The Effect of Inserting a Flat Glass Plate into the Optical Path Downstream from a Lens

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ABSTRACT AND SUMMARY

If a flat glass plate is inserted, perpendicular to the optical axis, into the path of rays from a lens heading to form an image, its effect is to shift the point of convergence of the rays (the point at which an image is formed) away from the lens by an amount depending on the thickness of the plate and its index of refraction.

In this article, we derive the expression for the amount of this shift.

INTRODUCTION

In this discussion, we will treat *paraxial* rays—rays that are very nearly parallel to the axis. As a result, their angle of incidence at interfaces with optical elements will be very small. They focus just like other rays, but by considering only them, we can enjoy an approximation that is useful for small angles: that the sine of an angle, the tangent of the angle, and the angle itself (expressed in radians) are all equal.

In our notation, we will use u to represent the slope of a ray (that is, the ratio of its change in distance from the axis to the "horizontal" distance as the ray passes between two points). We will use y to represent the height of a ray, above our reference axis, at some particular point in its travels.

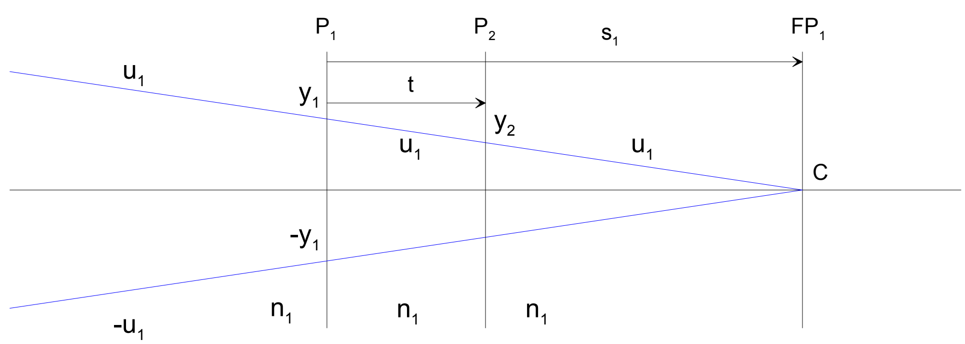

Figure 1. Basic setup

In figure 1, we see two rays, downstream from an image-forming lens, preparing to converge at the focal plane, FP_1. They are symmetrically disposed, and so their slopes are equal but of opposite sign. Actually, u_1, the slope of the ray that we follow, has a negative sign, since its height decreases as it progresses to the right.

We define two planes, P_1 and P_2, which here have no physical significance, but which later will be the boundaries of the glass plate (of thickness t) we will insert into the ray path.

The symbol n represents the index of refraction of the medium. Here, the medium is all the same, and so in all three "regions" (delineated by our two currently-arbitrary planes) the index is noted as n_1. (Ordinarily air is the medium, and thus the actual value of n_1 is very nearly 1.)

We assume that the slope of the top ray as it arrives in our scenario is u_1. Although for graphic convenience we show the ray with a modest slope, it is actually meant to be a paraxial ray (of infinitesimal slope) so we can enjoy the approximation mentioned above. The ray maintains that slope throughout its travel.

In the figure, as a result of the symmetry of the two rays shown, the point of convergence (by definition, on the focal plane) will be on the axis.

We can calculate the distance, s_1, from plane P_1 to the point of convergence as:

$$s_1 = \frac{y_1}{u_1} \tag{1}$$

just from the definition of slope.

We will also note that as the ray crosses plane P_2, its height above our reference axis is given by

$$y_2 = y_1 - u_1 t \tag{2}$$

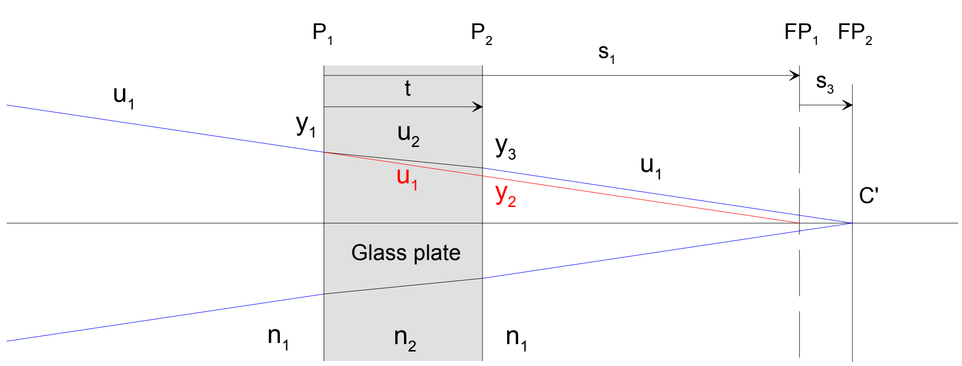

Figure 2. Glass plate inserted

In figure 2, we have inserted a glass plate of thickness *t* between planes P_1 and P_2. We will assume that the index of refraction of the glass in the plate is n_2.

Our subject ray is refracted (bent) when it crosses the interface between the air and the glass. Because the new index of refraction is greater than the old one, the ray is bent "toward the normal", where *normal* means a line perpendicular to the interface surface. In this situation, this means that the slope of the line in the plate is less than it was when it arrived. We call the slope of the ray inside the plate u_2.

(Note that for reference we also show, in red, the continuing path of the unrefracted ray, carried from figure 1.)

The new slope can be calculated via Snell's law. With the "paraxial" approximation at work, that can be expressed without trigonometric functions as:

$$u_2 = u_1 \frac{n_1}{n_2} \qquad (3)$$

Let us note at this point that, when the ray exits the plate (back into air, with index of refraction again n_1), its original slope, u_1, is restored. (We can accept this by symmetry; the application of equation 3 again would prove it.)

We can now calculate the height of the ray when it exits the plate as:

$$y_3 = y_1 - u_2 t \qquad (4)$$

Substituting for u_2 from equation 3 gives us:

$$y_3 = y_1 - u_1 \frac{n_1}{n_2} t \qquad (5)$$

If we compare the difference in the heights of the ray at P_2 in the two cases (from equations 5 and 2), we get:

$$y_3 - y_2 = \left(y_1 - u_1 \frac{n_1}{n_2} t \right) - (y_1 - u_1 t) \qquad (6)$$

Simplifying gives us:

$$y_3 - y_2 = u_1 t \left(1 - \frac{n_1}{n_2} \right) \qquad (7)$$

Since, for all practical purposes, n_1 is 1, we can rewrite this as:

$$y_3 - y_2 = u_1 t\left(1 - \frac{1}{n_2}\right) \tag{8}$$

The expression on the right side shows how much the descent of the ray has been delayed by the presence of the glass plate.

Now, from this point on, the slope of the ray is the same (u_1) as it would have been without the plate. Thus delaying its descent by the mentioned amount in effect delays its arrival at the axis (the point of convergence) by that amount divided by the slope for the reminder of the journey. Thus the delay in convergence is given by:

$$s_3 = \frac{u_1 t\left(1 - \frac{1}{n_2}\right)}{u_1} \tag{9}$$

Simplifying gives us:

$$s_3 = t\left(1 - \frac{1}{n_2}\right) \tag{10}$$

The expression on the right side gives us the displacement in the point of convergence caused by the insertion of a glass plate of thickness *t* and index of refraction n_2. It is positive for displacement to the right.

Since n_2 is greater than one, the expression will be positive. Thus, the insertion of the plate increases the distance from the lens to the point of convergence—the image distance). In the figure, we label that point as the location of FP_2, the new plane of focus.

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