

# The Proper Pivot Point for Panoramic Photography

Douglas A. Kerr

Issue 3  
February 23, 2008

## ABSTRACT

When doing panoramic photography with a conventional camera, multiple, slightly-overlapping shots of the overall scene are taken by pivoting the camera in steps, and the images are joined to make a single large-scope image. In order to be able to properly join the images, we must avoid parallax shift between them. To do so, the camera must be pivoted about the camera's center of perspective, which turns out to be the center of the entrance pupil of the lens.

It is widely, but incorrectly, said that the proper pivot point is "the nodal point" of the lens.

In this article we discuss the optical principles involved, and demonstrate why the center of the entrance pupil is the proper pivot point.

## MULTIPLE-IMAGE PANORAMIC PHOTOGRAPHY

### Introduction

By using multiple-image technique, panoramic images (generally thought of as images whose field of view, horizontally and/or vertically, is very great) can be produced with conventional cameras.

In this technique, multiple, slightly overlapping images are taken of the scene from the same point by rotating the camera horizontally (and/or vertically) in steps between the shots. The multiple images are then joined by a process which, in effect, involves cropping them to bring corresponding points of the scene to the abutting edges and properly aligning the images. Today, digital image manipulation software is available which automates this process, making multi-image panoramic photography readily available to the photography enthusiast.

In this article, we discuss an important consideration in enabling the successful joining of the multiple images—the selection of the proper point about which the camera must be pivoted between shots.

We begin by reviewing some important optical principles and terms.

### Parallax shift

Suppose that we were to take multiple images for this purpose but did not use a tripod, and in fact were so careless that we moved considerably to the side between one shot and the next. The result would be what is called *parallax shift* between the frames.

Suppose that in our first shot, near the right edge of the image, there was a fence post in the scene fairly near the camera, perfectly aligned with a tree trunk at a greater distance.

Now suppose that, when we took the second frame (for which we assume we rotate the camera to the right), we also inadvertently moved the camera position a bit to the right. Now, the fence post and the tree would both appear on the left side of the image, but the camera would have “seen around” the fence post, and it would appear on the image not aligned with the tree trunk but to the left of it. It is this phenomenon that we describe as *parallax shift*.

If we decide to make the “seam” between these two frames along a line passing just left of the tree trunk, we find that in the resulting composite the fence post appears twice—in front of the tree trunk and also just to the left of it.

To avoid this unsatisfactory result, we must of course take care to keep the left-to-right position of the camera in the same place as we rotate it. But if we are to be precise in doing so, we must ask, “Which exact place on the camera must not move from left to right?” For example, if we pivot the camera to the right about its tripod screw, the front of the lens will move to the right, while the back of the body will move to the left.

The answer is that the camera’s “center of perspective” must not move from left to right—that is, we must pivot the camera about that point. But what is that?

### **The center of perspective**

The *center of perspective* of a camera is the point at which the camera seems to be with respect to the relative alignment of the different elements of the scene. It is in effect the “peephole” through which the camera sees the world. It is where a human observer’s eye would need to be to experience the same perspective as the camera does.

The center of perspective of a camera turns out to be at the center of the *entrance pupil* of the lens (as we will demonstrate later in this article).

### **The entrance pupil**

In a camera lens, there is ordinarily an *aperture stop*, an opaque plate with a generally-round circular opening in it. Typically, the diameter of the hole can be varied, usually with a multiple blade “iris”, so as to “throttle” the incoming light in order to control exposure.

In simple box cameras, the aperture stop is often placed in front of the lens, which itself may have only a single element. In more sophisticated cameras, though, the aperture stop is located inside the lens, perhaps behind the first group of lens elements.

If we look into such a lens from the front, we “see” the aperture stop—but not in its real size or distance from us. In optical terms, we see the “virtual image” of the

aperture stop which is created for us by the lens elements between us and the stop—just as is presented to us by a magnifying glass or a telescope.

This virtual image of the aperture stop is called the “entrance pupil” of the lens.

When we look into the matter of the “collection” of light by the lens from a small patch on the object, it is the area of the entrance pupil—not the area of the physical aperture stop itself—which is of importance. And it is for this reason that the  $f$ /number of the lens, which tells us about the effect of the lens on exposure, is the ratio of the focal length of the lens to the diameter of the entrance pupil—not to the diameter of the physical aperture stop (as is often, but incorrectly, thought).

It should also be apparent, when we use the metaphor that the camera “sees the world through a peephole”, that the entrance pupil is that peephole.

