Douglas A. Kerr

Issue 1 January 25, 2024

ABSTRACT

The "Hall effect" is a phenomenon in certain materials (notably semiconductors) where, if a current is passel along the x axis of a "slab" of the material, and a magnetic field (in the sense of a magnetic flux density) is applied along the z axis, a voltage appears across the slab along the y axis. This principle is used, for example, to sense the DC current in a conductor in a "clamp-on DC ammeter" (by sensing the magnetic field caused by that current in the conductor).

In certain "electronic" watthour meters, a Hall effect device is used not only to sense the current flowing through the conductors through the meter, but within itself to perform the multiplication of instantaneous voltage and current values that underlies the measurement of electrical power (and in turn, of electrical energy).

1 BACKGROUND

1.1 Introduction

In this section, before getting to our actual topic, I will review certain principles of electrical engineering that are fundamental to the measurement of the electric power flowing through a circuit.

1.2 In a DC circuit

I first note that in a "pure" DC circuit the voltage and current (at least over a short period of our interest) are constant. That having been said, the power flowing from a source into a load in a DC circuit is given by:

$$P = VI \tag{1}$$

where P is the power, V is the voltage across the load, and I is the current through the load. I use upper-case letters here to mean that these are "constant" values.

Since P, being constant, it is also the average power (over the period of our interest).

The SI unit of power is the *watt* (W), the unit of voltage is the *volt* (V), and the unit of current is the *ampere* (A).

Copyright © 2024 Douglas A. Kerr. May be reproduced and/or distributed but only intact, including this notice. Brief excerpts may be reproduced with credit.

The energy provided into the load over some period is the time integral of the (instantaneous) power; thus:

$$E = \int_{t} P dt$$
 (2)

where E is the energy conveyed over the period of interest. The conceptual SI unit of energy is the *watt-second*, but that unit has its own name, the *joule* (J). Substituting from Equation 1 into Equation 22 gives:

$$E = \int_{t} V l \, dt \tag{3}$$

Of course with P constant over the period of interest, that becomes, for period T:

$$E = TP = TVI \tag{4}$$

1.3 In an AC circuit

Here we recognize that the voltage and/or current are continuously changing during even a short period of our interest, perhaps following a "sine wave" pattern at some frequency (as we typically assume for AC power distinction situations).

The instantaneous power at some specific instant is given by:

$$\rho = vi \tag{5}$$

where the lower-case letters indicate that these are instantaneous values.

Although in electronic circuit design and some other areas we are indeed concerned with the instantaneous power, when speaking of the power consumed by some load device (a toaster, perhaps), our interest is in the time average (time mean) power (averaged over some period of time, perhaps only one cycle of the AC waveform), thus:

$$P = \overline{(vi)} \tag{6}$$

where the overbar signifies the mean of the quantity beneath (the time average being implied by the narrative)..

The use of the upper case for P here means that this is the average power.

And if we are concerned with energy (perhaps in that we have to buy it from a power utility), we again find that energy (over some perhaps long period) is the time integral over that period of the average power, or:

$$E = \int_{t} P \, dt \tag{7}$$

But since the (long term) time integral of the (short term) time average of some value is the (long term) time integral of that value itself, this becomes

$$E = \int_{t} v i \, dt \tag{8}$$

2 WATTHOUR METERS

2.1 General

Most of us encounter "electric energy meters" as the device used by our power utility to tot up all the energy we have consumed over some period so they can bill us for that.

While in scientific work the unit of energy (electrical or otherwise) is the joule (J), in the world of buying and selling electrical energy, the customary unit is the watt-hour. Since there are 3600 seconds in an hour, it follows that one watt hour is 3600 joules. And thus it is no surprise that the energy meters I speak of here are called "watthour meters".

Of course, even though the watt-hour is not an SI unit, is admits of the use of SI multiple prefixes. Because of the quantities involved in the reckoning of energy in the context I mention, the unit in which a typical watthour meter reports the energy flow through it is the kilowatt-hour (kWh). One kWh is of course 1000 watt-hours (1000 W-h), and so is 3.6 megajoules (3.6 MJ).

2.2 Electromechanical watthour meters

For many decades, the watthour meters use for metering energy usage were electromechanical. They operate on a very clever principle. They involve a special type of (AC) induction motor, with a disk rotor and two sets of windings on an odd-shaped magnetic core surrounding the rotor.

One set of windings, with a few turns of large wire, carries the actual current passing through the meter to the "load". The other set, with many turns of fine wire, is connected to the circuit passing through the meter so it is exposed to the voltage across the circuit, and thus the current through that winding set is proportional to that voltage.

Simplistically, the magnetic field caused by the current in one winding set induces a voltage in the rotor disk, which in turn leads to a current in the rotor disk. That current interacts with the magnetic field produced by the current in the other winding set to develop a torque on the rotor disk. That torque is proportional to the product of the currents in the two windings.

A magnetic brake provides a resisting torque on the rotor disk proportional to the rotational speed of the rotor. Thus the system reaches equilibrium at a rotational speed proportional to the (short term average) power through the meter.

