

Photographic Photometry 101

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Issue 2
December 25, 2007

ABSTRACT

Photography deals with light, and we are concerned in many technical ways with light and its behavior. Especially in matters of exposure, exposure metering, and the like, our discussions often involve *photometry*, the discipline of describing the “strength” of light. Our discussions are often hampered by inadequate or incorrect understandings of the different concepts of the strength of light and the terms, quantities and units that are involved. This article provides a concise review of photometry as it applies to photographic matters. It also gives an introduction into how the f/number of a lens affects photographic exposure.

INTRODUCTION

Photometry

The term “photometry” means, literally, the measurement of light. It is in particular the discipline of measuring what I will call, for the moment, the “strength” of light. This includes not only the making of actual physical measurements, but also the definition of the physical properties that are involved and of the quantities used to describe them. Many topics in photography involve issues and concepts in this area.

The strength of light and dimensionality

What I call broadly “strength of light” (an intentionally non-specific term) actually embraces many separate photometric quantities, different not because they have different units (although they do), but because they represent different physical concepts and, as a result, have different “dimensionality”.

Dimensionality is what distinguishes, for example, *area* (whose dimensionality is “length squared”) from *volume* (whose dimensionality is “length cubed”); it is what distinguishes *velocity* (whose dimensionality is “length per unit time”) from *acceleration* (whose dimensionality is “length per unit time squared”).

This distinction is not an abstract mathematical one, but reflects a fundamental difference in the physical nature of the properties being “measured”: we recognize area is a different property than volume.

For example, we would not try to explain the amount of carpet needed to cover a floor in terms of volume.

But the distinctions between the different photometric quantities are often not understood. This can greatly hamper discussions of various technical matters. One result is that statements are made, or questions asked, that have no meaning (such as "what fraction of the luminance of the scene does an f/2.0 lens let through to the film?").

This article

A major thrust of this article is to clarify the significance of the various photometric quantities and the physical phenomena to which they apply.

With that background in hand, we will then move on to some important concepts in photographic technology that revolve around photometric matters, including the matter of the significance of the f/number of a lens and its role in photographic exposure.

PHOTOMETRIC QUANTITIES

General

In this section we will discuss several photometric quantities that are of interest in the field of photography.

Units

Each quantity has its own unit(s) in which it is expressed numerically. As in almost every technical field, different units have been used at different times for a particular quantity, and even today different units will be found in different contexts. In scientific work today, it is preferred to use the units of the International System of Units (SI). All our discussions here will be in terms of the applicable SI units.

Caveat

There are exquisite subtleties in the formal definitions of many of these photometric quantities. I will often gloss over some of these in the interest of giving the reader the most direct view of the concept involved.

Perceptual nature of photometric quantities

All of these quantities apply only to visible light, and are based on the perception of the human eye. In particular, the definitions of all these quantities take into account the different response of the eye to light components of different wavelengths across the visible spectrum. The term "luminous", which appears in the names of most of the

quantities (as a separate word or as the source of a root), is a reminder of this fact.

Radiometric quantities

For each of the quantities we will discuss, there is a parallel quantity whose definition does not depend on human eye spectral response, but rather on directly definable physical properties such as "power". These are known as the "radiometric", in contrast to "photometric", quantities. They are of importance in dealing with, for example, infrared or ultraviolet radiation, or radio waves. We will not discuss the radiometric quantities here.

Names

Often, there are several names for the same photometric quantity. We will use here the one that is most widely used in practical photometric work.

These are not always the names used formally in the International System of Units. Many of the SI formal names for photometric quantities are the name of the corresponding radiometric quantity with the qualifier "luminous" attached.¹ For example, what we will call here "illuminance" is called formally, in the SI, "luminous irradiance".

For each quantity whose formal SI name is not the one we use, we will give the formal SI name at the end of the discussion.

The chain of definitions

The definitions of certain photometric quantities depend upon the definitions of others. I will present them in a sequence such that, at each stage, we have already covered all the applicable ingredients.

In fact, in that vein, we will be starting below with a quantity from the realm of solid geometry, which is involved in the definition of our first real photometric quantity.

