

ABSTRACT

The familiar kind of steam locomotive is propelled by a reciprocating steam engine. Central to the operation of any such engine is a valve system, which admits steam to the cylinder during certain portions of the cycle and exhausts the cylinder to the atmosphere during other portions.

In a steam locomotive, valve systems that are robust and free from the need for delicate adjustments are desirable and are generally used. They nevertheless follow ingenious principles to approach ideal engine performance over a range of operating conditions.

In this article, we postulate a simple mathematical model which, it will turn out, closely follows the actual operation of most of these valve systems, and we see how it produces a beneficial plan of steam admission and exhaust.

Two appendixes then review the construction and operation of two classical "valve gear" systems, showing analytically how they in fact closely fulfill the abstract mathematical relationship between piston and valve movement we assume in the body of the article, thus closing the circle.

INTRODUCTION

The locomotive's engine

The familiar steam locomotive is driven by a reciprocating two-cylinder double-acting steam engine¹. *Double-acting* means that the piston in a cylinder is actively driven, by steam pressure, in both directions.

The piston motions in the cylinders on the two sides are separated in phase by 90°. Thus, when the piston on one side is at or near one of the two limits of its travel (in or near a "dead center" position), when it can exert little torque on the driving wheels, the piston on the other side will be near the middle of its stroke, when it can most effectively exert torque. The working of the two cylinders produces a fairly-uniform torque on a continuous basis.

¹ Of course, we often call a locomotive an "engine", but here I separate the terms.

Figure 1 shows a typical “contemporary” (design ca. 1915) steam locomotive. A few key parts have been identified.

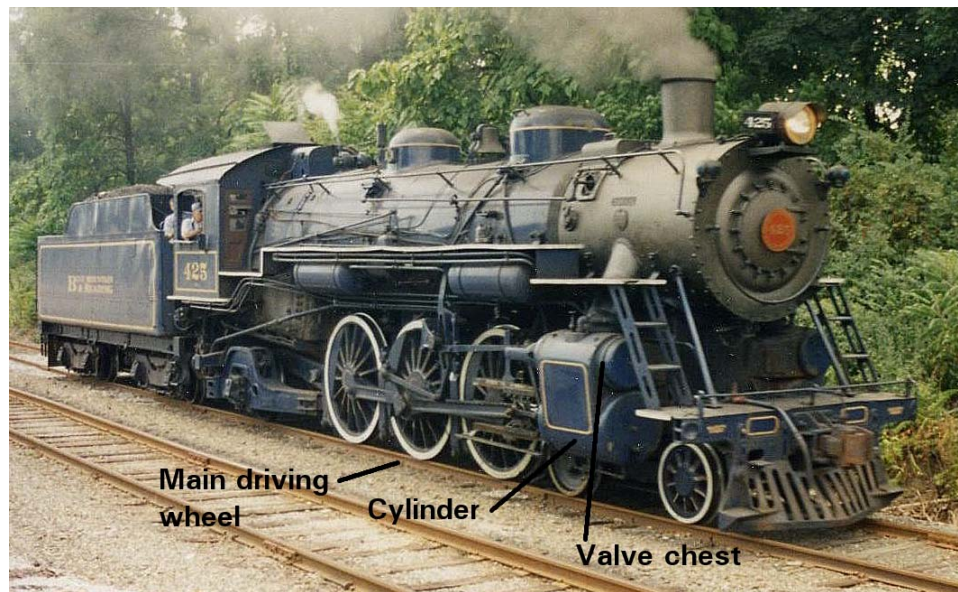


Figure 1. Blue Mountain and Reading Railroad 4-6-2 “Pacific” Locomotive
“Johnson bar in the forward company notch”²

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Steam engine valve systems

Any reciprocating steam engine depends on a *valve system*. Its job is to admit steam, under substantial pressure, to the cylinder space on one side of the piston during certain portions of the cycle, and to connect that space to the atmosphere (for “exhaust”) during other portions of the cycle.

The desirable plan for timing these periods of admission and exhaust is governed by complex thermodynamic and mechanical considerations. For each combination of operating speed and torque load on the engine, there is a specific plan that gives the highest efficiency: that is, the lowest consumption of steam (and thus of fuel) to maintain the predicated speed and output torque.

The valve system generally can be considered as comprising two aspects:

- The valves themselves. These open and close various passages as required to permit admission and exhaust.
- The valve operating mechanism (often known as the *valve gear*). This manipulates the valves.

² Curious about that? Read on.

In stationary steam engines (typically used for pumping water, driving industrial machinery en masse through systems of overhead line shafts and leather belts, and generating electrical power) ingenious and complex systems of valves and valve gear were used that could attain very high efficiencies. These, however, were generally ill-suited for locomotive application, as their delicate parts could not be adapted to a robust form, tolerant of the environment on the exterior of a locomotive, and as well they typically required periodic adjustments, not desirable in a locomotive context.

Locomotive valve systems

Thus, locomotive designers developed equally-ingenious valve systems, amenable to robust construction, that could approximate the ideal plans of steam admission and exhaust, even over the wide range of operating conditions of a locomotive. Among other key properties, the actual valve portion of these systems comprised, for one cylinder, a single element, moved in linear, oscillating movement by the valve gear.

Our outlook

In this article, we will gain insight into the valve systems of many locomotives by first recognizing the typical effect of the valve itself on the various steam paths as it moves through its range of movement.

We then adopt a hypothetical, fairly simple, mathematical model of the motion of the valve itself as the “engine” goes through a cycle of operation—one we happen to know closely follows the actual behavior of most widely-used valve gear designs. We then see the implications of this model on the flow of steam in the engine, and the implications of that on the mechanical working of the engine.

Then we will examine how varying a single parameter of the mechanism (controllable by the engine driver) will shift an important property affecting the efficiency of the engine, allowing optimization of efficiency as the circumstances of operation vary during a “run”.

Finally, in two appendixes, we will examine the principles of construction and operation of two “classical” locomotive valve gear systems, the *Walschaerts* and the *Stephenson*, and analytically show that, for both, their operation closely fulfills the hypothetical model assumed in the earlier work.

THE LOCOMOTIVE ENGINE

The cylinder and crank mechanism

As mentioned above, the typical locomotive has a two-cylinder engine, one cylinder on each side. Here, we will only examine the cylinder and its supporting arrangements on the right-hand side; those on the

left-hand side are identical except for mirror-image symmetry and the 90° phase difference mentioned earlier.

In most steam locomotives, the cylinders are at the front, and we will observe that in our illustrations.

Figure 2 shows the basic concept in semi-schematic form, and shows some of the terminology and notation we will be using:

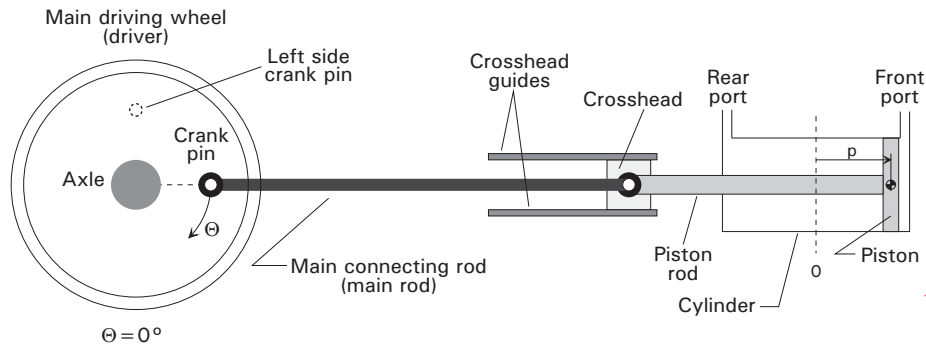


Figure 2. Basic cylinder concept

A connecting rod (the *main rod*) links the rear end of the piston rod to an offset *crank pin* on the driven wheel (the *main driving wheel*, or *driver*) on this side. The wheel is attached solidly to an axle, which runs in bearings, and which, on the left side, carries (solidly attached) the complementary driving wheel. As noted earlier, the two crank pins are separated in orientation by 90° (could be either way, depending on the design).

The joint between the piston rod and the main rod is carried by a block, the *crosshead*, that slides between a pair of guide rails running fore-and-aft. The purpose of this is to take the up or down reaction force caused when the main rod is at an angle, so as not to force the piston rod itself up or down, which could exacerbate wear on the bushing (equipped with a sealing gland) through which the piston rod passes to exit the cylinder at the rear. We will not show the crosshead and its guides in further illustrations.