Therefore the overall rotation of the rotor over some period is proportional to the overall energy through the meter during that period.

The rotation of the rotor disk is carried through a gear train to the train of pointers on the "register", from which the accumulation of energy is read.

This figure shows a classical electromechanical watthour meter from the front:



Figure 1. GE electromechanical watthour meter

Enough about watthour meters for now.

3 THE HALL EFFECT

3.1 Discovery

The *Hall effect* is named in honor of Edwin Hall, who in 1879 (years before the electron was discovered!), while working on his doctorate at Johns Hopkins University, discovered a fascinating phenomenon, which he studied and characterized at length.

3.2 The underlying physics

Since that time, the underlying physics has been further understood. I will not trouble the present reader with that, but will move to a practical appreciation of the phenomenon.¹

3.3 The effect in operation

I will work from this figure, which shows a hypothetical "Hall effect device":



Figure 2. Hall effect device

The shaded object is a "slab" of an appropriate material (today typically a semiconductor). A current generator, labeled I, causes a current (*I*) to flow along the **y** axis of the slab. A magnetic field (in the sense of a magnetic flux density) B exists along the **z** axis of the slab. The result is that a voltage, $V_{\rm H}$ (the "Hall voltage") appears between two points along the **x** axis of the slab.

We observe that voltage with the voltmeter shown at the top (which we assume has a very high impedance, so it draws essentially no current from this mysterious voltage source).

If in fact the current direction, the magnetic field direction, and the axis between the points between which we determine the voltage are all mutually orthogonal, as the figure suggests, then:

$$V_{\rm H} = {\rm K}/B \tag{9}$$

where K is a constant that depends on the material and geometry of the slab.

¹ The reader who would like to learn about the underlying physics is commended to the excellent Wikipedia article, "Hall effect".

4 THE ELECTRONIC WATTHOUR METER

4.1 General advantages

As clever as the electrometrical watthour meter is, it has its disadvantages. For one this, its manufacture involves hundreds of parts, many of them very "fussy", which have to be made and then assembled, the latter a process that cannot much be mechanized.

If special data capture is needed (as needed for the more complicated energy pricing schemes now in use), that is complicated to do, or for some desirable types of data, impractical. And of course the mechanisms wear out with use.

Not surprisingly, in recent years "electronic" watthour meters have come into widespread use. They have no moving parts, and can be given enormous complexity and flexibility at modest manufacturing cost.

Often, these are housed in the same general style of "fishbowl" enclosure used for classical electromechanical watthour meters, with the same mechanical mounting dimensions. They typically also have the exact same "plug-in" connections, and so an electronic meter can be put in the place of an electromechanical one in just a couple of minutes as an electrical utility upgrades its metering situation. We see a typical one from the front in this figure:



Figure 3. Itron electronic watthour meter

4.2 A further wrinkle

A further issues when we have a home (or small business, but I will say "home" for simplicity) with a solar electrical energy system that is arranged to interact with the utility company's electrical supply "grid". In general, such a system strives, at each moment, to deliver as much AC power as it can, considering the present solar irradiation on its photovoltaic (PV) panels (perhaps limited by the throughput limit of the inverter that converts the DC from the panels into AC for use.

Almost inevitably at certain times that generated power is greater than is, at the time, being used by the various loads in the home. The result is that then the "excess power" (and thus energy) flows **from** the home **to** the electric grid of the power utility.

How the homeowner gets credit for that is a complicated topic far beyond the scope of this article. But from our point of interest, the schemes typically rely on the watthour meter, over the period between two successive "readings", changing its indication by the **net** of the energy flow into the home from the grid. For the billing period, that net amount might even be negative (that is, more energy is sent into the grid than is taken from the grid.

Thus the meter has to "run backwards" at any time in which the power flow is **into** the grid **from** the home.

There is good news and bad news with respect to a traditional electromechanical watthour meter being able to play this game. The good news it that the basic concept of the meter is wholly symmetrical. When the power flow is in the "forward" direction, its motor turns "forward" and its reading advances. When the power flow is in the "reverse" direction (which did not formerly happen to it), its motor turns "backward" and its reading recedes. So far this sounds like just what we need. This should be a true "net energy flow" meter.

But there is a clinker. While many steps are taken in the design of an electromechanical watthour meter to minimize friction, some remains, and this can cause a small (but not negligible) shortfall in its reckoning of energy.

To overcome that, there is a feature in the design of the "motor" of the meter that produces a small forward torque (assuming that the normal **voltage** input is present) even when the **current** through the meter is zero. This can be "tweaked" with an internal adjustment so that it just compensates for the friction.

The problem with this is that when the meter runs backwards (something it really wasn't originally designed to do), this compensating torque is still in the "forward" direction. But now this does not work to overcome the friction but to actually cause a further shortfall in the meter's reckoning of the "reverse direction" energy.

This is of course not a large error, but meters used to "charge for energy" (which includes various ways to deal with the "buying and selling" that goes on when the home has a solar energy system) are required to have a high accuracy (said to provide "revenue grade metering"). And when operation in the "backward" direction is involved, the typical electromechanical meter cannot meet those standards.