### **About “the nodal point”**

In a large number of publications devoted to photographic technique, it is stated (incorrectly) that the proper pivot point for multiple-image panoramic photography is “the nodal point” of the lens. The discerning student of optics should be immediately suspicious of this since, with the exception of the fanciful “thin lens” used to illustrate optical principles, a camera lens has **two** nodal points.

The nodal points of a lens are defined this way: If we have a light ray that arrives at the front of the lens headed for the first nodal point, then it will emerge from the rear of the lens along a line that starts at the second nodal point and is parallel to the line along which the ray arrives.

Thus a ray that arrives headed toward the first nodal point is not “deflected” (in its angle of travel) by the lens; its path, in effect, just holds up its lateral motion for the longitudinal distance between the nodal points. (For this reason, the distance between the two nodal points is sometimes called the “hiatus distance” of the lens.)

The locations of the nodal points of the lens are not involved in the matter of the elimination of parallax error by the use of the proper pivot point.

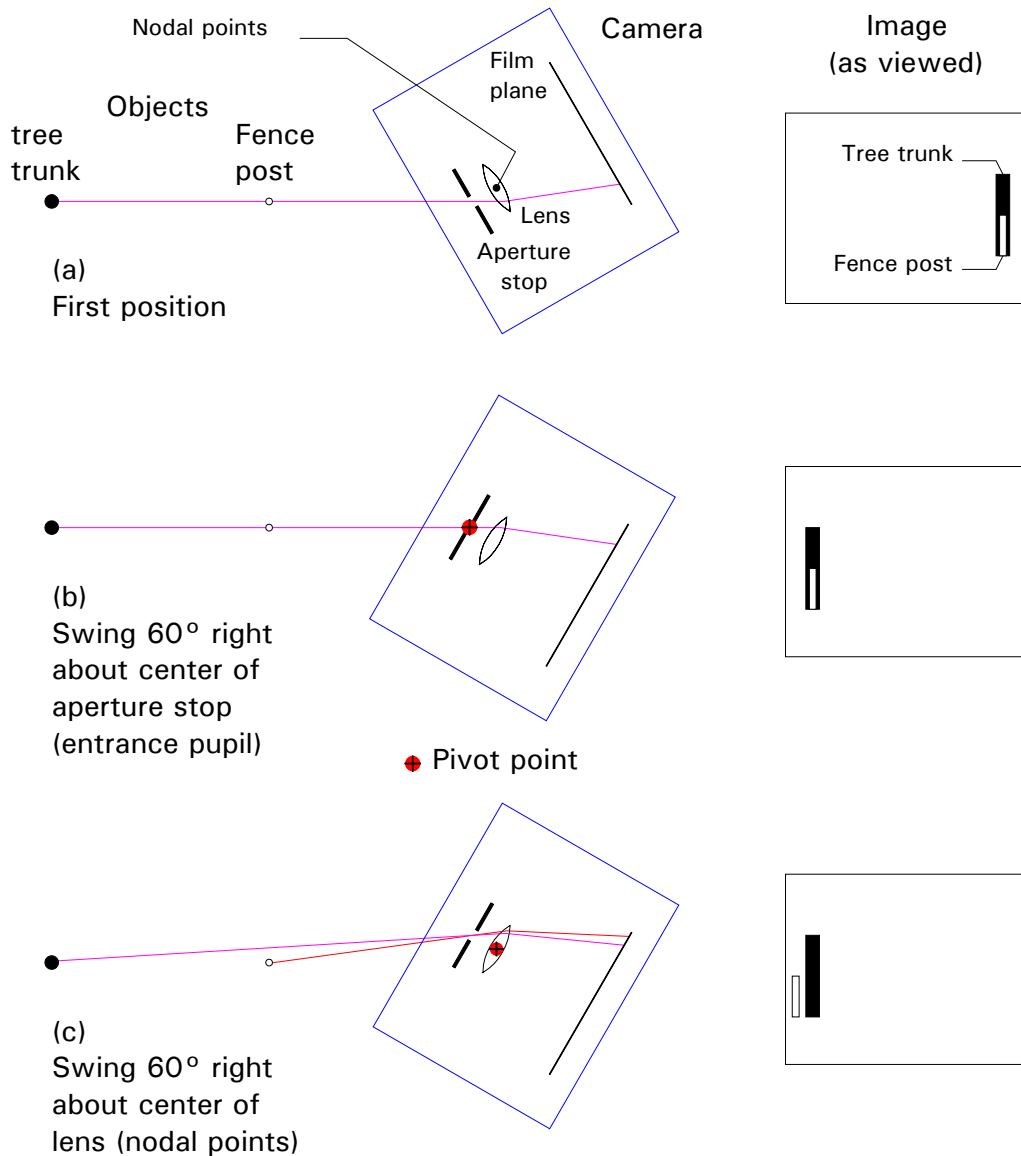
### **The Bottom Line**

The proper pivot point for multi-image panoramic photography is at the center of the entrance pupil of the lens. It is not either nodal point of the lens.

