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Plus and minus cylinder notation in ophthalmology and optometry

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ABSTRACT AND INTRODUCTION

The prescription for a corrective eyeglass lens of the most common form describes a composite lens, one component of which is a lens whose refractive effect is uniform in all directions (called, in the vision care fields, a "sphere" lens), and the other component of which is a lens whose refractive effect is only in a certain direction (called a "cylinder" lens). We state the refractive power of these two hypothetical component lenses and, for the cylinder lens, the angular direction of its axis (its refractive power is at 90° to its axis).

The same corrective lens can be described by (a) a sphere lens of a certain refractive power, along with a cylinder lens of a certain "plus" (positive) power, with a certain axis direction, or (b) a sphere lens of a different power, along with a cylinder lens of the same power as before but of the "minus" (negative) sign, with its axis direction 90° from that before.

In writing a prescription that will describe a certain corrective lens an ophthalmologist will invariably write it describing the lens with the cylinder component in terms of a plus power. But an optometrist will invariably write it describing the lens with the cylinder component in terms of a minus power).

The story behind this odd situation is fascinating and complex, and is described in this article in considerable detail.

1 CAVEAT

I am not an eye care professional, nor do I have any formal training in the practice in that field nor in its own unique branch of optical science. The information in this article is my own interpretation of the results of extensive (mostly quite recent) research into the available literature, through the prism of my own scientific and engineering background and outlook.

2 BACKGROUND

2.1 Accommodation

The term *accommodation* is used in the field of vision science to refer to the eye's ability to focus on objects at different distances. Ophthalmic lenses, as found in eyeglasses and contact lenses, are in part intended to overcome deficiencies in the eye that prevent the person from fully utilizing that capability.

In a "completely normal" eye, when the accommodation mechanism is relaxed, focus is (ideally) at infinity, or at least, at a great distance. As accommodation is exerted, the focus distance moves closer to the eye. The nearest focus distance usually increases with age. For a 30-year old, a near focus distance of 11 cm from the front of the eye is often considered "normal"

2.2 Refractive errors of vision

The classical vision "defects" (often described as "refractive errors") are:

<u>Hyperopia</u> ("far-sightedness") is the deficiency in which the total range of accommodation is "offset outward", such that distant objects (even at "infinity") can be focused, but the near limit is not nearly as close as is normal. From a theoretical standpoint, the far limit is "beyond infinity", although since there are no objects there that is not of any value to the person. But to the person, the significant effect is that near objects cannot be seen clearly.

<u>Myopia</u> ("near-sightedness") is the deficiency in which the total range of accommodation is "offset inward", such that close objects can be focused on but the far limit is short of infinity. To the person, the effect is that distant objects cannot be seen clearly.

Note that in both these it is assumed that the person still has the normal "span" of accommodation; it has just been shifted from the desirable place (so one "end" is forfeit).

The basic cause of these two defects is that the focal length of the eye's lens system (which comprises two lens elements, the cornea and the "crystalline lens") is not appropriate for the distance from the lens system to the retina. The cornea is most often the principal villain in this.

<u>Presbyopia</u> ("old person's vision") is the deficiency in which the eye is able to make less than "normal" (perhaps no) change in the distance at which it focused".

It may be combined with *hyperopia*, which case the far limit of the range of vision is “beyond infinity”, and the near limit may still be a large distance. Or it may be combined with *myopia*, in which case the far limit may be at a modest distance, and the near limit not much closer.

In “full blown” presbyopia, the eye cannot change its vision distance at all, so the near and far limits become the same (and what distance that is can be considered as a manifestation of myopia or hyperopia).

The basic cause of presbyopia is decline in the effectivity of the eye’s mechanism for changing the focal length of the crystalline lens.

Astigmatism is a refractive defect that is not a flaw in accommodation. It is basically caused by some part of the eye’s lens system (most often the cornea) not having the same power in all directions (from its not having rotational symmetry).

In astigmatism, the eye cannot, in the same “state of accommodation”, focus on a line at a certain distance running in one direction and a line at the same distance running in a different direction.

2.3 Refraction

I in the eye care field, *refraction* refers to the quantification of the vision defects of an eye, or more to the point, developing the “prescription” that is the specification of a corrective lens that, hopeully, will usually to the end of developing a prescription for a corrective lens (eyeglass lens or contact lens).