Notation

In our analytical work, we will (refer to figure 2):

- Reckon the position of the piston in terms of its distance p (a signed number) from its mid-stroke position. The small target symbol on the piston centerline represents the point on the piston we use as the datum for this, and the dotted line shows the mid-stroke position (the scale origin). The scale is such that the range of piston motion runs from -50 units to $+50$ units; thus, one unit

corresponds to 1% of the total piston travel (a unit widely used in technical work in this field³).

- Reckon the angular position of the driver (Θ —upper-case Greek *theta*) using the crank pin as our datum with “to the right” (forward on the locomotive) being zero, as is done in most analytical work. We however will consider increasing angle to be the result of clockwise rotation from the reference point (the opposite of the usual analytical convention), so that positive rotation corresponds to forward motion of the locomotive (to our right).

The system in motion

We now imagine the engine in operation, in the forward direction (clockwise rotation of the driving wheels, Θ increasing). One might wonder, looking at figure 2, how this can happen, given that the system is in the “front dead center” position, where force on the main rod cannot impart any torque to the driving wheels. Of course, it is the cylinder on the other side (which, at the instant depicted in figure 2, has $\Theta = +90^\circ$ or $\Theta = -90^\circ$, depending on the design, with the piston at approximately mid-stroke) that propels the initial movement.

In any event, in figure 3 we see the system with the driver having rotated 90° clockwise from the reference position:

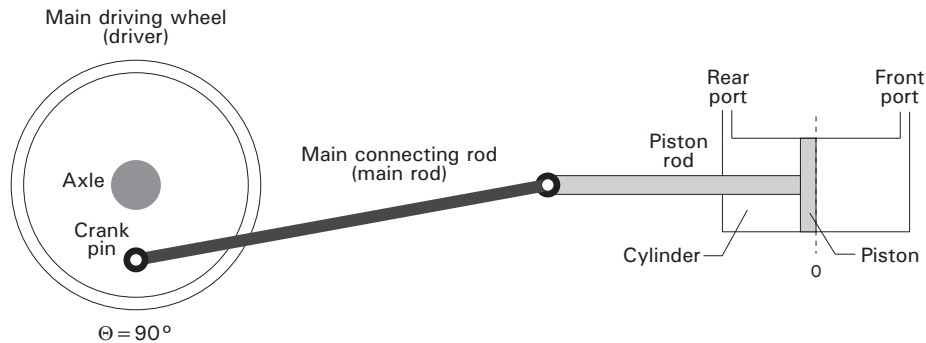


Figure 3. Situation with the driver at 90°

We note that if the position of the piston actually followed $\cos \Theta$, it would be precisely at mid stroke in this situation ($\cos 90^\circ = 0$). But we see that instead it is a small amount to the rear of mid stroke. This is of course a result of the angle of the main rod at this point.

³ Commonly the scale in other writings runs (in % of total travel) from one end of the travel, not from the center as I do here. My convention simplifies the trigonometry.

This is an important consideration in the detailed design and analysis of locomotive systems, but for most of this article we will ignore it and assume sinusoidal motion of the piston.

Thus, we will assume that the position of the piston, p , is given by:

$$p = k_1 \cos \Theta \quad (1)$$

where k_1 is equal to the radius from the axle center to the center of the crank pin.

Management of the steam

Of course, in actual operation, the locomotive is not moved by my postulating changes in Θ . Rather, it moves because of torque imparted to the driving wheel by the force of the piston when one face is subject to steam pressure and the other to atmospheric pressure.

In order for this to happen, it is necessary to have a system of valves that will admit steam under pressure (from the locomotive boiler) to the space on one side of the piston at the appropriate portion of the cycle, while allowing free flow from the opposite side of the piston into the atmosphere.

In figure 4, we see this illustrated with the locomotive traveling forward and the driver at an angle of 90° .

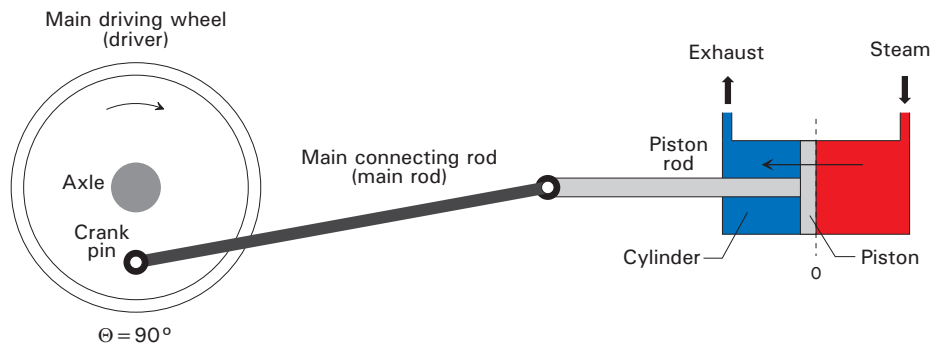


Figure 4. Steam and exhaust with the driver at 90°

Steam is admitted to the space in front of the piston, and the space behind the piston is opened to the atmosphere.

In figure 5, we see this a half a revolution later, with the driver at an angle of 270° :

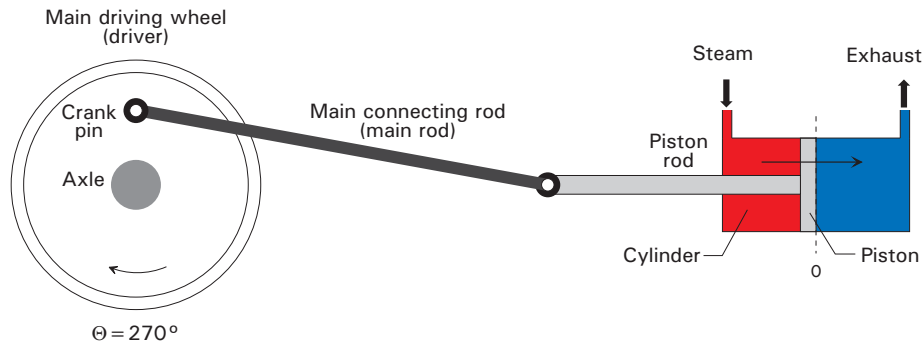


Figure 5. Steam and exhaust with the driver at 270°

Here, with the piston seen at the same position, but now moving forward, we see steam admitted to the space behind the piston, and the space in front of the piston open for exhaust, as required to continue to provide torque in the proper direction.

The cutoff principle

We may (rather naïvely, it turns out) imagine that ideally, steam would be admitted to the front side of the piston (the rear side being open to exhaust) during its the entire rearward stroke. The opposite would be true for the forward stroke.

In fact this is not desirable. Under such an arrangement, at the end of the stroke the cylinder space on the “steam” side of the piston is filled with steam at full boiler pressure, which contains a large amount of energy, part of it thermal (because of its temperature and because the water is in the form of vapor) and part mechanical (from its pressure).

When, at the end of the stroke, that cylinder space is exhausted to the atmosphere, all that energy would be lost.

Thus, in actual practice, we arrange for steam to be admitted to one side of the piston for part of the stroke but then to cut off the steam admission, the cylinder space now being blocked from either steam admission or exhaust passages for the remainder of the stroke. As the piston continues its stroke, the steam expands, the thermal energy being extracted from it as work done on the piston and delivered as energy to the driving wheels.

This is called the “cutoff” principle.

With the engine rolling along at speed, a certain average amount of torque on the driving wheels being required to overcome friction, wind resistance, and so forth, it may be practical to cut off steam admission very early on the piston stroke (perhaps as early as 15% of the stroke). But when accelerating the train, or pulling the train up a grade, a substantially greater average torque may be required.

If, under that circumstance, we can arrange for a later cutoff (perhaps as early as 85% of the stroke), a greater average torque will be delivered, although the efficiency of the engine (in terms of amount of steam required to deliver a certain amount of mechanical energy) will be decreased (a pragmatic tradeoff).

This situation somewhat parallels that in an automobile driven by an internal combustion engine. At a start, or when climbing a steep hill, we want a high gear ratio between the engine and the driven wheels, so as to provide the needed propulsive torque. But at speed, on level ground, we want a lower gear ratio so as to make most efficient use of the fuel in the engine.

Thus we have, in a gearbox or automatic transmission, provision for varying the gear ratio during different stages of vehicle operation.

Similarly, it is desirable for the cutoff fraction of the locomotive's steam engine to be controllable "on the fly".

The same is in fact true of *stationary steam engines*, such as were once used to pump water, power industrial machinery, or even generate electricity, as well as of their cousins, *marine engines* (used to propel vessels). There, even if the engine operated at a constant speed, the cutoff fraction needed to be changed with changing load on the engine to maintain optimal overall efficiency. In fact, in such engines, the maintenance of the proper speed (under control of a flyball governor) was typically done not by throttling the steam flow but rather by adjusting the cutoff.