Selected photometric quantities

Solid angle. Imagine looking at the universe through a telescope; our field of view is bounded by a circular cone with its apex at our location. Or imagine a camera with a rectangular frame. Its field of view is bounded by a rectangular pyramid with its apex at the camera's location. *Solid angle* is the property that describes "how

¹ To be precise, the formal SI symbols for all the photometric quantities have the subscript "v", indicating that these are photometric quantities, as distinguished from the radiometric quantity having the same symbol. ("V" is mnemonic for "visible".) We will use the symbols here without the "v" subscript.

much of the universe” is included within that cone or pyramid—not in the horizontal or vertical direction, but “altogether”.

The unit of solid angle is the *steradian* (sr). It is named by parallelism with the radian, the scientific unit of plane angle.²

The measure of the solid angle at the apex of a cone or pyramid in steradians is defined as the area the cone or pyramid would “cut out” on the surface of a sphere of radius one, with its center at the apex.

The symbol for solid angle is ω (lower case Greek *omega*).

Luminous flux. *Luminous flux* is the “stuff of light”. Its quantity is parallel with *power* in an electrical or radiometric context (subject of course to the matter of its definition taking into account the spectral response of the human eye). Luminous flux is a “flow rate” quantity (“flux” in fact means “flow”). (By comparison, power is the flow rate of energy.)

The SI unit of luminous flux is the *lumen* (lm). The symbol for luminous flux is Φ (upper case Greek *phi*).

The formal SI name for this quantity is *luminous power*.

Luminous intensity. *Luminous intensity* is a property of the light emitted by a “point source”, by which we mean a source whose area is infinitesimal. It is defined as luminous flux emitted per unit solid angle, in some direction of interest.

Why does “per unit solid angle” come into the picture? Couldn’t we just state the amount of luminous flux emitted in the direction of interest? Well speaking of “in a direction” from a point source implies only the flux that travels along a line (the line that defines that direction). A line has zero cross-sectional area, and thus cannot be a conduit for any finite amount of luminous flux. Thus, zero flux “travels along a line”, and so, from a point source, zero flux is emitted in any particular direction.

However, if we imagine a cone, no matter how small in solid angle, with its apex at the emitting point, and whose axis is the direction of interest, then there will be a finite amount of luminous flux emitted within that cone. The ratio of that amount of luminous flux to the solid angle of the cone, as the (solid angular) size of the cone gets smaller and smaller (but never all the way to zero) is the measure of the luminous intensity in the direction of interest.

² *Stereo* is a Greek prefix meaning solid; “stereophonic” sound was so called because it was seen as “solid sound”.

The SI unit of luminous intensity is the *candela* (cd). It corresponds to one lumen per steradian. It is almost exactly the same size as the older, non-SI unit, the *candlepower*.

The symbol for luminous intensity is I.

Luminance. *Luminance* is a property of the light emitted by a surface of finite size (said to be an “extended source”). It is defined as the amount of luminous intensity per unit of surface area—a concept that is a little hard to grasp. Perhaps this homily will help.

Suppose that we were going to construct a luminous surface by planting a lot of little point sources across the surface. Suppose that the little point sources each have a luminous intensity of 0.001 candela in the direction perpendicular to the surface. That means that each of them will emit 0.001 lumen per radian of solid angle in that direction.

Now suppose that we distribute 1,000,000 of these over a one square meter area of our surface (they would be on 1 mm centers). Then what is being emitted (at least, in the direction perpendicular to the surface) is 1000 lumens per steradian per square meter.

It is this quantity, luminance, that influences what our eye perceives as the brightness of the surface.³

As we would expect from the little thought experiment above, the basic unit of luminance is the lumen per steradian per square meter. Unfortunately, there is always the possibility of ambiguity when we have a unit with more than one “per” in it, especially if we try to write it “inline” with slants. In this case, it means:

$$\frac{\left(\frac{lm}{sr}\right)}{m^2} \quad (1)$$

which can also be written:

$$\frac{lm}{sr m^2} \quad (2)$$

and so can be written inline (clumsily) as $lm/(sr \cdot m^2)$ (which could be spoken “lumens per steradian-square meter”).

³ Often brightness as an actual synonym for luminance. But that is not rigorous, since formally *brightness* is actually the name of a separate quantity, one that recognizes the nonlinear human perceptual response to different values of luminance. (Quantitatively, brightness varies as about the cube root of luminance.)