Today that is most commonly done with a refractor, that scary mask-like instrument through which we look while having a refraction done at an ophthalmologist’s or optometrist’s office. It is essentially a corrective lens simulator. By manipulating the controls, the refractionist can introduce into the line of sight of the eye being examined a sphere lens whose power can be varied over a wide range, and or a cylinder lens, whose power can be varied over a modest range.

2.4 Lenses

We will be concerned here with two types of lens.

2.4.1 *Sphere lens*

In ophthalmic work, a *sphere lens* is any lens that is a figure of revolution, whether or not its surface is actually a portion of a sphere. A sphere lens exhibits the same refractive power along any direction.

A converging lens (which has a plus focal length) has a plus power. A diverging lens (which has a minus focal length) has a minus power.

2.4.2 *Cylinder lens*

In ophthalmic work, a *cylinder lens* is any lens that is a portion of a cylinder (most often a right circular cylinder). It exerts its refractive power in one direction. In the direction at right angles to that (the direction of its axis) it exerts no refractive power.

2.4.3 *Lenses for vision correction*

In the application of eyeglass lenses, a role is played by both sphere and cylinder refractive behavior.

To correct for hyperopia (farsightedness), we provide the effect of a converging lens (net convex, with a plus power) to shift the range of focusing ability "closer". (Photographers do the very same thing with an *auxiliary closeup lens* to allow their cameras to focus at a closer distance than they would otherwise be able to.)

To correct for myopia (nearsightedness), we provide the effect of a diverging lens (net concave, with a minus power) to shift the range of focusing ability "farther".

In astigmatism, the eye's lens has a different refracting power in different directions, the maximum in a certain direction and the minimum in the direction at right angles to that. (The specific direction varies from person to person, from eye to eye.)

We could compensate for that by using a cylindrical lens with a plus power equal to the difference between the eye lens' maximum and minimum power (in a given state of focus), with its cylinder axis aligned with the eye lens' direction of maximum power (so the power of the cylindrical lens adds to the less-than-ideal power of the eye's lens).

Or we would use a cylindrical lens with a minus power equal to the difference between the eye's maximum and minimum power (in a given state of focus), with its cylinder axis aligned with the eye lens' direction of minimum power (so the power of the cylindrical lens subtracts from the greater-than-ideal power of the eye's lens).[1]

That duality is at the heart of the topic of this article.

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[1] Note that the starting spherical power would have to be different in these two cases.

Now, for a person having, for example, both hyperopia and astigmatism, we can visualize a sandwich of two lenses, a spherical lens to shift the focusing range (to overcome the hyperopia), as was just discussed, and a cylindrical lens “trimming out” the difference in the eye lens’ refractive power in different directions (to overcome the astigmatism.

Of course, in reality, we make a single lens that does the same thing as that sandwich. One way (actually used in an earlier era) would be a lens with a spherical surface on one face and a cylindrical surface on the other.

Or we could visualize a lens with one face planar (“plano”) and the other face having a compound curve, with one curvature along a certain direction and a different curvature along the direction at right angles to that.

In fact such a compound curve is found at the surface of a recognized three-dimensional figure, the *torus*. (A doughnut is nominally a torus in shape, as is a “rootbeer barrel”.)

As a result, a lens having both spherical and cylindrical aspects to its refractive power, especially when conferred on one surface, is often referred to as a *toric* lens. This is the approach most often used today.

3 THE PRESCRIPTION

3.1 General

An eyeglass prescription is a specification for the lenses in the glasses. It is done in terms of the model we saw above, in which the overall refractive pattern of the lens is described in terms of the joint effect of two hypothetical lenses, one a sphere lens in this field) and one (only present if there is a correction for astigmatism) a cylinder lens.

3.2 Two views

We learned earlier of the duality of the cylinder lens. And the way the prescription is written can be in two forms, reflecting those two points of view of, hypothetically, how to make the same thing.

We can visualize a certain corrective lens as being, in its behavior, equivalent to the combination of:

- a spherical lens with power +1.00 D, and
- a cylindrical lens with power +0.50 D and axis 30°

which might be written in the prescription as as

+1.00 +0.50 x 30

or we can visualize the corrective lens as being, in its behavior, equivalent to the combination of:

- a spherical lens with power +1.50 D, and
- a cylindrical lens with power –0.50 D and axis 120°

which might be written in the prescription as as

+1.50 -0.50 x 120

Either of those two prescriptions specify the identical corrective lens behavior–the same corrective lens.