We haven't yet heard how an electronic meter measures power (and thus energy), but it is not hard to imagine that this can be made precisely symmetrical, so the meter can meet the standards of accuracy when running in either direction.

5 THE HALL EFFECT IN ONE MANUFACTURER'S ELECTRONIC WATTHOUR METERS

5.1 Circuit and operation

The manufacturer ltron, in their "Centron" line of electronic watthour meters (for example, as seen in Figure 3), uses a Hall effect device not only to sense the instantaneous current flow in the circuit conductors passing through the meter but also to perform the multiplication shown in Equation $5.^2$ We see the arrangement in a semi-schematic way in this figure:



Figure 4. Hall effect device in a watthour meter

Figure adapted from figure by Itron in one of their technical publications. Used here under doctrine of fair use.

² A similar approach may well be used by other manufacturers. I have no insight into that.

This is set up for a 120/240 V single phase system (as found in essentially all residences today).

The **Hall effect device** (HED) is shown in the center of the **core**, which is made of iron laminations, like a transformer core.

The L1 and L2 conductors of the power line each pass through one of the two lobes of the opening in the core, each in effect becoming a 1-turn winding. The joint effect of the currents in those two conductors. i_1 and i_2 , generates a magnetic field (in the sense of a magnetic flux density, *B*) across the two pole tips surrounding the HED. This field is proportional to the instantaneous sum of the two line currents (given the direction conventions I show on the figure).

The voltage across the two line conductors, v, is fed through resistor R1 to contacts on the the ends of the Hall crystal slab, causing a current proportional to the instantaneous line voltage along its length.

The result is a voltage, v_{H} , that appears "across" the width of the Hall slab (and picked up by contacts there). That voltage is given by:

$$v_{\rm H} = {\rm K}i_h b \tag{10}$$

where i_h is the longitudinal current through the Hall slab, b is the magnetic field through the slab, and, K is a constant depending on the material and dimensions of the slab.

But we know that:

$$i_h \propto v$$

and (11)
 $b \propto (i_1 + i_2)$

where \propto is the symbol for "is proportional to".

Putting all that together, we get:

$$v_{\rm H} = {\rm G} v (i_1 + i_2)$$
 (12)

where G is a constant that depends on various details of the design.

But it can be shown that (see Section 5.2):

$$\boldsymbol{\rho} = \boldsymbol{v}(i_1 + i_2) \tag{13}$$

So:

$$v_{\rm H} = {\rm G}\rho \tag{14}$$

That is, v_{H} is proportional to the instantaneous power passing through the meter.

Voltage $v_{\rm H}$, which is fairly small, is (amplified by a differential amplifier), and the resulting voltage is "strobed" periodically and digitized by an analog-to-digital converter, the result again being proportional to the instantaneous power through the meter (which can be either positive of negative). The time integral of that power value is calculated digitally. The result is used to drive a display reading in kWh.

There are of course many subtle things to be done in the course of this process to make it work as intended. These are beyond the scope of this article.

5.2 About the two120 V circuits

The astute reader will be aware that the instantaneous currents in the two "legs" of the incoming power line (L1 and L2) may well not be equal. That is because the various 120 V load circuits in the house are arbitrarily connected from either L1 or L2 to the neutral conductor, N, which we see however is completely ignored by the meter. At any time, the total loads on the two legs may be quite different.

To be specific, the instantaneous power being carried by the circuit comprising L1 and the neutral is the product of the current in L1 and the voltage from L1 to the neutral. At the same time, the instantaneous power being carried by the circuit comprising L2 and the neutral is the product of the current in L2 and the voltage from L2 to the neutral. The total instantaneous power at any instant is the sum of those two products.

Yet we do not have two separate Hall effect devices to reckon those two products. So how does that work out properly?

The secret ingredient is that L1 and L2, at any instant, with respect to N, have essentially equal but opposite voltages. That is because L1 and L2 come from opposite ends of the secondary winding of the power utility distribution transformer serving this house (and probably several others as well), and N is connected to the center of that winding.

The total instantaneous power through the meter is of course:

$$\boldsymbol{\rho} = \boldsymbol{\rho}_1 + \boldsymbol{\rho}_2 \tag{15}$$

where p_{L1} and p_{L2} are the instantaneous powers over the L1-N circuit and over the L2-N circuit, respectively.

That can be expanded to:

$$\rho = v_{L1N} i_{L1} - v_{L2N} i_{L2} \tag{16}$$

where v_{L1N} is the voltage from L1 to N and v_{L2N} is the voltage from L2 to N. (The minus sign is because of the conventions I have chosen for the directions of the currents.)

But because of what we just learned about the symmetry caused by the distribution transformer secondary:

$$v_{L1N} = \frac{v}{2}$$

and (17)
$$v_{L2N} = -\frac{v}{2}$$

so we can rewrite Equation 16 as:

$$\rho = \frac{v}{2}i_{L1} + \frac{v}{2}i_{L2} = v(i_{L1} + i_{L2})$$
(18)

which of course is just what the circuit in Figure 4 seems set up to do.

Quod erat demonstrandum.