In fact the safest way to write the unit is: $\text{lm}\cdot\text{sr}^{-1}\cdot\text{m}^{-2}$ (where minus signs on the exponents mean their ingredients are in the denominator, to the power indicated, -1 meaning that the ingredient itself is in the denominator). For the truly fastidious, the hyphens should be replaced by dots, explicitly indicating multiplication: $\text{lm}\cdot\text{sr}^{-1}\cdot\text{m}^{-2}$.

Fortunately, since the unit *lumen per steradian* has a special name, *candela* (cd), the SI unit of luminance is actually stated as the much more tidy *candela per square meter* (cd/m^2).

The symbol for luminance is L.

In our example, we seemingly considered the luminance as observed along a direction perpendicular to the surface. For reasons that are beyond the scope of this article, it turns out that, for a certain "classical" distribution of the luminous intensity from each hypothetical point source over different angles of departure, the perceived luminance of the surface will be the same from any angle of observation (from in front of the surface, of course).

Illuminance. *Illuminance* is a property of the light landing on a surface. It is defined as the amount of luminous flux landing per unit area of the surface.

The SI unit of illuminance is the *lux* (lx), which corresponds to one lumen per square meter.

The symbol for illuminance is E (a nice mnemonic is to think of "EE-lum-i-nance.")

The formal SI name for this quantity is *luminous incidence*.

We will hear a little more about illuminance after we introduce our final photometric quantity.

Luminous flux density. We rarely hear of *luminous flux density*. The property that it measures often pertains to a stage in the photometric chain (*i.e.*, in a light beam that is just about to strike a surface) where we rarely linger, and thus the equations that would give us this quantity get swept into the next equation (giving illuminance).

Recall that illuminance is a measure of the "strength" of the illumination of a surface. It is defined as luminous flux per unit area, where the area is reckoned on the surface of interest.

Luminous flux density is also defined as luminous flux per unit area, but here the area is reckoned on an imaginary plane perpendicular to the direction of travel of the beam.

The symbol for luminous flux density is M . The SI unit is the *lumen per square meter* (which in this case we usually do not call by the derived name, lux, reserving that for illuminance.)

An important application of luminous flux density is to describe the “strength” of a traveling beam of light at a particular place in its travels (maybe just before it lands).

The formal SI name for this quantity is *luminous exitance*.

THE RELATIONSHIP BETWEEN LUMINOUS FLUX DENSITY AND ILLUMINANCE

The relationship between the luminous flux density of a beam and the resulting illuminance on a surface it strikes is given by this equation:

$$E = M \cos \Theta \quad (3)$$

where M is the luminous flux density of the “beam” of light just as it is about to land on the surface, E is the illuminance it causes on the surface, and Θ (upper-case Greek *theta*) represents the *angle of incidence*: the angle the direction of travel of the beam makes with a line perpendicular to the surface (the “normal” to the surface).

The $\cos \Theta$ factor comes from a very simple geometric reality. Suppose we consider, within a beam of light, a 1 cm square “pencil”, in which travels 10 lumens of luminous flux—a luminous flux density of 10 lm/cm² (not the standard SI unit, but handier for our exercise). Suppose this beam of light arrives at an angle of 45° to the normal. Then this 1 cm x 1 cm “pencil” hits (and thus illuminates) an region on the surface that is 1 cm x 1.414 cm in size. (Imagine a 1 cm square wood rod. Cut the end at a 45° angle. The dimensions of the cut surface will be 1 cm x 1.414 cm.)

Since the whole 10 lumens of flux contained in our “pencil” lands over a region on the surface whose area is 1.414 cm², the illuminance on the surface is 7.07 lumens/cm².

The “dilution” factor, 0.707, is of course the cosine of 45°.

REFLECTANCE

When a light beam falls on a diffuse reflecting surface (a surface that is not mirror-like), the kind of surface had by most objects we photograph, then for each infinitesimal region of the surface, some or all of the luminous flux landing on that region is emitted from the surface, “scattered” over all angles in the hemisphere “in front of” the surface. An *ideal diffuse reflecting surface* (said to be “Lambertian”) exhibits a certain distribution of the emitted flux over the various angles of departure. For such a surface, the emitted flux will cause the

surface to exhibit a luminance that is constant regardless of angle from which the surface is viewed (so long as the view is “from in front of” the surface). This is true regardless of the angle from which the arriving light comes.

