiation, and thus the concepts of radiometry apply to it just as to other forms of electromagnetic radiation—at least when we are not interested in the potency of the light from the perspective of human perception, but only with respect to the laws of physics.

When we are interested in the potency of light from the perspective of human perception, however, we enter the parallel field of *photometry*.

The quantities and units of photometry parallel those of radiometry, with the important difference that photometric quantities reflect the differing “sensitivity” of the human eye at different wavelengths. At any given wavelength, there is a standardized relationship between the corresponding radiometric and photometric quantities.

PHOTOMETRIC QUANTITIES AND UNITS

There are several different circumstances to which the general notion of the potency of light apply. Each has its own dimensionality and its own SI unit.

We will in many cases also mention the most-common of the non-SI units for the quantity.

⁸ The particle *ster* comes from the Greek and means “solid” (The term “stereophonic” was based on the concept that this referred to “solid sound”).

Luminous flux

Luminous flux is the quantity that characterizes the total “potency” of a “body of light”⁹. That body of light could be, for example, the total luminous output of a lamp, or the portion of that total luminous output that passes out through a nearby window, or the total amount of sunlight falling on a region of specified area.

The SI unit of luminous flux is the *lumen* (lm). The lumen is not one of the fundamental SI units. (Logically it should be, since it is at the bottom of the photometric food chain—see a further discussion of this under *Luminous intensity*.)

Luminous flux is parallel to the concept of *power* in radiometry, and has the same dimensionality.

At any given wavelength, there is a specific relationship between the luminous flux of a body of light (in lumens) and the power in the body of light (in watts). In fact, the modern definition of the lumen is (indirectly) based on that relationship at a wavelength of 555 nm, where 1 lumen is equivalent to 1/683 watt.¹⁰

A common non-SI unit of luminous flux is the *spherical candlepower*.¹¹

Luminous flux is often represented by the symbol Φ (upper-case Greek letter *phi*).

Luminous intensity

The potency of light emission in a particular direction from an emitter of very small size (from the perspective of the viewer)—a “point source”—is the *luminous intensity* in that direction. Its dimensionality is *luminous flux per (unit) solid angle*. It is in effect the solid-angular density of luminous flux. Luminous intensity is one of the seven fundamental physical quantities (but see below).

⁹ The word *flux* means “flow”.

¹⁰ This actually occurs through the definition of the *candela*, the unit we will encounter next.

¹¹ A hypothetical lamp exhibiting a *luminous intensity* (see the next section) of one *candlepower* in every direction is said to have a total luminous output of one *spherical candlepower*.

The SI unit of luminous intensity is the *candela* (cd). The candela corresponds to *one lumen per steradian* (lm/sr). The candela is one of the fundamental SI units.

Luminous intensity is parallel to the concept of *radiation intensity* in radiometry, often encountered in work on radio propagation.

It may at first seem peculiar that solid angle is involved in the concept of luminous intensity. Couldn't we just speak of the amount of luminous flux emitted in a certain direction? In fact, **no** flux flows in a particular direction—that is, along any particular line from the source¹². To have flux, we must have something for it to flow through, a non-zero solid angle. (It can be as small as we wish to contemplate, but not of zero size.)

Thus the indicator of the potency of emission (in a particular direction) is the ratio of (a) the luminous flux flowing through a certain solid angle to (b) the solid angle, in the limit as the solid angle approaches zero, where the solid angle we think of is centered on the direction of interest.

Note that the concept of *luminous intensity* does not involve distance from the emitter. It describes the emission, not the effect the emission produces at some distant point. (It of course influences the effect produced at a distant point, as we'll see shortly under *illuminance*.)

A common non-SI unit of luminous intensity is the *candlepower* (cp) (sometimes called *beam candlepower* to distinguish it from *spherical candlepower*, which is a unit of luminous flux). For all practical purposes, the *candlepower* is numerically equivalent to the *candela*.

Luminous intensity is one of the seven fundamental physical quantities identified under the SI. It really shouldn't be since (at the present time) it is directly relatable to power and thus to a combination of certain other fundamental physical quantities. But before 1979, it had an independent definition in terms of a certain physical phenomenon, qualifying it to be a fundamental quantity. Today it keeps that status out of "seniority".

As a fundamental physical quantity, it isn't the best choice among the photometric quantities. *Luminous flux* would have been a more logical choice, since it is at the bottom of the "photometric food chain".

¹² Recall that a line is infinitely thin, and as such does not constitute a "conduit" for luminous flux any more than an infinitely-thin pipe could convey any water.

However, when the fundamental quantities were originally being selected, it was much more practical to measure luminous intensity than luminous flux, so *luminous intensity* got the mantle out of pragmatism.