### **A DEMONSTRATION**

Figure 1 allows us to see why the proper pivot point for panoramic photography—the center of perspective of the camera—is located at the entrance pupil of the lens, and not at either nodal point of the lens (depending on the detailed design of the lens, though the entrance pupil may be located at the same place as a nodal point).

To simplify the presentation, and hopefully illuminate the “peephole” metaphor, I assume a simple camera with single-element “thin” lens, whose two nodal points we can treat as if they were both in the very center of the lens, with an aperture stop in front of the lens. In Appendix A, I give a more general demonstration.



**Figure 1. Rotation of camera with “in front of the lens” aperture stop**

Note that with the aperture stop in front of the entire lens, the entrance pupil is identical to the physical aperture stop.

The test scene comprises two objects, a fence post fairly near the camera and a tree trunk at a greater distance. In the illustration, both are shown at unreasonably close distances to make the figure sufficiently compact and to make the phenomenon being discussed more readily visible.

We will assume focus on the distant object<sup>1</sup>, and assume a small aperture, as would probably be used in this situation to assure that the near object is included within the depth of field (that is, so it is also “adequately” focused).

In part (a) of the figure, we see the camera aimed at the scene such that the images of the two objects fall on the right side of the image. (Remember that the image as seen is reversed from that in the camera.) The left-to-right location of the camera is such that the two objects’ images are perfectly aligned.

We trace a single ray from a particular point (at the center of) each object, the “chief” or “principal” ray, which passes through the center of the aperture stop. These rays from the two objects are shown in the same color since in this case they follow the same path.

(Note that since in this situation the chief ray does not pass through the first nodal point of the lens. It is “refracted”, leaving the lens at a smaller angle to the axis than that of its arrival, thus its “bent” appearance. Other ray tracing, not shown on the figure, allowed us to determine the precise amount of this refraction. This refraction is part of the process by which the various rays emanating from a certain place on the object, all passing through the aperture stop, are converged to a focus at a point in the image. (But we are not concerned here with the rest of that process.)

In part (b) of the figure, we have pivoted the camera  $60^\circ$  to the right in order to capture the next frame of our panorama. The camera was pivoted about the center of the entrance pupil (which in this camera is the center of the aperture stop). The images of the two objects now appear at the left side of the image, but still coincide—there has been no parallax shift to spoil the joining of the two images.

In part (c) of the figure, we have fallen victim to the misconception about “the nodal point” of the lens. We have again rotated the camera  $60^\circ$  to the right from its original position, this time about an axis passing through both nodal points of the lens. Here again, the two objects appear on the left side of the image, but this time not coincident—we have a parallax shift, which will spoil the joining of the two images (as discussed earlier in the section on parallax shift).

Thus we see that the proper pivot point for multi-image panoramic photography is the center of the entrance pupil of the lens.

*Quod erat demonstrandum.*

### **The more general case**

It can be demonstrated that this same conclusion emerges if the entrance pupil is not in front of the lens element (and thus coincident with the physical aperture

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<sup>1</sup> Focus issues do not show up here, but that condition is needed as a basis for the precise ray paths shown.

stop) but rather is inside a multi-element lens as the virtual image of the physical aperture stop—the general case for most camera lenses in which we are interested. This case is investigated in Appendix A (with the same conclusion).

But through the case described above, we can intuitively note that the camera, regardless of the lens design, can only see things as they can be seen through the entrance pupil, the camera's only "peephole on the world"—the concept that we really see at work in the figure above.

## **DETERMINING THE LOCATION OF THE ENTRANCE PUPIL**

### **Published tests**

Many publications on panoramic photographic techniques (the same ones that wrongly tell us that the proper pivot point is "the nodal point" of the lens) describe practical tests for finding the location of "the nodal point". Are these tests useless to us, given that we need to know the location of the entrance pupil?

Not at all. That's because most of these tests actually determine the location of the entrance pupil!

### **Two ways to test**

There are two practical ways to determine the location of the entrance pupil in a camera lens.

