Lens Principal and Nodal Points

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ABSTRACT

In discussions of photographic lenses, we often hear of the importance of the *principal points* and *nodal points* of a lens. Here we describe these concepts.

Predicting lens behavior

We can predict the theoretical behavior of a photographic lens by contemplating the paths taken by rays of light passing through the lens, using geometry and trigonometry in connection with the fundamental law of refraction (Snell's law). If the lens is thin—that is, if its center thickness is small compared to its focal length—the geometric model is very simple.

We will be considering here convex (converging) lenses.

Any light ray traveling parallel to the lens axis, but not along the axis itself, will be refracted (bent) toward the axis in passing through the lens. It will intersect the axis at a point behind the lens called the *rear focal point*. (We arbitrarily consider the light to be coming from the "front" of the lens.)

The refraction actually takes place in two steps, one where the light passes into the lens at its front surface and the other where it exits from the lens at its rear surface. Since the lens is "thin", looking at it as a "black box" (meaning that we are not concerned with how things happen inside it) we just say that the refraction takes place in the plane of the lens.

Thick lenses

But many lenses of interest to us are not "thin". Even a single-element lens having a relatively-short focal length will be fairly thick, and of course a compound (multi-element) lens, such as most camera prime lenses, will be quite "thick" (we actually tend to call it "long").

Now the geometry becomes more complex.

The principal points and principal planes of a lens

Again treating the lens as a "black box", and considering as before a light ray arriving parallel to the lens axis but not along the axis, we again find that the ray emerges from the lens following a path bent toward the axis. As before, where it crosses the axis is described as the rear focal point.

If we were to plot the arriving and emerging rays on a drawing showing a "section" of the lens (still treating its as a black box) (see figure 1), and extend each line into the lens, we would find that the two lines intersect at a point inside the lens and (usually) more than halfway through it (that is, nearer the rear than the front). That point is known as **a** rear principal point of the lens (not **the** rear principal point).

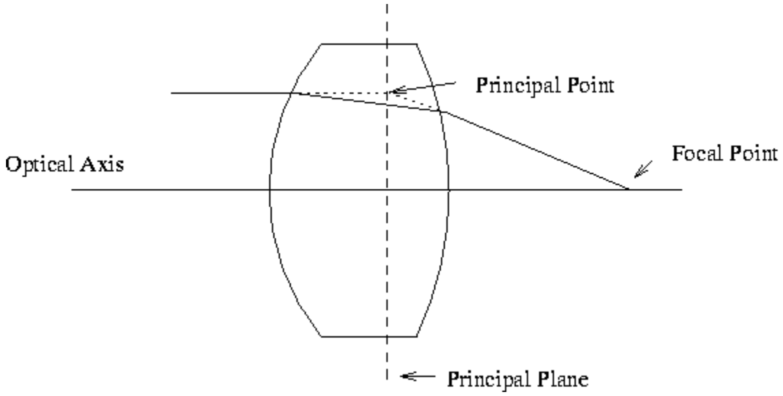

Figure 1. Principal points and planes

If we do this for many rays arriving parallel to the lens axis but at different distances from the axis, we will find different rear principal points. If life is simple, they will all lie in a plane perpendicular to the axis. This as called the *rear principal plane* of the lens.[1]

Ideally, the emerging rays from all these arriving rays will all cross the axis at the same *rear focal point*. These parallel rays can be thought of as all coming from a point on an infinitely-distant object, (the point in the center of the camera's field of view). and the situation described is how such a point is brought to focus on the film plane. A classical lens aberration, called *coma*, is the situation in which rays arriving parallel to the axis but at different distances from it, after emerging

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[1] Note that there is no principal point for a ray arriving along the axis; since such a ray is not refracted, the emerging and arriving ray paths are the same, and there is no intersection between them. Therefore, to be rigorous, the point on the principal plane where it intersects the axis is **not** a principal point!.

from the rear of the lens, cross the axis at slightly different points. Thus they do not collect at a single point on the film for any film plane location, leading to a blurred image.[2]

If we now consider a ray of light coming from the rear of the lens, traveling parallel to the axis but not along the axis, we find a situation symmetrical to what we had before. The ray emerges from the front of the lens along a path bent toward the axis. It intersects the axis at the front focal point of the lens. If we plot the arriving and emerging rays, extending the lines into the lens, we find that they intersect at a point inside the lens but nearest the front—a front principal point. If we do this for rays arriving parallel to the axis but at different distances from it, and again if life is simple, the collection of these front principal points makes up a plane, the *front principal plane*.