Stationary engine valve systems

There have been many types of valves used for this purpose, and an unbelievably wide range of mechanisms (often called the *valve gear*) for controlling their operation. Sometimes, the valves were of the *poppet* type (essentially as we find in an internal combustion engine, where of course their duty is a bit different). Other important engine types used rotary valves (such as on a French horn, or an expensive trumpet). Often, there were four separate valves for each double-acting cylinder, one for admission and one for exhaust with respect to each end of the cylinder.

The mechanisms for controlling the opening and closing of these sometimes involved arms operating on separate camshafts, and in

other cases involved systems of latches that would “trip” at a certain point to allow the valve to quickly close under spring pressure.⁴

These systems allowed for the timing of steam admission and exhaust to be precisely controlled, and varied, to closely meet the thermodynamic mandates for highest efficiency for any combination of operating speed and load torque.

But these arrangements of valves and valve-controlling mechanisms were very complex and inherently delicate. They provided for, and in turn demanded the use of, adjustments to precisely fulfill their roles.

We can get some insight into the intricacy of those valve systems from figure 1), which shows the valve gear of one of the many designs of the Corliss engine:

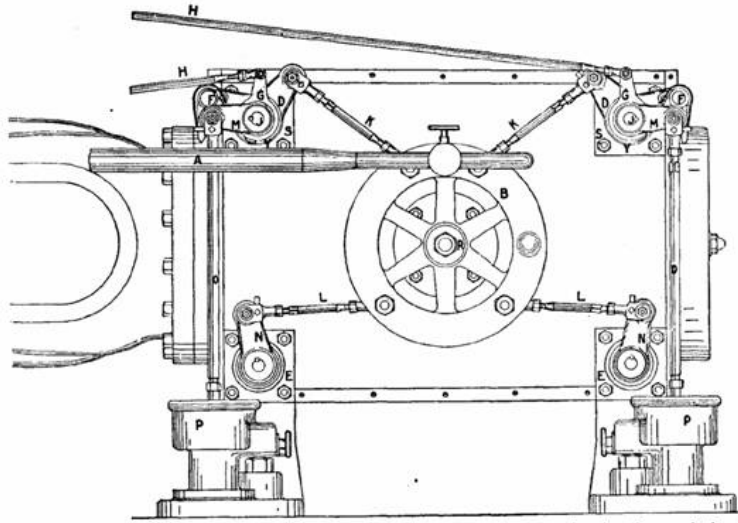


Figure 6. Valve gear of illustrative Corliss engine

The mechanisms at the upper left and right corners of the cylinder themselves have additional small moving parts not readily visible in this view. Note also the adjusting turnbuckles L, K, and D, and the adjustment at the top of the circular “wrist plate” in the center.

We can readily grasp that these mechanisms were not amenable to adaption to the context of a locomotive. There, we had to have mechanisms that could be made of large, strong parts, that would operate reliably in the presence of dirt, snow, or mud among the parts, and that would perform consistently without need of adjustment after manufacture.

⁴ But too-rapid closing had bad side effects, so in some engines using a “trip” mechanism for closing the valve, a pneumatic damper (“dashpot”) was used to limit the speed of closure. Beginning to sound complicated? Oh, my, yes.

Locomotive valve systems

As a result, a genre of valves and valve-operating systems (*valve gear*) came into almost universal use on locomotives that had, from our standpoint, these key properties:

- On one side of the locomotive (one cylinder), all control of the passage of steam into and out of the cylinder spaces at both ends was done by the movement of a single monolithic part (**the valve**).
- The motion of the valve was smooth, continuous, and oscillatory (approximating sinusoidal motion).

In particular, the preponderance of steam locomotives about 1905 onward use what is called in the locomotive field a *piston valve*; it would be called a *spool valve* in the field of hydraulic and pneumatic engineering.⁵

In figure 7, we see its basic configuration:

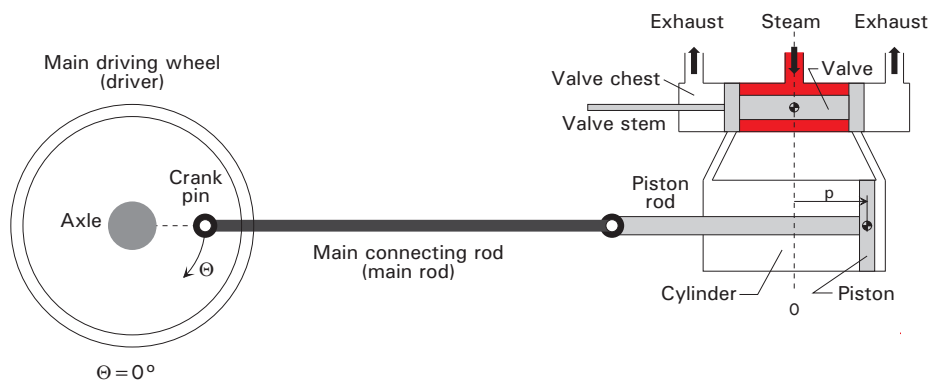


Figure 7. Locomotive piston valve

The valve comprises a cylindrical bore in a housing called the *valve chest*⁶ in which travels a cylindrical “spool” (the *valve*), movable from outside the housing by way of a *valve stem*. The valve spool has at its ends flanges of significance thickness. At the bottom of the valve bore there are two generally-rectangular openings (the front and rear ports), which communicate through channels with the front and rear ends of the interior of the cylinder itself.⁷

⁵ Earlier, sliding flat valves were almost universally used. But similar mechanisms were used to move them.

⁶ Sometimes, *steam chest*.

⁷ In some cases, these ports extend into channels that extend a considerable distance around the circumference of the cylinder wall, perhaps even all the way around.

In addition, near the center of the valve bore (perhaps near the top), there is a larger port, through which steam from the boiler (by way of the *throttle valve*) enters the valve. At each end, there is a larger port, which communicates with the atmosphere (this usually means through two pipes that join into a single *blast pipe* that shoots up the center of the smokestack, where it helps to encourage the outward flow of the boiler exhaust gases).⁸

We can see from the illustration that the “flanges” at the end of the spool are of such a size and relative location that with the valve in its “neutral” position (as shown) both front and rear ports are blocked, and we see that the valve would have to move a small distance one way or another to open either of those ports, a situation called “lap”. In our analytical work, we describe the position of the valve spool in terms of its position, v (a signed number) from the neutral position.

In figure 8, we see what happens if we move the valve from its neutral position:

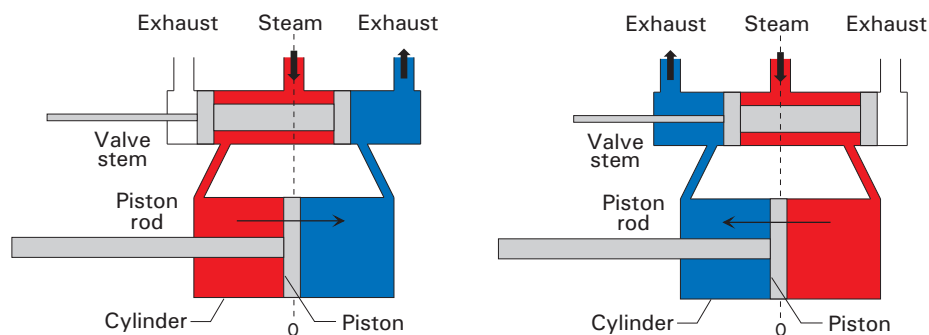


Figure 8. Valve action

On the left, we see what happens if we move the valve spool to the left ($v < 0$). After travel through a small “dead zone” (because of the *lap*), both front and rear ports are opened (not necessarily at precisely the same point), the rear port to the space between the two flanges of the valve (and thus to the steam supply), the front port to the space to the right of the valve spool (and thus to the exhaust path).

Thus, moving the valve to the left causes conditions to be set up that move the piston to the right under steam pressure.

⁸ In some designs the steam is led to the spaces outside the ends of the valve and the exhaust taken from the center. This is called an “outside admission” arrangement; the one we will discuss here is called the “inside admission” arrangement.

On the right, we see what happens if we move the valve spool to the right ($v > 0$). Here, the results are just the opposite, setting up conditions that move the piston to the left under steam pressure.

Thus we can imagine that, by manipulating the valve in proper synchronism with the rotation of the drive, we can cause the piston to be alternately driven in the two directions such that the driver is almost continuously given torque in the direction that will propel the locomotive forward.