#### The panoramic slide or panoramic mount

A *panoramic slide* is a device on which the camera is mounted, which is in turn mounted on the tripod. The slide can be adjusted to move the entire camera rig fore and aft until the proper pivot point falls over the tripod axis of horizontal motion. Once this has been properly adjusted, the camera is rotated between frames using the tripod horizontal movement. (This assumes that we are taking a panoramic series in which movement is only horizontal).

A *panoramic mount* is a device on which the camera is mounted, which in turn is mounted on a tripod. The mount itself allows changing the camera aim both horizontally and vertically (without using any of the tripod movements). The mount has provision for adjusting the position of the camera on the mount in two directions so the two axes of movement both pass through the desired pivot point.

With either of these, to test for proper adjustment of the axis (and we will for the moment assume that only horizontal motion is involved) we aim the camera at some scene in which there are near and far object features that overlap, located toward one side of the image. While watching the image in the viewfinder, we rotate the camera to a new aiming position in which these object features will lie on the other side of the image (just as we saw in part (b) of the figure).

If the near and far object features remain in the same alignment, then the location of the pivot axis is already properly set. If we see a parallax shift, then we must change the location of the pivot axis and test again.

In which direction? Assume that we start with the features on the right side of the image and then rotate the camera to the right (moving the features to the left of the image). If the near object's image now has shifted to the left with respect to the far object's image, the pivot axis is too far to the rear on the camera (as we saw in part (c) of the figure.) If the near object's image now has shifted to the right with respect to the far object's image, the pivot axis is too far forward on the camera.

### Visual determination

In most cases, we can make a sufficiently-accurate determination of the location of the entrance pupil by just looking at it. Recall that what appears to us to be the aperture stop when we look into the front of the lens is actually (by definition) the entrance pupil. Of course, what we see is just a "mirage", but that's what the entrance pupil is when the aperture stop is inside the lens—a virtual image. Do not be concerned by the fact that you can't tell from in front of the lens exactly where the physical aperture stop itself is—that's not what we are interested in, just where it looks like it is.

The first task is to be sure which of the various circular outlines we see when peering into the lens is actually the entrance pupil. If the camera provides a way to stop down the aperture, that will of course make it immediately clear which is the correct feature. On a single lens reflex camera with a depth of field preview button, set the aperture to a moderate value and, while looking into the lens, press the button. You will see the entrance pupil get smaller.

Then judge its apparent "depth" from the front of the lens. This is then the location of the entrance pupil.

### **Variation of the location with lens adjustments**

Note that the location of the entrance pupil may change with focal length setting (on a zoom lens) and even with focus setting (depending on the focus system that is used). The change will ordinarily be small, and such small changes should not be a matter of great concern.

### **How accurate do we have to be?**

In reality, unless there are objects that lie quite close to the camera, the parallax shift caused by even a substantial error in selecting the position of the panoramic pivot axis will be negligible.

### **SO WHAT'S WITH THIS NODAL POINT THING, ANYWAY?**

It's hard to reconstruct the original misunderstanding about the proper pivot point for panoramic photography being "the nodal point" of the lens. I suspect it goes back to this:

The laboratory test to determine the location of the nodal points of the lens (any practical lens has two of them) involves swinging the lens (but not the rest of the

“camera”) about a certain axis and observing whether the image of a single point of light at an infinite distance (actually a virtual object created for us by an instrument called an “optical collimator”) moves on the (unmoving) focal plane. We may actually use a special microscope focused on the focal plane to observe whether there is any movement. We adjust the pivot axis until swinging the lens does not make the image move. Then, the axis about which we have just swung the lens passes through the second nodal point of the lens.

To determine the location of the first nodal point, we turn the lens end-for-end so the first nodal point takes the role of the second nodal point, and repeat the same test.

This is quite different than the test to determine the location of the proper pivot point for panoramic photography, in the following respects:

- The lens is swung, but not the whole “camera”—not the “film” or an instrument that takes its place at the focal plane.
- The criterion is not (a) the maintenance of the relative position of the images of two objects, at different distances, on the focal plane, while those images both move, but rather (b) the maintenance of the absolute position of the image of a single object (at infinite distance) on the (fixed) focal plane.

Still, the fact that both tests involve (a) swinging some part of the optical system, and (b) observing something about the effect of that on the image, may have made those who don’t pay too much attention to details get the two tests confused. And this in turn perhaps led to the notion that the proper pivot point for panoramic photography lies at “the nodal point”.

Interestingly enough, when those who say that the pivot point is located at “the nodal point” are reminded that a lens has two nodal points, many will say, “Well, sure—I mean the front one”. But some insist that it is the rear one—again perhaps a result of the confusion over the test, since the test for the nodal point in fact finds the rear one.