Luminous intensity is often represented by the symbol I .

Luminance

Luminance tells us the “brightness” of the light emission from a source whose size (from the perspective of the viewer) is not insignificant—an “extended source”, as seen from a certain direction. Its dimensionality is *luminous intensity per unit area*, or *luminous flux per unit solid angle per unit area*. Note that the area of concern here relates to the “projected” area as seen from the direction in which we are defining the luminance, not necessarily the actual surface area involved.

In terms of the SI, the unit of luminance is the *candela per square meter*¹³ (cd/m^2) The unit can also be expressed as the *lumen per steradian per square meter* ($\text{lm}/\text{sr}\cdot\text{m}^2$).

We can perhaps best understand the concept and dimensionality of *luminance* by imagining that the emitting surface is populated with a very large number of point sources, each emitting with a certain *luminous intensity* (in terms of *lumens per steradian*) in some direction of interest. The brightness of the surface, as seen by a human observer from that direction, is proportional to both the luminous intensity of these point sources (in that direction) and to how tightly-packed they are—how many of them there are per square meter of the surface (again, per square meter of the projected area as seen from the direction of the viewer.)

Luminance is often called “brightness”.¹⁴

In radiometry we are rarely concerned with emission from an extended source, and so the radiometric concept which is parallel to luminance (*radiance*) is not often encountered.

¹³ This compound unit does have its own name, the *nit*, but that name is not actually a part of the SI and is not too widely used.

¹⁴ However, in some work “brightness” is used to describe the human perceptual response to luminance, which is not proportional to luminance.

Photographic light meters of the “reflected light” type measure the *luminance* of the scene (often, the average luminance over the field of view of the meter).

A common non-SI unit of luminance is the *foot-lambert*¹⁵.

Luminance is often represented by the symbol *L*.

Luminous exitance

Luminous exitance describes the total amount of luminous flux per unit area leaving an extended surface. It does not relate to observation from any particular direction.

The SI unit of luminous exitance is the *lumen per square meter*.

The quantity luminous exitance is perhaps the least-frequently encountered of all the “potency of light” quantities. It is most often encountered as an intermediate result in deriving the relationships among other “potency of light” quantities.

Luminous flux density

Luminous flux density tells us the amount of luminous flux per unit area as light crosses some plane in space perpendicular to the direction of flow. Its dimensionality is *luminous flux per unit area*.

The SI unit of luminous flux density is the *lux*, which corresponds to the *lumen per square meter*.

Luminous flux density is not spoken of frequently in photometric work, although the quantity often appears as an unnamed intermediate result in deriving the formulas for other quantities. It is parallel to the concept of power flux density (PFD) encountered in the general study of electromagnetic radiation, especially in matters relating to radio propagation.

Illuminance

Illuminance tells us the “potency” of light falling on a surface. As with luminous flux density, its dimensionality is *luminous flux per unit area*. In this case, the concept of “unit area” pertains to the actual area of

¹⁵ Despite its appearance, this is not a compound unit, the product of *foot* and *lambert*. Rather the word “foot” distinguishes this unit, which corresponds to $1/\pi$ candela per square foot, from the *lambert*, which corresponds to $1/\pi$ candela per square cm.

the surface—not, for example, to the area as projected in the direction from which the light might be coming.

If the direction of the arriving light is normal (perpendicular) to the surface, then the illuminance is the same as the luminous flux density of the light as it arrives at the surface. However, light arriving from a direction other than normal produces a smaller illuminance than the luminous flux density of the light—the luminous flux density times the cosine of the angle of incidence (measured from the normal: the direction perpendicular to the surface).

The SI unit of illuminance is the *lux*, which corresponds to *one lumen per square meter*.

The illuminance created on a surface at a distant location from the emission from a point source is proportional to the luminous intensity of the source in the direction toward the location, inversely proportional to the square of the distance from the source to the location, and of course proportional to the cosine of the angle of incidence (one form of the “inverse square law”).

Photographic light meters of the “incident light” type measure *illuminance*.

Illuminance is sometimes called “illumination”.

A common non-SI unit of illuminance is the *footcandle*, which corresponds to one lumen per square foot.

The *luminance* (brightness) of an object, in most cases of interest, is proportional to the total *illuminance* it receives from light sources and also to its *reflectance*. (Information on reflection is found in Appendix A.)

The parallel concept in radiometry is rarely encountered.

Illuminance is often represented by the symbol *E*. (Mnemonic: think “E-luminance.”)

Photometric energy

Not, strictly speaking, one of the concepts of the “potency” of light, but important nevertheless is *photometric energy*. It is the photometric parallel to *energy* in radiometry. Its dimensionality is *luminous flux times time*. Its SI unit is the *lumen-second* (lm-s).