And, getting a little ahead of our story, we can visualize manipulation of the valve with a complementary plan that will result in the driver being almost continuously turned in the opposite direction, propelling the locomotive to the rear. In fact, we can do this in such a way that the cutoff fraction can be varied by the engine driver.

A mathematical model

Here, we will articulate a basic mathematical model of the motion of valve operation—one which, it turns out, is very nearly followed by most of the valve mechanisms in actual use in practical stem locomotives through essentially the entire history of the field.

We noted earlier that the movement of the piston **nearly** follows a sinusoidal motion. It turns out that almost all the common mechanisms used to move the valve also impart to it a **nearly** sinusoidal motion.

While the departures of both motions from true sinusoids is of considerable importance in locomotive design, for our purposes here we will ignore this wrinkle and assume in our model that the motions of both piston and valve are sinusoidal. That having been said, the two motions can be approximated by these expressions:

For the piston:

$$p = a_1 \cos \Theta \quad [1]$$

where p is the valve position, as we previously defined, and Θ is the position of the driver crank, as we previously defined. The coefficient a_1 is in fact equal to the radius to the crank pin from the axle.

For the valve:

$$v = Ra_2 \sin \Theta + a_3 \cos \Theta \quad (2)$$

where v is the position of the valve spool (measured from its “neutral” position), and a_2 and a_3 are determined by the geometry of the mechanism. R is a dynamic parameter, varied over a range of -1.0 to $+1.0$ by the engine driver by moving the *reversing lever* in the cab

(often called, for reasons that no longer seem able to be reconstructed, the “Johnson bar”).

Its most obvious job (as we might guess from its name) is to arrange the valve gear so that the locomotive’s engine will run either forward or in reverse. Specifically, $R = +1.0$ produces forward operation, while $R = -1.0$ produces reverse operation.

But it turns out that if we set the reversing lever to, for example, $R = +0.5$, the locomotive will still be propelled forward, but with an earlier cutoff. If we set it to $R = +0.3$, we will have forward operation with an earlier cutoff yet. We will see shortly how that comes about.

Thus, in operating the locomotive, when preparing to get underway (forward), the driver will set the reversing lever to $R = +1.0$ (often said, by virtue of the usual physical arrangement in the cab, to be “putting the Johnson bar in the corner”).

Then, as the speed of the train increases, the driver will move the reversing lever to smaller positive settings (often called “notching up⁹”), thus reducing the cutoff.

The position giving the earliest practical cutoff (the smallest usable value of R) is sometimes colloquially called “the company notch”, the implication being that the railroad company is best served, fiscally, by the high efficiency (low fuel consumption for each mile traveled) under that situation.

The ellipse diagram

We can learn a great deal about the working of this arrangement by plotting the valve position against the piston position and following this relationship for an entire cycle of operation. For an obvious reason, this plot is called the *ellipse diagram* of the particular valve arrangement. We see an example as figure 9:

This shows the actual relationship for a certain hypothetical valve gear system. It does not make the “true sinusoidal” motion assumption; one effect of the realities of the actual relationship is that the “watermelon” of curves is not symmetrical.¹⁰

⁹ In order to prevent the reversing lever from moving on its own as a result of reaction forces on the mechanism it controls, the lever is equipped with a pawl that will drop into one of many **notches** in a sector. A grip at the top of the lever lifts the pawl out of the sector and allows the reversing lever to be moved “to a new notch”.

¹⁰ The curves in this figure were in fact taken from plots done by an excellent program for the simulation of locomotive valve gear systems developed by Charles Dockstader. They were done for a Walschaerts type valve gear system (see Appendix A) of certain typical dimensions and parameters.

The x axis represents the piston position, on a scale of -50 to $+50$ (so that each increment corresponds to 1% of the full piston stroke, which is also from $+50$ to -50). The y axis is the valve position (but no numerical scale is given).

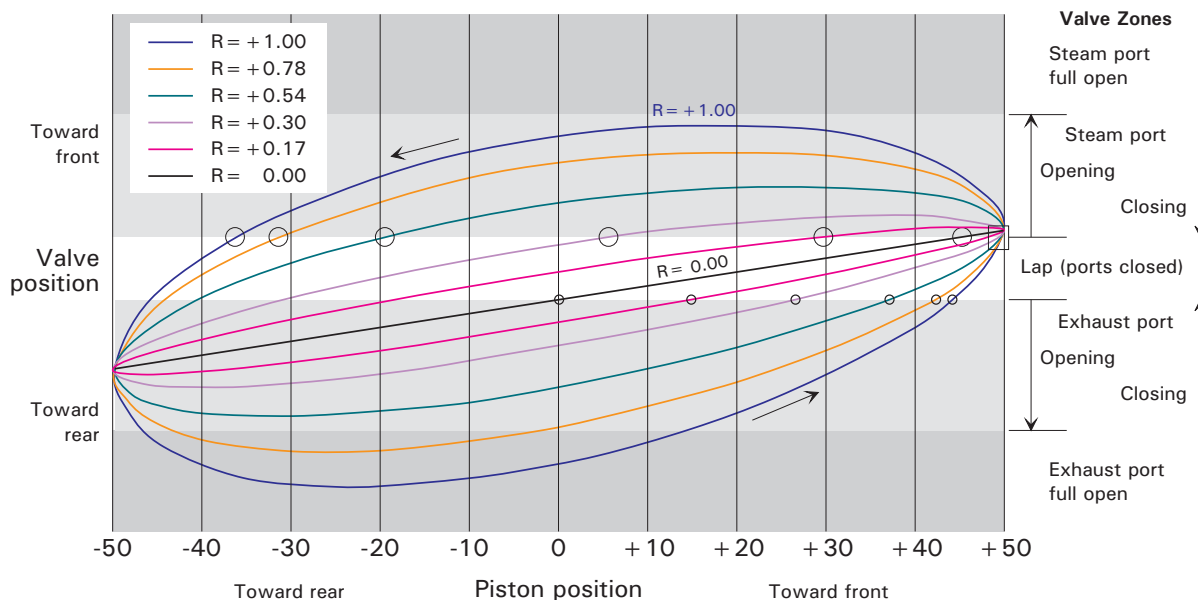


Figure 9. Ellipse diagram

The reversing lever and the parameter R

Each of the oval curves is the locus of all points describing the relationship between piston and valve position, over an entire cycle of rotation of the driver, for a given setting of the reverse lever, characterized by the parameter R . For forward operation (as on this chart), the operating point traverses the curve of interest in a counter-clockwise direction.

The curve labeled $R = +1.00$ represents the reverse lever being set to what is considered the "full forward" position. The curves labeled with lesser positive values of R represent "notched up" settings of the reverse lever, where the cutoff has been made earlier for greater efficiency when the greatest torque is not required.

The curve labeled $R = 0.00$ represents the center position of the reversing lever. Curiously enough, with the lever in this position, the engine will develop a small amount of torque, always in the direction consistent with its current motion (more on this shortly).

All the other curves on this chart represent "forward" settings of the reverse lever, and the chart shows the conditions pertaining to the front portion of one cylinder; that is, managing the steam admission to, and exhaust from, the chamber in front of the piston. There would be an almost identical figure for the portion behind the piston. And

there would be two almost identical charts for operation in reverse (with R having negative values except for the special case of $R=0$).

The vertical range of the chart (valve position, v) is divided into zones, distinguished by different colors of shading, and identified at the right-hand side. With the valve in a central range the front cylinder port (the one of interest on this chart) is blocked. Steam is not admitted, nor is there any path to the exhaust. This is often said to be the "lap" range, since the valve flange "overlaps" the cylinder port (although "lap" has a precise numerical meaning, related to this notion).

If we move the valve in the "forward" direction (upward on the chart), after a period where nothing changes, we enter the top light gray zone. At the very beginning (bottom) of that zone, the front port of the cylinder is just "cracked" to begin the admission of steam into the cylinder. As the valve moves farther in this direction, the port is further opened. When we cross into the top zone (dark gray), the port is completely uncovered and steam admission is "wide open".

If we return to the starting position of the valve, and then move it in the rearward direction (downward on the chart), after a period where nothing changes, we enter the bottom light gray zone. At the very beginning (top) of that zone, the front port of the cylinder is just "cracked" to begin the exhaust of steam. As the valve moves farther in this direction, the port is further opened. When we cross into the bottom zone (dark gray), the port is completely uncovered and the exhaust passage is "wide open".

Simple inspection of the chart (let's concentrate on the $R=1.00$ curve) shows that steam is admitted to the front side of the piston during much of the rearward stroke of the piston, and the front side of the piston is open to the atmosphere for exhaust during most of the forward (return) stroke of the piston—basically just what it would seem that we need for ongoing operation.