But of course it is neither.

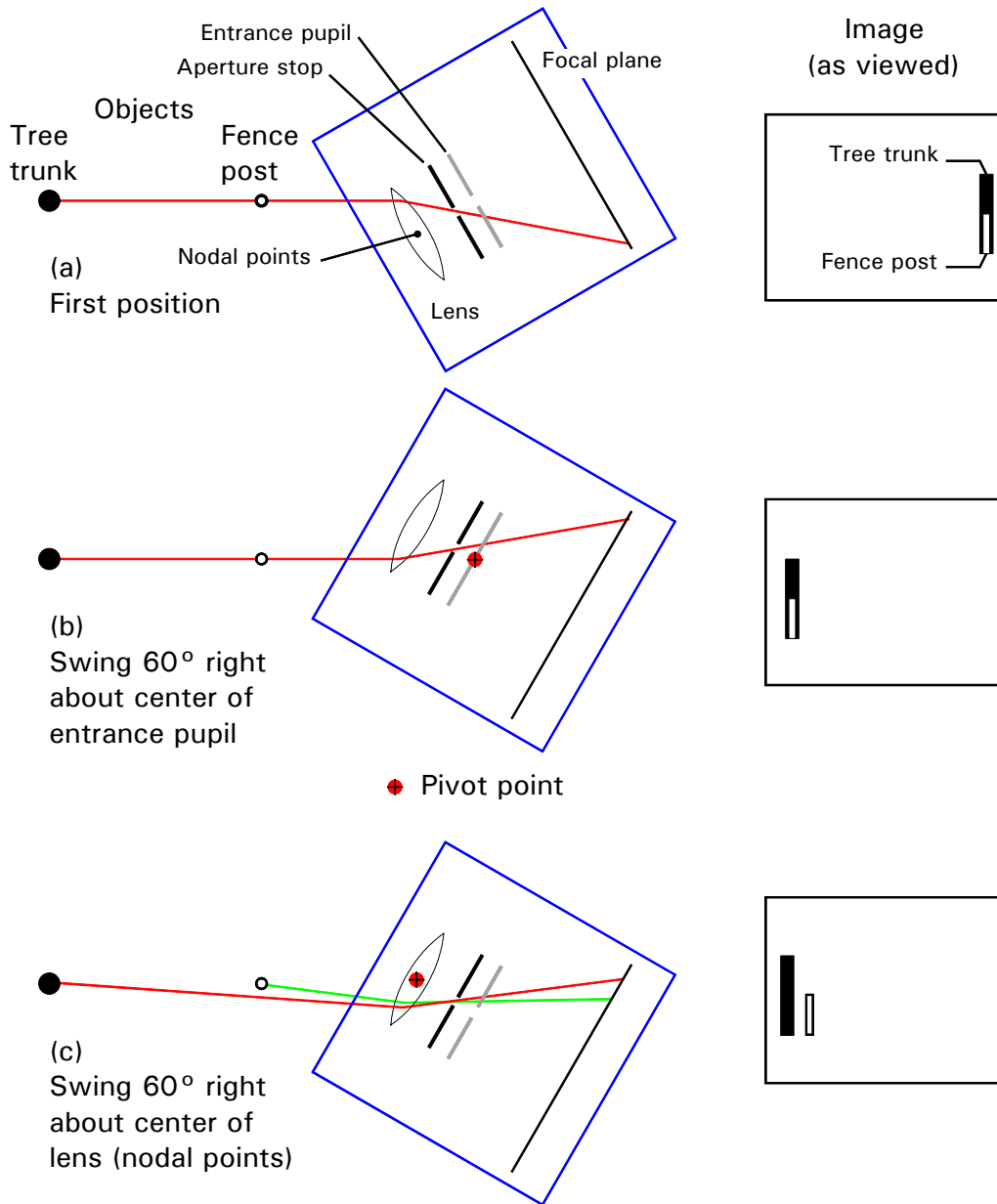
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**APPENDIX A**

**The Hard Way**

For the benefit of those who might think that the model used in the body of the paper, assuming an aperture stop in front of the lens, lacks generality, I will now give the demonstration "the hard way", using a model with the aperture stop behind the lens (again, a simple thin lens), for which situation the entrance pupil is not the same as the aperture stop (in diameter or position).



**Figure 2. Rotation of camera with "behind the lens" aperture stop**

The relationship between the diameters and locations of the aperture stop and the entrance pupil are governed by the famous Gauss focus equation and the resulting relationship for magnification (just as the relationship between any object and its image). They are shown to scale in that regard on the figure.