Photometric energy is often represented by the symbol *Q*.

Exposure

In photography, the quantity *photometric energy per unit area*, or *illuminance times time*, determines the effect of the light on photographic film. As such, it is often called “exposure” (although, unfortunately, that term is also used with another meaning as well in the field of photography.¹⁶ Its SI unit is the *lumen-second per square meter* ($\text{lm}\cdot\text{s}/\text{m}^2$) or *lux-second*.

Exposure is often denoted with the symbol E (as in the famous “Hurter and Driffield” curve portraying the response of photographic film, the “D [vs.] log E” curve.¹⁷

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¹⁶ That other meaning is the combination of exposure time (shutter speed) and effective relative aperture (“f/number”). Its dimensionality is *time*, and its unit is the *second*. (Relative aperture is dimensionless and unitless.)

¹⁷ Sometimes the symbol H is used for exposure, avoiding the conflict with E for illuminance. The reason is that some people thought that the “H & D” (Hurter and Driffield) curve must be a plot of D vs. H (where D was density). When the scientists needed a better symbol for exposure, they just legitimized the myth!

APPENDIX A

Reflection

Introduction

Most objects we observe do not emit light but rather reflect light from another source, such as the sun or a lamp.

Many situations in which we are concerned with describing the potency of light relate to the effects of reflection.

Kinds of reflection

Surfaces may exhibit two kinds of reflection. *Specular reflection* is the reflection afforded by a mirror. A light ray striking a specular reflecting surface is reflected as a ray. The angle at which it arrives and the angle at which it leaves (both measured with respect to a line from the point of "impact" and perpendicular to the surface) have the same magnitude but are opposite in direction.

Diffuse reflection is the reflection afforded by most surfaces we encounter. In diffuse reflection, when light strikes a surface, the reflected light departs in every direction on the same side of the plane.

Many real surfaces exhibit "mixed" reflection, a combination of the specular and diffuse types. A piece of fairly shiny metal, or a refrigerator with a glossy finish, gives reflection of this type.

An "ideal" diffuse surface obeys what is called *Lambert's law*. Three of its important properties are:

1. The distribution of luminous intensity of the reflected light (for any given exit angle) is not affected by the angle from which the incident illumination comes¹⁸,
2. If we consider any tiny region of the surface (which we can treat as a point source), the luminous intensity from it in any direction is proportional to the cosine of the angle between that direction and a line from the point perpendicular to the surface. [This isn't really of much importance, but it leads to property 3.]

¹⁸ Of course, the actual value of the illuminance (for any given luminous flux density of the arriving beam) is affected by the angle of incidence.

3. The luminance of any part of the surface is constant regardless of the direction from which the surface is observed

In view of property 2, we might expect that the luminance (brightness) of an illuminated “Lambertian” surface would vary with the angle from which we view the surface. However it doesn’t—it is the same for any angle of view (as noted in property 3).

The reason has to do with the concept of *projected area*. If we observe a certain region on a surface not “head on” but rather from an angle, the area of the region will appear smaller. The decrease in apparent area goes as the cosine of the angle, measured from “head on”, at which we view the surface.

Now let us consider our illuminated surface under the model discussed above in connection with luminance: the surface in effect harbors an enormous number of tiny “point sources”, each emitting with the same luminous intensity in any given direction. The luminance of the surface is the product of that luminous intensity and the area density of these point sources (the number per unit area)—**area as we see it**.

Now, for our Lambertian surface, as we observe it from an increasing angle, the luminous intensity for each point source decreases as the cosine of the angle of view (per point 2, above). But the area density of the point sources increases, since there are the same number of those sources in what we see as a smaller area; the increase is inversely proportional to the cosine of the angle of view. These two effects cancel out, and thus the luminance is the same from any direction of view.

A Lambertian surface will not necessarily reflect all the luminous flux incident on it, and it will not necessarily reflect the same fraction at all wavelengths.¹⁹ The fraction of light reflected by a Lambertian surface, taking the differing response of the eye at different wavelengths into account, is called the *reflectance* of the surface²⁰ and is represented by the symbol ρ (lower-case Greek letter *rho*). (Sometimes R is used.)

If we illuminate a Lambertian surface of reflectance ρ with light having illuminance E (in lux), the reflected light will have luminance L (in cd/m^2), as follows:

¹⁹ Gray paint, for example, reflects far less than all the light incident on it, and red paint does not reflect the same fraction of the light for all wavelengths.