Cutoff in action

A closer look at the chart shows the matter of cutoff of the steam admission. Again following the $R=1.00$ curve, we note that as the piston progresses past $+10$, the port begins to slowly close. When the piston reaches a position of about -37 , the curve crosses into the white zone. At that point, admission of steam is totally cut off (a small circle highlights this point). This is said to be "cutoff at 63%", since it occurs after 63% of the stroke of the piston (that is, from $+50$ to -37 , the full stroke being from $+50$ to -50).

Next, let's look at the green curve, for $R=+0.54$. (The engine driver has "notched up" the reversing lever, needing less torque now and looking for higher efficiency.) In that case, as the piston moves

beyond about +35, the port begins to slowly close. When the piston reaches a position of about -19, the curve crosses into the white zone. At that point, admission of steam is totally cut off (a small circle highlights this point). This is a much earlier cutoff, said to be at “at 31%”).

Later in the run, the driver may further “notch up” the reversing lever to $R=0.17$, putting us on the cerise curve. Now we see that cutoff occurs with the piston at about +31: a 10% cutoff.

In fact, the driver may in fact operate with an even smaller value of R (I just didn’t draw any more curves—they get too crowded), leading to an even earlier cutoff, and even greater efficiency (albeit with a fairly small available torque—perfectly fine if we are just rolling along at speed and don’t encounter any upward grades).

Note that with the reversing lever “centered” ($R=0.00$), cutoff is quite early (perhaps at 5% of piston travel in the figure), probably not practical for operation, but it would provide some torque in whatever direction the locomotive is traveling.¹¹

Other wrinkles

Consideration of the chart reveals some other important aspects of valve operation.

It’s hard to see given the scale of the drawing, but in the region inside the small rectangular box (when the piston is nearly fully to the right), we see that the curves cross the line into the “steam port opening” zone before the piston reaches its rightmost position. This situation, called “lead”, is not counterproductive, since the steam that is admitted will be part of the total charge the cylinder receives. It is in fact advantageous in the overall scheme of engine efficiency.

The amount of lead, incidentally, is described not in terms of how far from the end of the stroke is the piston when admission begins, but rather how far open is the cylinder port when the piston reaches the end of its travel. In the valve mechanism on which this chart is predicated, this is essentially the same for any value of R (a so-called “constant lead” valve gear).

Another curiosity occurs during the “return” part of the stroke of the piston (forward in this example, for the front portion of the cylinder). Consider for a moment the $R=+0.80$ curve (blue). It shows that the

¹¹ We are reminded of the small AC motors used in aquarium pumps and the like, which are angularly symmetrical, and will run equally happily in either direction. They develop torque in the direction in which they happen to be running. Typically they will start, randomly, in either direction.

cylinder space is first opened to the atmosphere for exhaust at a piston position of about -46 (when the curve first crosses from the lap zone (white) into the exhaust port opening zone (light gray)).

When the piston reaches about $+44$, the curve crosses back into the white (lap) zone. Now the front cylinder space is closed off—no more residual steam can exhaust.

As a result, for the next little while, the residual steam is compressed, the energy coming from the kinetic energy of the locomotive. This might seem to be counter-productive, but in fact the energy put into that residual steam by this (said to be the “compression” phase of operation) will be delivered back as mechanical energy during the expansion of the steam on the forthcoming stroke.

Note that in the case of lower values of R (a reversing lever setting for an earlier cutoff), such as for the cerise curve ($R = +0.17$), we see the onset of the compression phase is much earlier (a piston position of about $+15$). The onset of compression for each curve is shown by a small circle.

The compression phase is a mixed bag, with both positive and negative implications on operation (which are well beyond the scope of this article). In reality, it is mostly a side effect of our use of a quasi-sinusoidal motion of the valve.

Lap and lead

We saw that “lead”, qualitatively, refers to the fact that the admission of steam to one cylinder end begins before the piston has actually reached that end of its travel. Quantitatively, this is not expressed in terms of how far the piston is from the end of its travel when the admission port begins to open, but rather the amount (in terms of valve travel) by which the admission port is open when the piston does reach the end of its travel.

The amount by which the valve must move from its “neutral” position before a port begins to open is called the “lap”. The term of course comes from the fact that the valve flange “overlaps” the edge of the port by some distance, and must move by that distance before the port starts to actually be uncovered. With respect to the admission port, this is called the *outside lap*; with regard to the exhaust port, it is called the *inside lap* (these terms are reversed for an outside admission scheme).

The amount by which the admission port is open when the piston reaches the end of its travel (the *lead*) is just the amount by which the valve has moved by this time minus the outside lap. Said another way, the amount by which the valve has moved by the time the piston reaches the end of its stroke is the (outside) lap plus the lead.

Although it is a little hard to see from figure 9 owing to its scale, for the system shown the lead is constant for any setting, R , of the reverse lever.

If we look at equation 2, which we assume to represent the behavior shown on the chart (and of course we know it doesn't quite), we can see why this is so:

$$v = Ra_2 \sin \Theta + a_3 \cos \Theta \quad [2]$$

When the piston reaches the end of its travel (we are assuming forward travel in this discussion), $\Theta=0$, and thus $\sin\Theta=0$ and $\cos\Theta=1$. Thus the position of the valve, v_{fdc} , at that point (*fdc* is evocative of "front dead center") is given by:

$$v_{fdc} = a_3 \quad (3)$$

Thus the lead plus lap is constant, and since the lap is constant (being a property of the valve dimensions), the lead is constant (that is, independent of R).

In some situations, it is considered desirable for the lead to vary with the "cutoff setting" (R), and so the valve gear mechanisms may be made so as to not so closely fulfill equation 2 (in particular, so that v_{fdc} does depend in part on R). Potential motives for this are beyond the scope of this article (beyond the author's ken, in fact).

The role of the second term

I started by postulating a description of valve motion with two sinusoidal components, one in $\sin \Theta$ and one in $\cos \Theta$, and as we have seen (and will see further), this in fact approximately represents the situation in the majority of locomotive valve systems. But I never explained why the early workers settled onto this model.

The attraction of sinusoidal motion is that it can be readily approximated with crank or eccentric mechanisms, which in turn lend themselves to robust construction, attractive in the locomotive context. Thus we had, in effect, an attractive implementation looking for a way to perform the needed function.

In fact, reliable operation of a steam engine can be attained with a valve motion that follows this easy-to-implement relationship:

$$v = Ra_2 \sin \Theta \quad (4)$$

and early workers were initially attracted to this approach.

Figure 4 shows the ellipse diagram for such a valve motion:

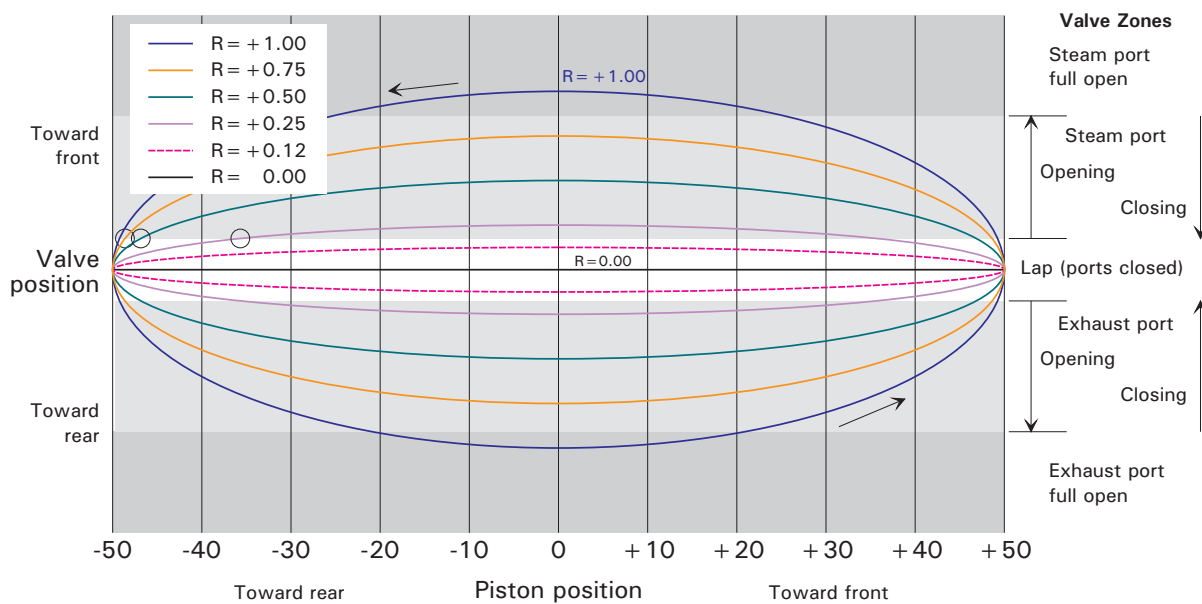


Figure 10. Pure sinusoidal valve motion

It, by the way, for convenience (mine), is based on true sinusoidal motion of both valve and piston; thus the curves are true ellipses.