Before we proceed, let me note that (by definition) any ray which, as it approaches the lens, heads for the center of the entrance pupil, will, as it actually travels through the interior of the lens, pass through the center of the physical aperture stop. (This is the "chief ray".)

We in fact follow the chief rays from a central point of each of the two objects we consider.

In part (a), we see the camera aimed at our familiar test scene such that the images of the two objects fall on the right side of the image, and are aligned.

In part (b), we swing the camera to the right—correctly, about the entrance pupil—until the objects are at the left of the frame. Their images are still aligned in the image—the rays from the two in fact follow the same path. Thus there is no parallax error to spoil the use of these two images to create a panoramic photograph.

Have I cheated and picked rays from the two objects that force this to be so? No—no other significantly-different rays would pass through the aperture stop. The rays I show have been picked for me by the requirements of the laws of optics.

In part (c), we have also swung the camera to the right, but this time about the nodal points of the lens (at the center of our "thin" lens). Now we see that the only rays from the two objects that can pass through the aperture stop follow different paths, and end up in different places on the image. The images of the two objects are no longer aligned in the overall image. Thus there is a parallax error, and the two images cannot be successfully joined into a panoramic photograph.

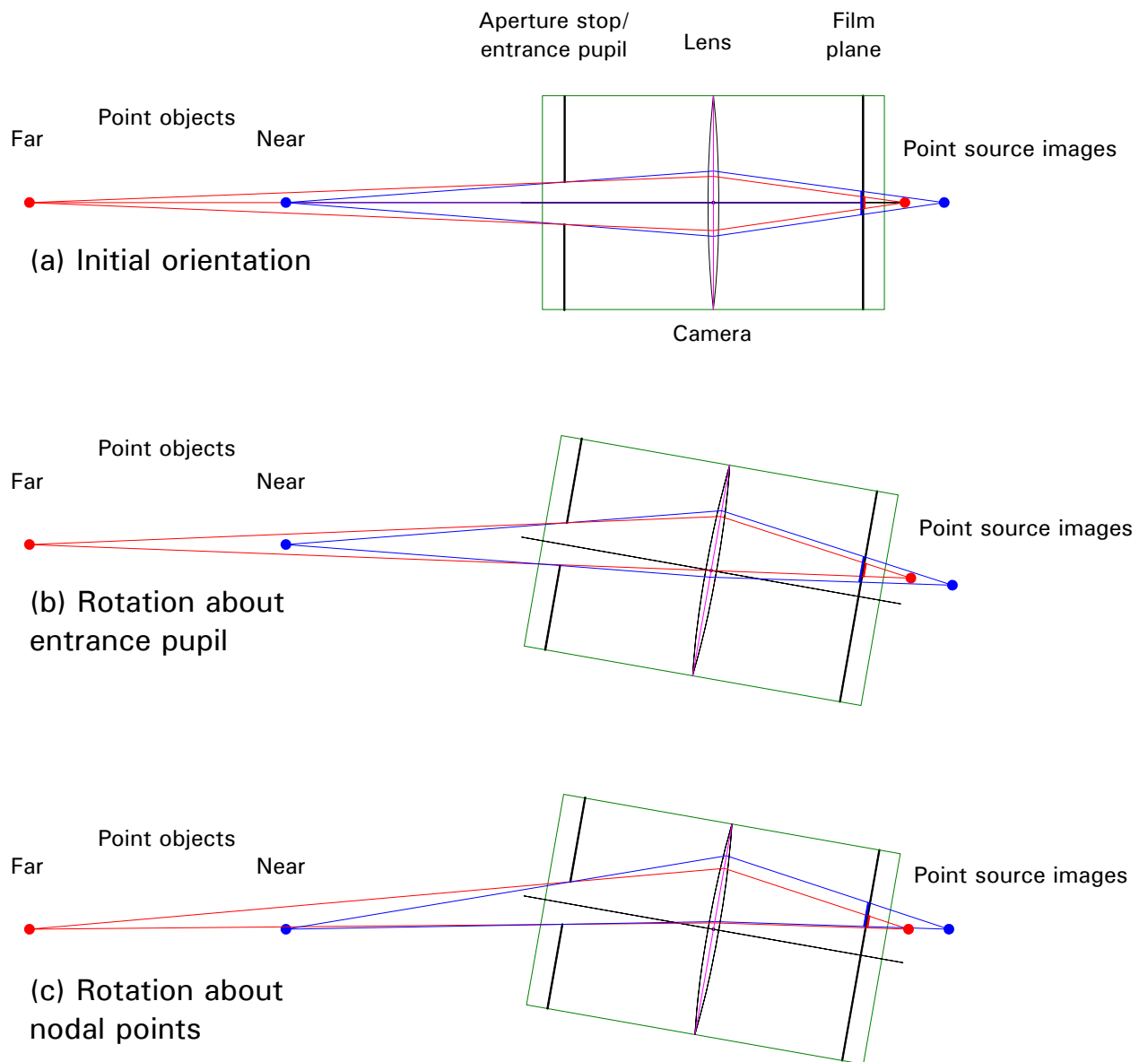
*Quod erat demonstrandum.*

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**APPENDIX B  
LARGER APERTURES**

In the body of this article and in Appendix A, we assume a very small aperture. This allowed us to essentially treat only the path of the “chief ray”—the ray that passes through the center of the aperture stop (and thus the entrance pupil).

In figure 3, we look at the more general case, in which the aperture has a realistic size.



**Figure 3. Moderate-sized aperture**

We use a little different setup here. In order to simplify some of the “ray tracing” construction, we assume an entrance pupil located in front of the lens by precisely the focal length (that is, located at the first focal point of the lens). This does not in any way disturb the result.

We must here immediately note that, with a modest-size aperture, depth of field considerations means that, as we consider objects at different distances, not all of them will be in perfect focus. (Maybe none of the objects we consider are, depending on where the camera is focused for the entire panoramic process.) In the interest of simplicity of the work, we assume focus at infinity. Again, since in the general case, neither of the two objects we consider are in perfect focus, this will serve us as well as anything.

Further, to allow greater specificity, we treat only a point on each of the objects. In part (a) of the figure, we see the camera aimed “straight ahead”: its axis passes through both of the object point we consider. Point images of the two object points are conceptually produced, but they would actually fall behind the film plane (because of the focus situation).