In such a case, the luminance of the surface is given by:

$$L = \frac{\pi}{4} RE \quad (4)$$

where L is the luminance exhibited by the surface, E is the illuminance of the arriving (“incident”) light, and R is the *reflectance* of the surface. If the surface reflects all the luminous flux incident on it, R is one; if it reflects only half of the incident luminous flux, R is 0.5, etc.

Sometimes the symbol ρ (lower-case Greek *rho*) is used for reflectance.

THE ROLE OF THE LENS APERTURE

From a photometric standpoint, the lens takes each infinitesimal region in the scene (which region will have a certain luminance) and from it forms an infinitesimal region in the image on the focal plane having a certain illuminance.⁴ We will be interested in what affects the relationship between the “source” scene luminance and the resulting image illuminance.

Most lenses contain an *aperture stop*, which is an opaque plate with a (roughly) circular opening someplace in the path through the lens. Usually, the diameter of the opening can be varied (the plate usually involves some type of “iris” construction).

If we look into the front of the lens, we think we see the aperture stop, but because of the effect of the lens elements in front of it, we do not see it with its actual diameter nor location along the lens axis. What we think we see is a virtual image of the aperture stop. That virtual image is called the *entrance pupil* of the lens. Although it does not physically exist, in photometric analysis it seems to be the window through which the lens collects light from the scene.

There is a corresponding virtual image of the aperture stop “seen” from behind the lens, called the *exit pupil*.

⁴ Note here that since luminance and illuminance are measures of different physical properties, the statements we sometimes hear about “what fraction of the scene luminance ends up on the focal plane” are misguided and meaningless.

In a lens of symmetrical design, the entrance and exit pupils are of the same diameter, and they are located (along the lens axis) in the same places as the first and second principal points of the lens, respectively. We will assume that situation here, which allows us to ignore a complication that would only serve to confuse the presentation.

We will also assume object and image points lying on the lens axis, so as to sidestep the matter of "illuminance falloff". We will further assume that the lens has "100% transmission"; that is, all the light it collects is properly delivered to the image, none being lost by absorption, reflection, or scattering.

If we then follow a long trail of photometric algebra (which I will spare the reader here!), we find that the image illuminance on the focal plane (the location of the film or digital sensor) is very nearly given by:

$$E_f = \frac{\pi L_s D^2}{4Q^2} \quad (5)$$

where E_f is the illuminance caused on the focal plane at some point on the image; L_s is the luminance of the corresponding "source" point of the scene; D is the diameter of the entrance pupil; and Q is the distance from the second principal point of the lens to the focal plane (D and Q being in the same units).

To get us ready for our next conclusion, I will rewrite equation 5 this way:

$$E_f = \frac{\pi}{4} \left(\frac{D}{Q} \right)^2 L_s \quad (6)$$

If focus is at a substantial distance, Q becomes very nearly equal to f , the focal length of the lens (exactly equal to f for focus at infinity), and so in this situation equation 6 becomes essentially:

$$E_f = \frac{\pi}{4} \left(\frac{D}{f} \right)^2 L_s \quad (7)$$

Thus the quantity D/f tells us (subject to the various assumptions and limitations I have introduced along the way) the effect of the lens in transforming the luminance of the scene into illuminance on the focal plane. Its inverse, f/D , is defined as the f /number of the lens. Rewriting equation 7 in terms of the f /number (for which we use the symbol N), we get:

$$E_f = \frac{\pi}{4} \left(\frac{1}{N^2} \right) L_s \quad (8)$$

Thus we see why we can use the f /number as the indicator of how the lens transforms scene luminance to focal plane illuminance. But remember this is only strictly true for focus at infinity, 100% transmission, and so forth—otherwise, it's just a handy approximation.

PHOTOMETRIC EXPOSURE

Early workers in the field of photography discerned that the effect of light on a particular photosensitive surface was approximately determined by the product of the illuminance on the surface and the length of time that it persisted. Today we call that quantity the *photometric exposure*. Its SI unit is the lux-second. The standard symbol today is H .