3.3 Practice

It turns out that almost invariably a prescription written by an *ophthalmologist* will be in the first of those forms (called the "plus cylinder" form), and a prescription written by an *optometrist* in the second of those forms (called the "minus cylinder" form).

This is in fact the centerpiece of this article.

4 TWO REFRACTION TECHNIQUES

4.1 Introduction

It turns out that in the same way the two forms of the prescription define the same corrective lens, the determination of the ideal lens behavior for corrective the vision of a certain eye during refraction can be done either using only minus power cylinder lenses in the refractor or of only plus power cylinder lenses.

The preference to use one or that other practice, which will be pivotal to the topic of this article, involves two basic approaches that are commonly used when doing a refraction of the eye. So I will discuss those first before proceeding.

I will actually start by describing a concept that figures into both of them.

4.2 Accommodation by the eye

As suggested earlier, the human visual system, faced with an out-of-focus image, tries to attain good focus by *accommodation*; that is, by changing the shape of the *crystalline lens* of the eye, done by way of the *ciliary muscle*, which surrounds the lens capsule.

When refraction is done, with test lenses of different powers being introduced into the subject's line of vision, the eye's attempt to accommodate, by shifting its focus state, can interfere with the maneuvers of the refraction procedure. Thus we must in general in some way disable, or frustrate, the eye's attempt at accommodation.

Now, for the two different approaches to refraction.

4.3 Clycloplegic refraction

One approach is to instill into the eyes a *cycloplegic medication*[2], which seriously disables the ciliary muscle (induces *cycloplegia*), and thus keeps the eye's focus state fixed. That state is with the ciliary muscle relaxed, which puts the lens into its greatest focal length (as for focus on an object at a great distance, ideally and theoretically at an infinite distance). This technique is called *clycloplegic refraction*.

It turns out that, with cycloplegia in effect, the cylinder aspect of the refraction can be essentially equally-well conducted with either plus or minus cylinder lenses.

But, at the time the practice of refraction was being "normalized", another consideration led to a preference for using plus cylinder lenses in cycloplegic refraction.

I will put off explaining that "other consideration" just now so as not to slow down the real story. I will discuss it in section 5.3.

4.4 Manifest refraction

In this technique, clycloplegia is not used. The name suggests that this is the refraction "as seen" (manifest), meaning without changing the eye's behavior (such as with cycloplegia).

Here, we still need to avert interference with the measurement process by the eye's action of accommodation, as it tries to maintain good focus. This is done by a clever ploy, involving what is spoken of as "fogging" the eye.

The process is very complicated to explain. I discuss it in Appendix A.

But the bottom line is that this process works most handily if minus cylinders are used.

4.5 The implications

The use of minus cylinder technique in refraction leads most directly to the use of minus cylinder notation in the prescription, and the use

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[2] This medication is usually also a *mydriatic*—it causes the eye's pupil to dilate (open wide). This has its own advantage in the refraction process.

of plus cylinder technique in refraction leads most directly to the use of plus cylinder notation in the prescription

So we can begin to guess that ophthalmologists for some reason, probably prefer to use cycloplegic refraction, while optometrists, for some reason, prefer to use manifest refraction.

5 WHY THE DIFFERENCE BY PROFESSION?

5.1 The regulatory factor

Above we saw the basic differences between clycloplegic and manifest refraction. And overall, clycloplegic refraction was the easiest to actually do—the "maneuvers" we less tricky than for manifest refraction (even when the latter was done with minus cylinder lenses).

However, under the medical practice laws of the various states (which differed greatly), there were (still are) are certain things that a licensed ophthalmologist (who had to start off with "MD" or "DO" training) can do that a licensed optometrist (whose training, while very extensive, is not that of an MD or DO) may not do.

And in many states, at an earlier time, one of those prohibited things was to prescribe or administer "medication". The definition of "medication" varied from state to state, but in many cases included that which was used to induce cycloplegia.

So, in those states, licensed optometrists, unable to induce cycloplegia, were forced to us the less-convenient manifest form of refraction.

> The licensed optometrist associations in the various states were typically supportive of relieving those restrictions. By now, we should be able to imagine that the licensed ophthalmologist associations in the various states were not in general supportive of relieving those restrictions.