The cones of light heading for convergence at these point images form “blur circles (circles of confusion) of finite size on the film plane. If we consider the apparent “position” of such a blur circle, we essentially think in terms of its center, and we see that the centers of the two blur circles are coincident. Thus we would adjudge the two points to have “aligned” images on the film.

To make it easy to visualize this situation, I have drawn little colored bars showing the extent of the two blur circles. (They are drawn slightly shifted from the film plane so they can be separately seen.)

In part (b) of the figure, we rotate the camera counterclockwise (“to the right”) about an axis located at the entrance pupil. The images of the two object points have of course shifted with respect to the film plane. But note that the two blur circles are still (at least very nearly) centered on each other. Thus, even in the face of these blurred images, we still would conclude that the two images are still “aligned”.

In part (c) of the figure, we rotate the camera about an axis through the center of the lens (and thus, for this thin lens model, through both nodal points). Now we see that the two blur circles, though still overlapping, are no longer mutually centered. Thus here we would conclude that the two images are “no longer aligned”.

This is of course the same conclusion we reached by earlier considering the case of the very small aperture.

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## APPENDIX C

### DEMONSTRATION IMAGES

In this appendix, we see some actual photographic images that illustrate the matter of the panoramic pivot point.

Here, the parts of the famous two objects (one near and one far) were played by two nail polish bottles, seen here:



**Figure 4. The test objects**

They were photographed with a Canon EOS 20D digital SLR, using a modified Canon EF 50 mm f/1.4 lens.

In this lens, the entrance pupil is located not too far from the first nodal point. To be certain that we can distinguish the behavior of the various pivot point locations discussed in the article, I wanted to have an entrance pupil that was in a substantially different location than either of the nodal points.

Therefore, I equipped the lens with an auxiliary aperture stop, a 6-mm diameter opening in a black paper disk held between two filter rings, mounted to the filter threads on the front of the lens. As an aperture stop, this would have an f/number of f/8.0. The “real” aperture stop was set to f/1.4. Thus, the auxiliary aperture stop was controlling. Then, because it is located in front of all lens elements, it is the entrance pupil.

The two objects were placed at distances of 20” and 40” from the front of the lens. The lens was focused on a temporary test target placed at 30” from the front of the lens. Thus, neither of the objects would be imaged in precise focus. (In any case, both could not have been).

The camera was mounted on a modified Adorama two-axis macro focusing rail on a pan-tilt tripod head. This allows the camera to be moved back and forth with respect to the rotation axis (the pan axis of the tripod head). It also allows the camera to be moved from side to side.

For each set of images for a certain pivot axis, the camera was first aimed directly at the two objects and the lateral position of the camera adjusted so that the images of the two objects in the viewfinder were precisely aligned.