²⁰ Also called its *albedo*, especially if we are speaking of astronomical objects, such as planets and their satellites.

$$L = \frac{1}{\pi} \rho E$$

This relationship is derived by integrating the luminance over the entire hemisphere above the surface (embracing all directions in which the light can be reflected), and equating it to the total reflected light per unit area (the *luminous exitance*). The pi (π) gets into the deal in the integration process.

If we do this for the most common non-SI units, the *foot-candle* for illuminance (illumination), and the *foot-lambert* for luminance (brightness), the relationship becomes:

$$L = \rho E$$

Why is there no π here? Because the definition of the foot-lambert has the $1/\pi$ built in to make this important equation simpler!

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APPENDIX B

Photometric units and conversions

Use and notation

To convert a quantity from the unit on the left to that on the right, multiply by the constant just to the right of the equals sign. To convert a quantity from the unit on the right to that on the left, divide by that constant.

The preferred SI unit is shown in bold. Its symbol is shown in parentheses.

In square brackets after the right-hand unit is the basis of the constant.

Alternate, equivalent names for the unit are shown below the preferred name.

Luminous flux, Φ (upper case Greek *phi*) or F

Dimensionality: From a physics perspective, luminous flux is at the bottom of the food chain, and can be considered a fundamental quantity. (It is not, however, the fundamental photometric quantity of the SI, which is the *candela*, for historical reasons.)

The SI unit is the **lumen**. We can think of the lumen as being $1/4\pi$ the overall luminous flux emitted by today's equivalent of the old "standard candle".

$$1.0 \text{ spherical candlepower} = \mathbf{12.57 \text{ lumen (lm)}} \quad [4\pi]$$

Luminous intensity, I

Dimensionality: luminous flux per unit solid angle ($d\phi/d\omega$).

The SI unit is the **candela**, defined as one lumen/steradian. It is the modern "candlepower".

$$1.0 \text{ candlepower} = \mathbf{1.0 \text{ candela (cd)}} \\ \text{lumen/steradian}$$

Illuminance, E

Dimensionality: luminous flux per unit area ($d\phi/da$).

The SI unit is the **lux**, defined as one lumen/m².

$$1.0 \text{ footcandle} = \mathbf{10.76 \text{ lux (lx)}} \quad [1.0 \text{ m}^2 = 10.76 \text{ ft}^2]$$

$$\text{lumen/ft}^2 \quad \text{lumen/m}^2 \text{ (lm/m}^2\text{)}$$

$$\text{meter-candela}$$

$$1.0 \text{ lumen/cm}^2 = \mathbf{1.0 \times 10^{-4} \text{ lux}} \quad (1.0 \text{ m}^2 = 10^4 \text{ cm}^2)$$

Note that the quantity *photometric flux density* is expressed in the same units as illuminance.

Luminance, L

Dimensionality: luminous intensity per unit area (dI/da); luminous flux per unit solid angle per unit area.

The SI unit is the **candela/m²**. A non-SI synonym is "nit".

$$1.0 \text{ candela/ft}^2 = \mathbf{10.76 \text{ candela/m}^2 \text{ (cd/m}^2\text{)}} \quad [1.0 \text{ m}^2 = 10.76 \text{ ft}^2]$$

$$\text{(nit)}$$

$$1.0 \text{ footlambert} = \mathbf{3.426 \text{ candela/m}^2 \text{ (cd/m}^2\text{)}} \quad [10.76/\pi]$$

$$1.0 \text{ lambert} = \mathbf{3183 \text{ candela/m}^2 \text{ (cd/m}^2\text{)}} \quad [10^4/\pi]$$

$$\text{(cm-lambert)}$$

$$= \mathbf{0.3183 \text{ candela/cm}^2 \text{ (cd/cm}^2\text{)}} \quad [1/\pi] \quad \text{(not an SI unit;}$$

$$\text{cited only to explain the basis of the alternate name "cm-lambert")}$$

$$1.0 \text{ m-lambert} = \mathbf{0.3183 \text{ candela/m}^2 \text{ (cd/m}^2\text{)}} \quad [1/\pi]$$

Note: The appearance of the $1/\pi$ in the conversion constants for the various "lambert" units of luminance reflects that those units were originally defined so as to avoid the appearance of π in the equation relating illuminance and luminance. (See below under Reflectance.)

Reflectance (ρ)

The reflectance, ρ (lower case Greek *rho*), of a diffuse reflecting surface tells the fraction of the incident light which is reflected.

If a "Lambertian" diffuse reflecting surface with a reflectance of ρ receives illuminance E , its luminance, L , will be the same for any direction of observation (above the surface, of course), and is given by:

For E in foot-candles and L in foot-lamberts:

$$L = \rho E$$

For E in lux and L in candelas/m²:

$$L = \rho E / \pi$$

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