5.2 In optometry

I mentioned above that in manifest refraction, which optometrists in most states were one forced to use, the minus cylinder procedure was handier than the plus cylinder procedure.

And the result of that was that the profession of optometry settled into the use of minus cylinder technique in refraction. And follows that to this day.

5.3 In ophthalmology

But that doesn't tell us why, in the profession of ophthalmology, where the practitioners have essentially always been free to use cycloplegia, the profession settled into the almost exclusive use of plus cylinder technique. Maybe this is the reason.

"In the day", it was generally the view that the prescription was not just a specification for the overall optical performance of the eyeglass lens but in fact a recipe for making it. And in that era, when the eyeglass lens was made, it was common to have a sphere surface (which determined the sphere power) ground into the back surface of the lens, and a cylinder surface (which determined the cylinder power), if applicable, ground into the front of the lens.

And at the time, it was at the time for various reasons considered desirable for this cylinder surface to be convex (a plus cylinder) rather than concave (a minus cylinder). (Remember, either kind can be used to get the desired effect.) For one thing, a concave surface on the front of a an eyeglass lens makes the glasses look really funny.

So, given the view of the time that the prescription was a "recipe" for making the lens, working backward from that it became desirable for the prescriptions to be written in "plus cylinder" form.

And, again working backward, since the prescription notation was most handily derived directly from the refraction procedure, it became most common to use plus cylinders in that process.

And so it became standard practice, in the field of ophthalmology, to use plus cylinder practice in refraction.

Maybe.

In any case, in modern times it was no longer considered that the prescription was a "recipe" for making the lens, but rather a specification for its optical behavior. And various strategies are used to actually make the lens. So that notion for preferring plus cylinder notation, and thus plus cylinder technique, is long obsolete.

But the die was cast for the profession of ophthalmology.

6 IN REFRACTORS

6.1 Introduction

In a refractor, the sphere lens that can be introduced into the line of sight can have its power varied over a wide rage, spanning both minus and plus signs. This does not mean that there are perhaps over one hundred sphere lenses in the refractor, but rather there are perhaps two sets of 12 sphere lenses each (held on two disks), and one lens from each set is put in place to compose a certain power.

The same plan is used for the cylinder lens section of the refractor, with perhaps 5 lenses on each of two disks, the combinations of one lens on each disk giving 25 different cylinder lens powers.

But in most cases, for the individual refractor specimen, its cylinder power can only be with the minus sign or the plus sign. The two different "versions" of a given refractor model are said to be the "minus cylinder" and "plus cylinder" versions.

The reason for this is that, for a refractor used by an optometrist or an ophthalmologist, only one sign of the cylinder lenses would be used. By only supporting powers of one sign or another, the number of individual lenses cylinder needed in the refractor is reduced, with advantages in cost, size, and weight.

For whatever reason, earlier refractors were seemingly all made in only the "minus cylinder" form.

And today, for someone looking to acquire a modern refractor on the used market, they will probably find far more minus cylinder machines offered than plus cylinder machines. Perhaps more optometrists retire, or go out of business, than ophthalmologists.

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Appendix A

"Eye fogging" in manifest refraction

A.1 Introduction

In manifest refraction, we do not use cycloplegia to disable, or at least frustrate, the eye's attempt to accommodate as different cylindrical powers are put in place with the refractor.

Rather, in most practice, a clever ploy is used instead, one that is spoken of as "fogging the eye".

A.2 A graphic convention

As lens system that has astigmatism will not bring the arriving rays of light from a point on the viewed "object" to a point (creating a point image. But it does create two images of that pint, both of them lines, located at different distances from the lens.

In our 2-dimensional drawing, we will show then using a form of the oblique projection, thus:

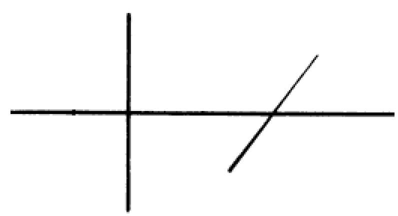

Figure 1. Line images in oblique projection

Now, as we would reduce the degree of astigmatism in that lens (perhaps by neutralizing it with another cylinder lens), we would expect these two line images to move closer to each other, and get shorter, until when all the astigmatism had been neutralized, they coalesce into a point image, thus:

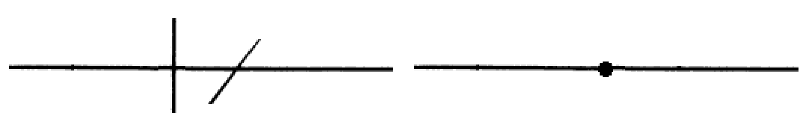

Figure 2. Astigmatism (l) partly neutralized, (r) fully neutralized

The point image is by convention shown with a finite diameter, otherwise we couldn't see it at all in the figure at all.