Then two images were taken, one with the camera aimed to the left so that the two objects appear near the right edge of the frame, and one with the camera aimed to the right so that the two objects appear near the left edge of the frame.

Image pairs were taken for these pivot axis positions:

- With the axis passing through the entrance pupil (at the auxiliary aperture stop). This was done with the aid of drafting instruments.
- With the axis passing through the estimated position of the first nodal point of the lens. This is very difficult to measure without specialized instruments, which are not available here. Thus we used a SWAG<sup>2</sup>.
- With the axis passing through the second nodal point. Its location was calculated, with respect to the location of the focal plane (for which there is a fiducial mark on the camera body) by using the standard focus equation. There is a small uncertainty owing to uncertainty in the location of the first nodal point (since one of the inputs to the calculation is the focus distance measured to the first nodal point).

The images shown here were cropped to retain only the portion that includes the test objects (up to the original frame edge)

Here we see the two images for rotation about the entrance pupil:



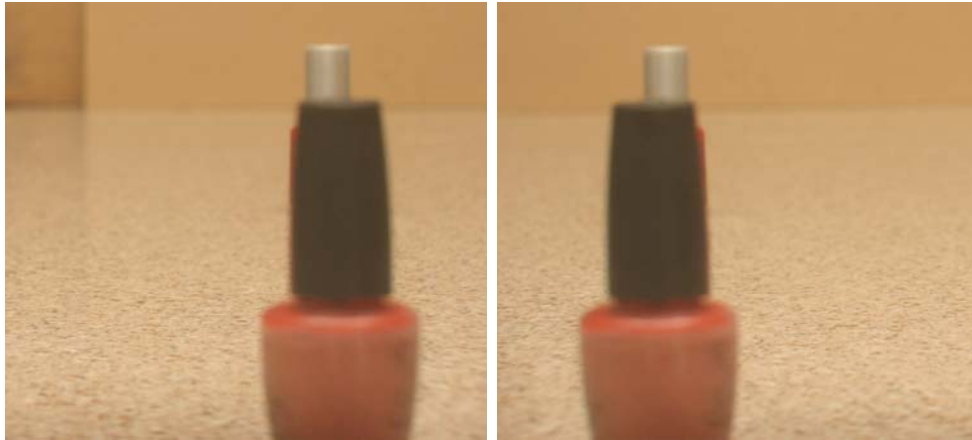
**Figure 5. Rotation about the entrance pupil**

As you can see, there is no significant parallax shift between these two images. They would “stitch well” into a multi-image panorama. (I have not attempted to pre-correct the slight vertical displacement.)

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<sup>2</sup> “Scientific wild-assed guess”.

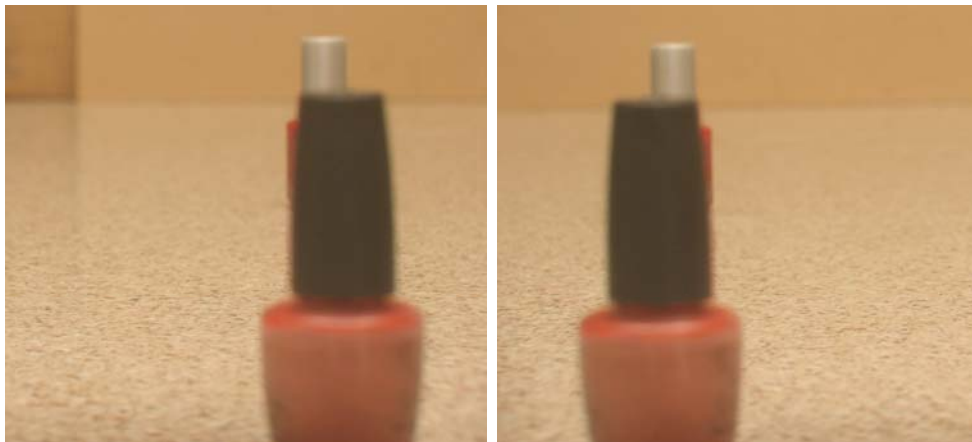
Here we see the two images for rotation about the assumed location of the first nodal point:



**Figure 6. Rotation about the first nodal point (more-or-less)**

This result shows substantial parallax shift. This scheme would not do well for a multi-image panorama.

Here we see the two images for rotation about the second nodal point:



**Figure 7. Rotation about the second nodal point**

This result shows substantial parallax shift. This scheme would not do well for a multi-image panorama.

These results are of course all consistent with what is discussed in the body of this article and Appendix B.

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