Although this will in fact make the engine run reliably, it doesn't go very far in meeting our more sophisticated needs. In particular, we see that change in R cannot give us relatively-early cutoff points. In fact moderate values of R , which slightly advance cutoff, give equally delayed onset of admission and never open the admission ports a substantial amount. Smaller values do not open the admission ports at all. The curve for $R = +0.12$ (cerise) is shown dashed; with that setting, no steam is ever admitted, and no torque would be developed at all.

Adding the second term (in $\cos \Theta$), as in equation 2, shifts the phase of the (quasi) sinusoidal motion of the valve (advancing it in time, actually), to what we see in figure 9. This then, as we saw, makes it practical to reliably get early cutoff points by varying R . In addition, it advances the time of commencement of admission (in fact, introducing the feature of lead), which is advantageous in overall engine operation.

Thus, designers saw a valve motion that (fairly) well met their desires regarding the efficient administration of steam and was well suited to being implemented with robust mechanisms.¹²

¹² Did I mention that these guys were mainly Scottish?

SUMMARY

We see that with the use of a quasi-sinusoidal movement of the locomotive valve, performable by a straightforward and robust mechanism, can provide the necessary steam admission and exhaust management for operation in either direction, additionally allowing the engine driver to vary the "steam cutoff" in order to strike the optimum balance, during different aspects of locomotive operation, between developed average torque and fuel efficiency.

THE APPENDIXES

In Appendixes A and B, we examine two classical locomotive valve gear systems, the *Walschaerts* and the *Stephenson*. We will show, analytically, that they both fulfill, approximately, the valve motion function described in equation 2. Thus, we recognize that they, at least approximately, implement the concepts of valve and engine operation assumed and discussed in the body of the paper.

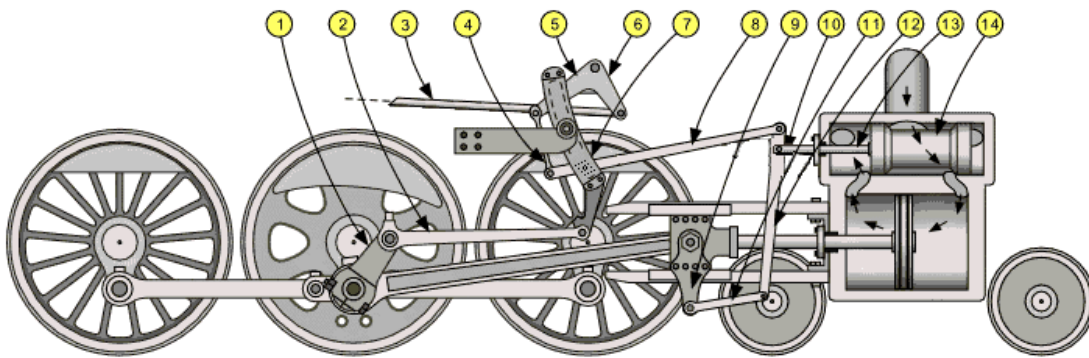
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APPENDIX A

The Walschaerts Valve Gear

One of the best known valve gear arrangements is one derived from an 1844 design by Egide Walschaerts of the Belgian State Railway, and is generally known by his name.¹³ It has remained in use on a large fraction of steam locomotives until the present time.

Here we see an illustration of a typical implementation of the Walschaerts valve gear, this on a 4-6-2 "Pacific" locomotive broadly similar to that we saw in figure 1.



1. Eccentric crank	4. Lifting link	8. Radius bar	12. Combination lever
2. Eccentric rod	5. Lifting arm	9. Crosshead arm	14. Valve
3. Reach rod	7. Expansion link	11. Union link	13. Valve Stem

Figure 11. Walschaerts valve gear

Adapted by an original by R. A. Booty, made available via Wikipedia Commons under a Creative Commons Attribution-Share Alike 3.0 Unported license.

We will see that the way in which the Walschaerts gear performs (approximately) the relationship of equation 2 is very direct.

The eccentric crank and expansion link

Just to the right of the tip of the arrow for item 1, the eccentric crank, is the eccentric crank pin (not called out on this drawing), a pin located almost exactly -90° in phase compared to the main crank pin.

¹³ When a patent was issued on Walschaerts' invention, his name was stated as "Walschaert". Accordingly, many writers refer to this system as the Walschaert valve gear.

Recall that the fore-and-aft movement of the main crank pin is:

$$x_m = r_m \cos \Theta \quad (5)$$

where r_m is the radius to the main crank pin. We take this as approximating the movement of the piston, p .

Similarly, the fore-and-aft movement of the eccentric crank pin is:

$$x_e = r_e \sin \Theta \quad (6)$$

where r_e is the radius to the eccentric crank pin. This movement is conveyed to the following stage of the mechanism by way of the *eccentric rod*.

We so far ignored the matter of how the eccentric crank pin is mounted on the driver such that main crank pin does not crash into the eccentric rod as the driver rotates.

The trick is that the eccentric crank pin is not mounted to the driver itself. Rather, it is on the end of an arm (the *eccentric crank*, item 1), which is clamped (and keyed) to the outboard end of the main crank pin (that is, outboard of the end of the main rod). Thus, the eccentric crank pin, and with it the eccentric rod, operate in a plane entirely in front of (from the perspective of the drawing) the main crank pin and main rod, and clear of them. Nevertheless, from a geometrical standpoint, the eccentric crank pin acts as if rigidly mounted to the driver, about -90° from the main crank pin.

The eccentric rod carries the fore-and-aft motion of the eccentric crank pin (give or take some small intrusion from changes in the angle of the rod) to the bottom of a swinging element, the *expansion link* (item 7). It swings on a fixed pivot about halfway up its curved portion (on the end of a J-shaped bracket). This is the key element of a mechanical analog computer element which multiplies the fore-and-aft movement of the eccentric crank pin by a parameter, R , which can have both positive and negative values. (We recognize this as the parameter changed by the reversing lever.)

The output of this "multiplier" comes from a block (sometimes called a *die block*) that fits into a curved track in the curved portion of the expansion link. It is shown dotted in the figure, just in line with the *radius bar* (item 8). It can be raised and lowered in its position in the track by the mechanism comprising the *reach rod* (3), the *lifting arm* (5), and the *lifting link* (4).

The link lifts or lowers the rear end of the radius bar (8), which in turn lifts or lowers the block. And, as we might now suspect, the reach rod is moved by the reversing lever (not seen here) in the cab.

With the reversing lever in the full forward position ($R = +1.0$), the block is at its lowest position in the link track (shown in the figure), **below** the pivot of the link. In this case, as the lower end of the expansion link moves forward (under the influence of the eccentric rod—we see it here in the forward-most position), the block moves forward, and with it the radius rod.

Imagine that instead, the reversing lever had been moved to its full-reverse position. Now the radius rod would have been fully lifted, and with it the block. Now, as the input to the expansion link (at its bottom) again goes forward, rotating the expansion link counterclockwise, the block (now being in the expansion link track **above** its pivot) moves to the rear, carrying the radius rod to the rear.

Now let's move the reversing lever back into the forward zone but not all the way. Then the block will be in the track of the expansion link below the link's pivot, but not so far as before. We can easily see that now as the link is rotated counterclockwise (by forward motion of its input by the eccentric rod), the block will be moved forward (moving the radius rod), but not as much as before.

Thus, we see that the fore-and-aft motion of the radius rod, x_r , will (if we ignore those pesky irregularities caused by angles changing) follow this relationship:

$$x_r = R r_e k_1 \sin \Theta \quad (7)$$

where as before r_e is the radius to the eccentric crank and k_1 is a constant reflecting the geometry of the expansion link.

This is exactly the first term of the relationship for valve position that we have been assuming (equation 2), where now a_2 turns out to be $r_e k_1$.

The combination lever

The *combination lever*, 12, is another mechanical analog computer element, one that adds its two inputs, each scaled by a constant.