But for greatest clarity (I hope), I will continue to show the line images with a constant size as they move together and actually get shorter, even as they actually coalesce into a point image:

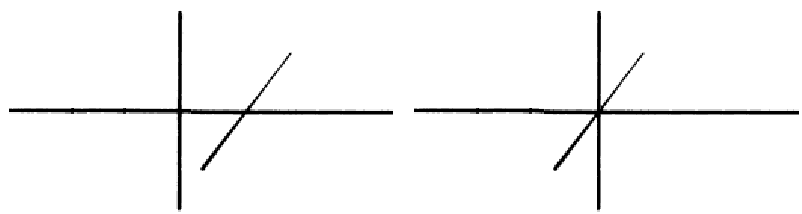

Figure 3. Astigmatism (l) partly (r) fully neutralized; graphic convention

The oblique projections of the two line images of constant length are in effect used as avatars for the actual (variable length) line images.

A.3 The procedure

A.3.1 Introduction

Let's first consider an eye whose lens system has a greater power in one meridian than the opposite one (that is, which has *astigmatism*), and which in addition has a "relaxed" focal length that is too great for the dimensions of the eye (that is, the eye suffers as well from *hyperopia*).

Imagine that this eye is regarding a point source of light at a great distance. Because of the inconsistency of refractive power in the different meridians, rather that the light rays from the point source being converged into a point image at some location, here (theoretically) two line images are formed. one (horizontal) at a location behind the retina, and the other (vertical) at a location farther behind the retina.

We see a fanciful oblique presentation of that in this figure [3].

[3] The figures in this section were adapted from those in a paper by Jay H. Kaufman, and may be original with him. They are used here under the doctrine of fair use.

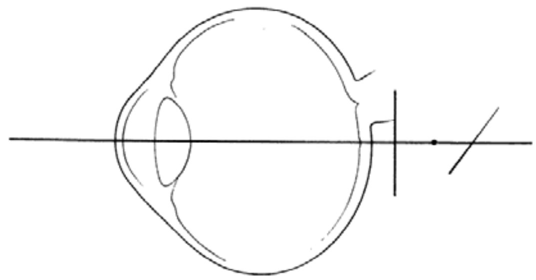

Figure 4.

This is predicated on the refractive power of the eye's lens system being greatest in the horizontal meridian and thus being the least in the vertical meridian.

Of course, no actual images can be formed behind the retina (which is opaque), but just imagine that this problem has been magically relieved for the moment.

A.3.2 "Fogging" the eye

The first step in the refraction process is to put a sphere lens of substantial power in front of the eye. The result is that our two heroes, the two line images, are now created farther forward, perhaps even, theoretically, in front of the eye, like so:

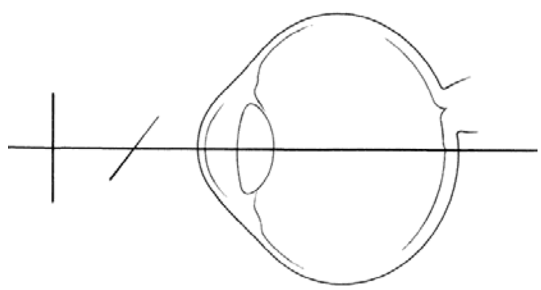

Figure 5.

Of course, these hypothetical images, being located in front of the lens, are "virtual" for the eye, but that doesn't make them of any less value to us at this point. The important thing is that they are so far removed from the retina that the image on the retina is gravely blurred, probably beyond recognition, as if the scene were being viewed through a thick fog. And in fact, this action is spoken of as "fogging" the eye's view.