Photometric exposure is defined by:

$$H = Et \tag{9}$$

where H is the photometric exposure, E is the illuminance, and t is the time the illuminance persists.⁵

Just above I said “approximately” because of the phenomenon of *reciprocity failure*. This says that the impact of 10 lux of illuminance for 2 seconds will not necessarily be exactly the same as the impact of 20 lux of illuminance for 1 second (both representing the same photometric exposure, 20 lux-seconds). But, over a reasonable range of exposure times, the proportional relationship is very nearly followed.

The story of “ H ” as the symbol for photometric exposure is interesting. The two researchers Hurter and Driffield published early papers on the response of photosensitive materials. They often presented their result on a graph that plotted *density* (a logarithmic measure of how opaque the film, after development, became in response to the impact of light—what I call the “exposure result”) versus the logarithm of the photometric exposure. This graph—with logarithmic scales on both axes—turns out to be nearly a straight line over a good portion of the range for many types of photographic materials.

The (logarithmic) measure of sensitivity is designated “ D ”, and the researchers used “ E ” for photometric exposure. Thus the curve was (and still is) widely spoken of as the “ $D \log E$ ” curve. But it is also often called the “ $H\&D$ curve”, referencing the researchers who first brought it to attention.

⁵ To be more precise, taking into account the possibility of the illuminance varying over the time interval, photometric exposure is the *time integral of illuminance*.

A problem with "E" for photometric exposure in the broader scientific realm is that it also was the symbol for *energy*, and, even worse, at a certain point, in photometric science, E was adopted for the quantity *illuminance* (one of the ingredients of photometric exposure).

So a new symbol was needed for photometric exposure. Many people, hearing the response curve called the "H&D" curve, thought that the curve must plot D versus H, and they knew D represented density, so they thought H must represent photometric exposure. That wasn't true of course, but when the boffins needed a symbol for photometric exposure to replace "E", they played a wonderful joke and chose "H".

THE ACTUAL PHOTOMETRIC EXPOSURE

Now we can look into what determines the photometric exposure in an actual camera shot. Recall that, by definition:

$$H = Et \quad [9]$$

Combining this with equation 8, we get:

$$H_f = \frac{\pi}{4} \left(\frac{1}{N^2} \right) t L_s \quad (10)$$

where H_f is the photometric exposure at a point on the focal plane; N is the f/number of the lens; t is the exposure time (shutter speed), in seconds; and L_s is the luminance of the scene at the corresponding source point. (H_f and L_s are in the SI units mentioned earlier.)

We can rewrite this as:

$$H_f = \frac{\pi}{4} \left(\frac{t}{N^2} \right) L_s \quad (11)$$

Thus we see that the properties of the camera that affect the relationship between scene luminance and image photometric exposure have come together in this factor:

$$\frac{t}{N^2} \quad (12)$$

which of course reflects the combined impact on photometric exposure of the shutter speed, t , and the f/number, N .

There is unfortunately no unambiguous name for this quantity (in its basic form), nor is there any standard scientific symbol for it. It is often called "exposure", but there is the risk of confusing it with photometric exposure (which is also often called just "exposure"). Here, I will use the coined term "exposure1" for this factor. (I often do

this in connection with my use of “exposure2” for photometric exposure, to contrast the two; however, in this article I call photometric exposure just that).

Under the APEX system, in which logarithmic expressions are provided for various factors involved in the matter of exposure, there is a representation for exposure1. It is called *exposure value*, and has the symbol Ev.⁶ It is defined by:

$$Ev = -\log_2 \frac{t}{N^2} \quad (13)$$

where \log_2 means the binary (base 2) logarithm. A larger value of Ev represents a lesser exposure; a change in one unit up or down in Ev represents a halving or doubling of exposure1 (what a photographer would call a “one stop” change).

It is tempting to use “Ev” to mean exposure1 in a narrative, qualitative sense (when no numerical value is to be stated). But this really doesn’t work out safely. For example, we might be tempted to say, “In that situation, we may wish to increase Ev from that recommended by the exposure meter.” Now, do we mean a greater exposure (a greater value of t/N^2), or a greater value of Ev (which implies a lesser exposure—a smaller value of t/N^2)?

So feel free to say, “exposure1”. I’ll know what you mean.

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⁶ Do not be confused by the unfortunate and confusing practice in which “Ev” is used to designate scene luminance.