One of its inputs, to its top, comes from the fore-and-aft motion of the radius rod (whose motion we have already examined). Its other input, to its bottom, comes via the union link from the motion of the crosshead. This is exactly the motion of the piston, and we have assumed it to be approximately $r_c \cos \Theta$. The output comes from the fore-and-aft motion of its intermediate pivot. That hangs on the end of the valve stem. Thus the output motion becomes exactly the motion of the valve, v (the desired end product of all this mechanical algebra).

If we call the length on the lever from the intermediate pivot to the top pivot l_t , the length from the intermediate pivot to the bottom pivot l_b ,

the top input motion x_t , and the bottom input motion x_b , then the expression for the output movement, x_o will be:

$$x_o = \frac{x_t l_b + x_b l_t}{l_b + l_t} \quad (8)$$

which we can rewrite as:

$$x_o = \frac{l_b}{l_b + l_t} x_t + \frac{l_t}{l_b + l_t} x_b \quad (9)$$

This shows that the output is indeed the sum of the two inputs, each scaled by a constant, those constants being determined by the (fixed) dimensions of the combination lever (l_t and l_b).

Now, substituting for x_t and x_b from our previous work, and recognizing that v , the valve movement, will be identically equal to x_o , we get:

$$v = \frac{l_b}{l_b + l_t} R r_e k_1 \sin \Theta + \frac{l_t}{l_b + l_t} r_m \cos \Theta \quad (10)$$

which we can rewrite as:

$$v = R a_2 \sin \Theta + a_3 \cos \Theta \quad [2]$$

We recognize this as the relationship we assumed in the body of the article (equation 2).

The two coefficients, a_2 and a_3 , are then seen to be:

$$a_2 = \frac{l_b}{l_b + l_t} r_e k_1 \quad (11)$$

$$a_3 = \frac{l_t}{l_b + l_t} r_m \quad (12)$$

Lead

If in fact an implementation of the Walschaerts valve gear closely follows our equation 2, as we suggest in this appendix, then it operates on a "constant lead" basis. This is essentially the most common situation.

The detailed dimensions can be varied (for one thing, the radius of the expansion link can be changed) so as to produce a lead which varies with the cutoff setting, R .

In the constant lead form, the constancy of the lead means that the sum of lap and lead is constant (since the lap is constant, being a property of the valve and port dimensions). In equation 2, that sum is precisely the coefficient of the second term of the valve movement, which term we know is directly contributed by action of the *combination lever*.

For that reason, the combination lever is sometimes known (especially in British practice) as the *lap and lead lever*. Despite that name, this lever does not produce the lap (which is a dimensional property of the valve), Rather, it produces the component of valve motion that, at its maximum, after allowing for the lap, constitutes the lead.

In fact, the combination lever also, by shifting (earlier) the phase of the sinusoidal motion of the valve, allows the variable amplitude of the first term in equation 2 (controlled by the engine driver via the reversing lever) to provide earlier values of cutoff than could otherwise be reliably attained (as was discussed in connection with figure 10 in the body of the article).

Heusinger

In Germany, this valve gear is often called the *Heusinger* valve gear after Edmund Heusinger von Waldegg, who, in 1849, independently invented essentially the same system (in a form, actually, closer to the eventually-evolved form of Walschaerts' design).

Summary

Thus we see that the Walschaerts valve gear (approximately) implements the expression for valve movement we assumed in the body of the paper. It does this with components that can be made robust, and is free from the need for any delicate adjustments in the field.

There are a number of other valve gear arrangements that essentially follow the same mathematical concepts, but the development of the two terms of our equation 2 is not as apparent in many of them.

A blast in the past

In figure 12, we get a good insight into the "robustness" of the Walschaerts valve gear as applied to a "contemporary" steam locomotive (vintage 1929).



Figure 12. Steam cleaning a CNW class "H" locomotive

Public domain

In this wonderful photo from April, 1943 by photographer Jack Delano, Mrs. Viola Sievers, one of the wipers at the Chicago and Northwestern Railway roundhouse in Clinton, Iowa gives a giant "H" class locomotive (4-8-4, "Northern" class) a bath of live steam.

The locomotive is equipped with Walschaerts valve gear.

Viola's current aim is just to the rear of the expansion link (we can see its pivot just above the steam plume).

On the lower right we see the eccentric crank and the eccentric rod (badly in need of Viola's cleaning touch).

This is a serious locomotive, with driving wheels 76" in diameter. It was 48'7" long (exclusive of tender) and weighed 498,000 lbs. The first five of its birth mates were equipped with Baker, rather than Walschaerts, valve gear.

It was made as part of a batch of 35 in 1929, and cost \$120,000.

APPENDIX B The Stephenson Valve Gear

The Stephenson valve gear is named in honor of Robert Stephenson who, with his father, George Stephenson, designed some of the earliest practical locomotives, and founded what is considered to be the first serious locomotive works.

In 1841, two employees of the works invented an improvement on the valve gear widely used at the time, and it is this arrangement we will discuss in this appendix.

Figure 13 illustrates the principle. It is not drawn to scale, and exaggerates certain things to make most clear the principles involved, and we don't show the driving wheels and the piston at all. We see it with the driving wheel set at $\Theta = 0$; that is, the piston is fully forward.

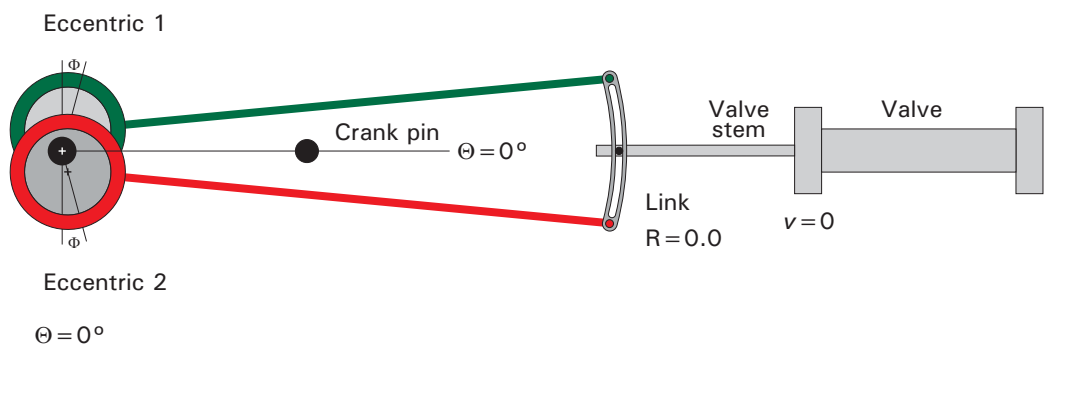


Figure 13. Stephenson valve gear in neutral, $\Theta = 0$

The valve motion is developed by two eccentrics mounted on the main driver axle. An eccentric is essentially a crank pin, one of such great diameter that it actually embraces the axis of rotation—in fact typically it embraces the axle on which it is mounted. A ring rides on the eccentric proper, and a rod attached to the ring delivers the reciprocating output of the eccentric.

When we get into our analytical work, we will as before assume that the output of the eccentric is sinusoidal.

The two eccentric rods go to the opposite ends of a curved link. In the slot of the link is a pin in the valve stem. The link is suspended by a link (not shown) that holds it at the altitude we see. As we should be able to guess by now, this suspension link is worked by the reversing lever in the cab. We see the link here in the “neutral” position, which we describe as before as $R = 0$.

We also see the driver assembly with the main crank pin fully forward ($\Theta = 0$).

The two eccentrics are mounted so that their centers (these play the same role as the center of a crank pin), observed with respect to the center of the driver axle (see the two small crosses), are at nearly $+90^\circ$ and -90° degrees to the angle of the main crank pin—but not quite. As we see, the actual angle (in magnitude) is less than 90° by an offset angle, Φ (upper-case Greek *phi*). We will later see the impact of this.

In figure 14, we see the system when the reversing lever has been put to the full forward position (“in the corner”), which we describe as $R = +1.0$. The driving wheels are still at $\Theta = 0$ (the piston fully forward).