The accommodation system, unsatisfied with this image not being on the retina, tries to move them back by an increase in the eye lens's focal length. but it runs out of range with the situation still almost as seen just above, maybe like so:

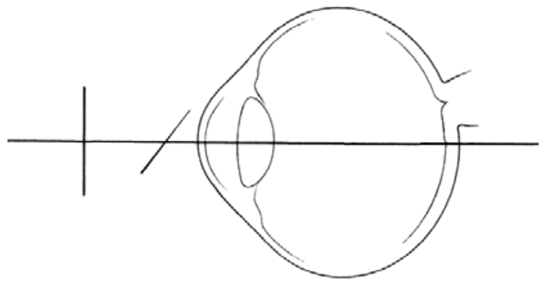

Figure 6.

The subject's view of the test chart is still severely "foggy".

We next reduce the power of the sphere lens, at one point leading to a situation like this:

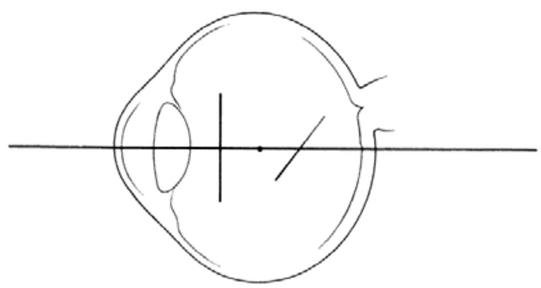

Figure 7.

Perhaps now the subject can begin to see the test chart, although still very badly out of focus. The accommodation system tries to move these images (on the average) onto the retina, but again that would require an increase in the focal length of the eye lens, and it is already at its greatest focal length. The accommodation system is "against the maximum focal length stop". And we want to keep it there.

We continue to decrease the power of the sphere lens until we get a situation like this:

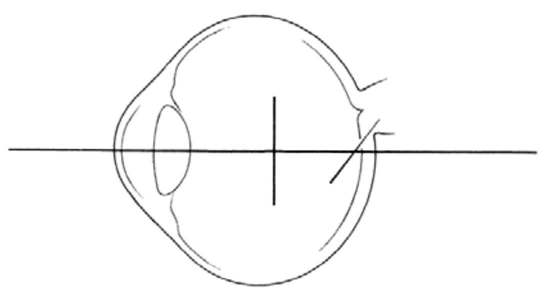

Figure 8.

Note that in reality the two line images now are smaller than in the prior figure. But my convention is to use the graphic presentation of the line images at constant size as the avatar for the entire situation downstream from the lens.

Now the rearmost (horizontal) line image lies almost on the retina (so the horizontal line image on the retina will be only slightly out of focus), but the frontmost line image (vertical) is still quite a way from the retina, so the vertical line image on the retina is substantially out of focus.

And again, the diligent accommodation system attempts to, “on the average”, put the two line images on the retina. But again, that would require an increase in the focal length of the eye lens, and it is already at its greatest focal length.

So the accommodation system remains frustrated, and thus cannot change the focal length of the eye’s lens as the remaining steps of the refraction take place.

Interestingly enough, although at this point both line images can be seen, the horizontal one a bit blurred, but neither as if “seen through a fog”, this situation is still described in optometric jargon as the eye “still being fogged”. That is really a metaphor for, “The accommodation system cannot do what it is inclined to do, which would mess up what we are doing.”

A.3.3 Neutralizing the astigmatism—minus cylinder lenses

Now we begin to neutralize the astigmatism. We put in place a minus cylinder lens of small power with its axis vertical (and thus its power is in the horizontal direction). Its effect is the reduce the discrepancy between the overall power of the lens system between the (in this case) horizontal and vertical meridians. The result is that the axial separation between the two line images is reduced (the vertical line image being moved toward the retina.

Then we perhaps have this:

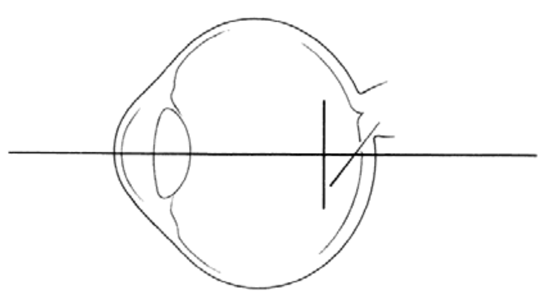

Figure 9.