The nodal points of a lens

Return for a moment to our “thin” lens. Consider a ray arriving at an angle to the axis and striking the lens at its center. It will emerge from the lens along a prolongation of the same path—it will not seem to have been bent. (In fact, it will have been bent twice, once when entering the lens at the front surface and once when exiting the lens at its rear surface, but these two refractions are of equal and opposite directions, and they occur essentially at the same place, since the lens is “thin”, so the net result is no apparent change in the path of the ray.)

Return now to our real (“thick”) lens, and again consider a ray arriving at an angle to the axis and striking the front face of the lens. The ray may emerge from the rear face at a different angle to the axis. However, if we shift the path of the arriving ray (keeping its same angle to the axis), we will find an arrival path such that the ray emerging from the rear of the lens is parallel to the arriving ray.

If we then plot these two rays on a side view of the lens (as seen on figure 2), the point at which that arriving ray intersects the lens axis will be inside the lens, nearer the front side than the rear, and is called the *front nodal point* of the lens (N1 on the figure). The point at which the path of the emerging ray intersects the axis is also inside the lens, nearer the rear than the front, and is called the *rear nodal point* (N2 on the figure).

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[2] Coma is inherent in any single-element lens having spherical surfaces (the easiest kind to make). Averting coma is one of the objectives of composite lenses and lenses with aspherical (non-spherical) surfaces.

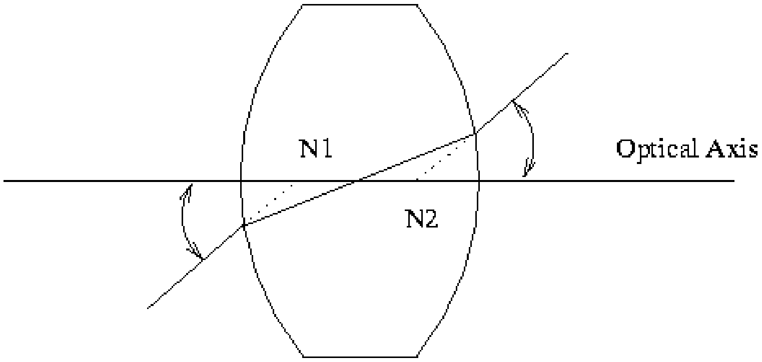

Figure 2. Nodal points of a lens

Thus, the front nodal point is the point, inside the lens, toward which an ray arriving at an angle to the axis must be aimed so that the emerging ray will emerge parallel to that arriving ray. The ray has not changed direction in passing through our thick lens, but it has of course been shifted to one side of its original path.

If the medium outside the front of the lens has the same index of refraction as the medium outside the rear (and in the cases of general interest to us this is so, since both are air), then the front and rear nodal points coincide with where the front and rear principal planes intersect the axis. (This is commonly stated as, "the nodal points coincide with the principal points", but of course there are many principal points, and in fact none of them lie on the axis!)

Significance in practical photographic work

A particular significance of the matter of the principal planes in practical photographic work is that in the equation for focus:

$$1/b + 1/m = 1/f$$

where b is the distance from the lens to the object, m is the distance to the film plane, and f is the focal length of the lens, b and m are measured from the front and rear principal planes, respectively.

Acknowledgement

The two figures in this article were taken directly from a paper by Itai Lahan and Shlomi Livne of the Technion University of Israel descrbing a research project involving a certain new optical concept for cameras.

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