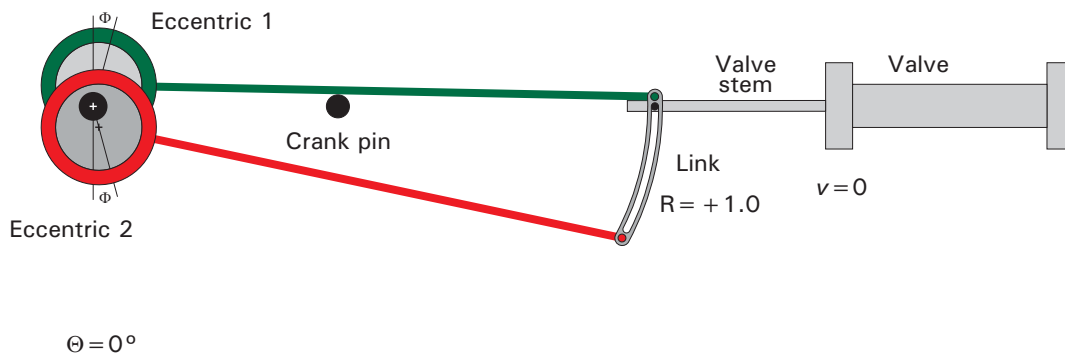


Figure 14. Stephenson valve gear in full forward, $\Theta = 0$

With $\Theta = 0$, the change in link position makes little or no change in the valve position.

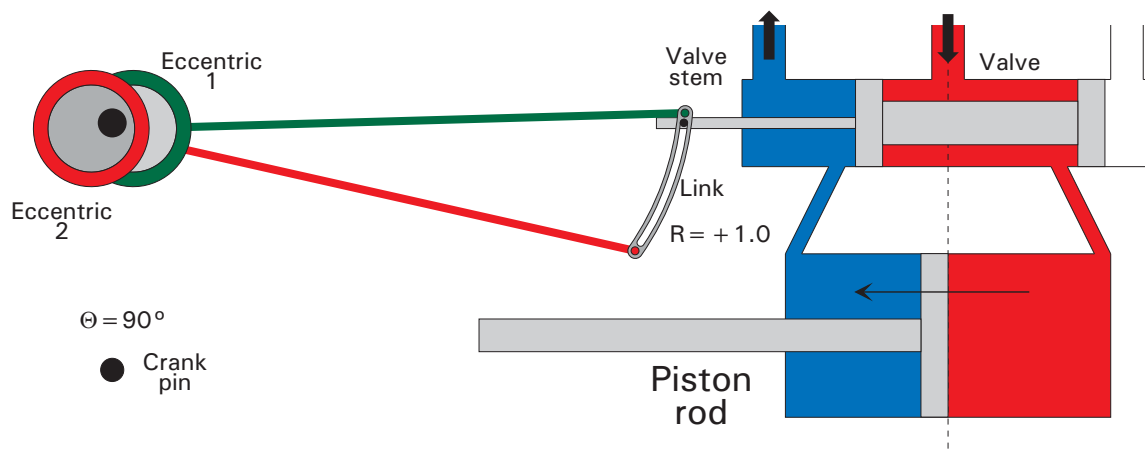


Figure 15. Stephenson valve gear in full forward, $\Theta = 90^\circ$

In Figure 15, we see the system after the locomotive has moved so that $\Theta = +90^\circ$; we've shown here the valve chest, cylinder, and piston.

Here we see that the valve spool has been moved forward from its neutral position ($v=0$), admitting steam as appropriate to drive the piston to the rear, needed at this point for the continuation of forward operation.

We will treat the link as a mechanical analog computer element that adds together the fore-and-aft motions of the forward ends of the two eccentric rods, scaled by factors that are varied with the link vertical position; that is, by the value of R . With the reversing lever in full forward position, the link is at its lowest position, which we describe as $R = +1.0$. With the reversing lever in full reverse position, the link is at its highest position, which we describe as $R = -1.0$.

The sources of the two input motions are the fore-and-aft motion of the centers of the two eccentrics, which we will call x_1 and x_2 . They are in fact sinusoidal with the rotation of the driving wheel.

Ignoring the matter of the changing angles of the rods, we assume that these are also the input motions to the top and bottom of the link.

The output of the summation is the fore-and-aft movement of the pin in the link slot. Since this pin is in the valve stem, the output of the summation is precisely the motion of the valve stem, v .

Under this outlook, the valve motion, v , is given approximately by:

$$v = \frac{1+R}{2}x_1 + \frac{1-R}{2}x_2 \quad (13)$$

We see that in fact x_1 and x_2 are summed after having been scaled by the factors $(1+R)/2$ and $(1-R)/2$, respectively.

For eccentric 1, we see the fore-and-aft movement, x_1 , is given by:

$$x_1 = r_e \sin(\Theta + \Phi) \quad (14)$$

where Θ is the variable representing the instantaneous position of the driver and Φ is a constant representing the offset of the angle of the two eccentrics from 90° ahead of and behind the angle of the main crank pin.

Similarly, for eccentric 2, we see that the fore-and-aft movement, x_2 , is sinusoidal and given by:

$$x_2 = -r_e \sin(\Theta - \Phi) \quad (15)$$

We will be interested in comparing the overall function for valve position given by this mechanism with the hypothetical one given in the body of the paper in equation 2. That is expressed in terms of the $\sin\Theta$ and $\cos\Theta$, where Θ is a variable

In order to move toward having our result for this valve gear expressed in those terms, for ease of comparison, equation 14 can be resolved into two components in the variables $\sin\Theta$ and $\cos\Theta$:¹⁴

$$x_1 = r_e \cos\Phi \sin\Theta + r_e \sin\Phi \cos\Theta \quad (16)$$

where $\cos\Phi$ and $\sin\Phi$ are constants, properties of the eccentric design.

Equation 15 can similarly be resolved into two components in $\sin\Theta$ and $\cos\Theta$:

$$x_2 = -r_e \cos\Phi \sin\Theta + r_e \sin\Phi \cos\Theta \quad (17)$$

Substituting into equation 13, we find that (subject to all our simplifying assumptions and caveats) the valve motion, v , is given by:

$$v = \frac{1+R}{2}(r_e \cos\Phi \sin\Theta + r_e \sin\Phi \cos\Theta) + \frac{1-R}{2}(-r_e \cos\Phi \sin\Theta + r_e \sin\Phi \cos\Theta) \quad (18)$$

Then, combining terms, we get:

$$v = Rr_e \cos\Phi \sin\Theta + r_e \sin\Phi \cos\Theta \quad (19)$$

This of course has exactly the form of equation 2:

$$v = Ra_2 \sin\Theta + a_3 \cos\Theta \quad [2]$$

if we set:

$$a_2 = r_e \cos\Phi \quad (20)$$

and

$$a_3 = r_e \sin\Phi \quad (21)$$

Lead

The Stephenson valve gear can be designed so that it closely follows our equation 2 (subject of course to the usual quibbles about the changes in angles of the links making various movement not truly

¹⁴ You might expect an electrical engineer to want to do this.

sinusoidal), in which case it would provide essentially a constant lead (lead independent of R).

However, as with the Walschaert gear system, the detailed dimensions can be varied (again, for one thing, the radius of the expansion link can be changed) so as to produce a lead which varies with R , and will thus be different for different cutoff values.

Comparison with the Walschaerts gear

Unlike the Walschaerts gear, in which the two terms of Equation 2 are clearly and separately developed, and then overtly added together, in the Stephenson gear the two components are developed at the same time by having the two eccentrics offset (by the angle Φ) from positions exactly 90° ahead of and behind the main crank position. This clever exploitation of trigonometry allows the Stephenson gear to be much simpler than, for example, the later-developed Walschaert gear.

The Walschaerts gear, however, has its own advantages, especially in implementation.

The common explanation of the implementation advantage of the Walschaerts gear goes like this (and of course I paraphrase):

The eccentrics of the Stephenson system are of necessity mounted inboard of the driving wheels. As locomotive sizes increased, axle diameters increased, requiring the eccentrics to have increasing diameter, and the rod end rings as well. In addition, the inboard location was not convenient from a standpoint of maintenance in the field.

In the Walschaerts gear, however, everything was outboard of the driving wheels, and the eccentric crank pin needed to only have the diameter required for purposes of strength and stiffness (far less than the needed diameter of the eccentrics of the Stephenson gear). In addition, in the Walschaerts gear, there was only one eccentric rod needed.

The slight flaw in the argument, however, is that one **can** construct a valve gear using the Stephenson principle but mounted wholly outboard of the driving wheels. It requires two eccentric links in a "zig-zag" arrangement, one mounted on the main crank pin and a second mounted at the outboard tip of the first eccentric crank pin. Having done that, there would no longer be a need for a large diameter ring end on the two eccentric rods.

But, although possible, this arrangement was not really attractive. Among other things, there were now more layers of stuff outboard of the driving wheels, potentially compromising the "clearance envelope" of the locomotive. For this and other reasons, the Walschaerts gear,

and its cousins, came to gradually displace the Stephenson gear from perhaps 1890 onward.

Summary

Thus we see that the Stephenson valve gear (approximately) implements the expression for valve movement we assumed in the body of the paper. It does this with components that can be made robust, and is free from the need for any delicate adjustments in the field.

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