W continue increasing the magnitude of the minus cylinder power until we have this:

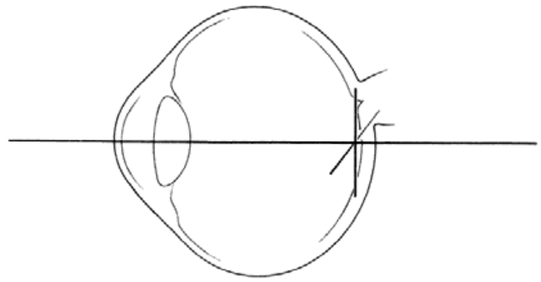

Figure 10.

In reality, the two line images have now shrunk to infinitesimal size, so it is a point image that is created. Still, as we recall, for consistency, I continue to show the two line images, at constant size, as an avatar for what is happening.

Here, the astigmatism is completely neutralized, but the image is not quite on the retina, and thus is still a little out of focus.

So we decrease the sphere power a little until we have this:

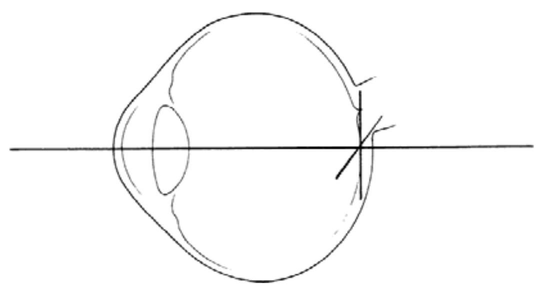

Figure 11.

Again, in reality, it is a point image that now lies on the retina, even though I show our familiar characters, the two line images, at constant size so they can be recognized.

We have neutralized all the refractive errors in the eye, and write down the sphere and cylinder power values we have in the refractor as the prescription for this eye.

A.3.4 Neutralizing the astigmatism—plus cylinder lenses

Now suppose we did this using plus cylinder lenses. We will start when we have again achieved this situation (as described earlier):

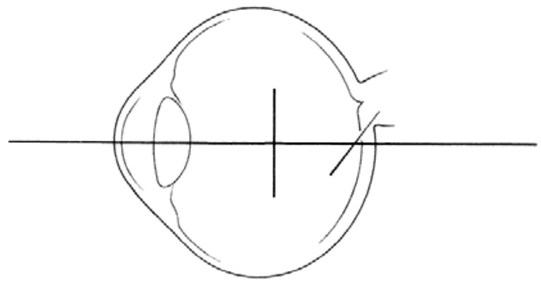

Figure 12.

Now we start to neutralize the astigmatism by putting a plus cylinder lens in front of the eye, with its axis horizontal (and thus its power is in the vertical direction). As before, its effect is the reduce the discrepancy between the overall power of the lens system between the (in this case) horizontal and vertical meridians. In particular, it increases the overall power in the vertical direction. The result is that the axial separation between the two line images is reduced (the horizontal line image being moved away from the retina).

We then might have this:

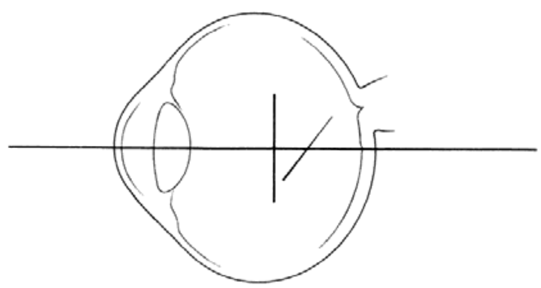

Figure 13.

But now neither of the line images are in very good focus, and it will be hard for the subject to report on changes seen with regard to the effect of the astigmatism. So we must now decrease the power of the sphere lens to again put the rearmost (horizontal) line image near the retina.

As we continue to increase the plus cylinder power toward the value that will completely neutralize the astigmatism, we will again need to decrease the plus sphere power to keep the entire image (here represented by the two line images) in a reasonable state of focus.

A.4 So what of it?

The clear difference between these two scenarios is that, if we neutralize the astigmatism with plus cylinder lenses, we have to keep fiddling with the sphere power, whereas if we neutralize the astigmatism with minus cylinder lenses, we don't. And that is a clear advantage.

Is that easier than using clycloplegia? Maybe so, maybe not.

But to an optometrist who was, at the time, prohibited from using clycloplegia, that was of no importance., The important point was that, using manifest refraction (no clycloplegia), the minus cylinder technique is the more convenient.

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