The Geometry of "Toward Mecca"

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ABSTRACT AND INTRODUCTION

Muslims worldwide are exhorted, with regard to their daily formal obligatory prayers, to face in the direction of The Kaaba, a stone structure located within the Sacred Mosque in the city of Makkah (often presented as "Mecca"), Saudi Arabia. It is considered the holiest place in Islam. The concept is often described in the popular press as praying "toward Mecca", thus the title of this article. While that mandate seems simple on the surface, when we consider that the Earth is not flat, we immediately run into the matter of how should "in the direction of The Kaaba" be interpreted if one is any significant distance from The Kaaba. Countless works have been written by Islamic scholars over the years on this matter. Two pragmatic premises for making the determination (giving quite different results) have been widely "taught" in modern times by different Islamic advisors. In this article we examine these two premises from a standpoint of the geometry they seem to imply. No attempt is made to judge which is the "most appropriate", although some personal observations are offered.

CAVEAT

I am not a Muslim by faith nor an Islamic scholar or "expert". My perspective on this matter is from a strictly secular basis, predicated on widely-presented descriptions of the two premises that are recommended.

I make no attempt to describe or recognize how often or at what times Muslims (adherents of whatever branch of Islam) are expected to pray in the formal, obligatory sense[1] to which the issue of "toward The Kaaba" pertains.

In the background material that follows, the various Islamic concepts and their descriptions in Arabic terms are not intended to be definitive nor claimed to be accurate. Rather, they are presented according to my best understanding solely to allow me to provide a concise context for the secular technical discussion which is the aim of this article.

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[1] These prayers are often described by the Arabic word *salah*.

BACKGROUND

Arabic words

Many of the items mentioned here are by tradition (or prescription). formally described by words of the Arabic language. I will not ordinarily present these in Arabic (for one thing, I have no skill in that language, editorial or otherwise), nor in the formal ALA-LC[2] transliteration using an augmented Roman alphabet. Rather, I will generally use one of the "popular" transliterations. Often, more than one such is in common use throughout the entire contemporary scope of journalism, reference books, and such. I will in each case choose one and use it consistently. I do not mean by so doing to suggest that this is the "most proper" transliteration (although in each case I believe I have justification for considering it "valid").

However, I will in many cases I give in a footnote the Arabic word itself, the ALA-LC transliteration (in italics), and other commonly-seen transliterations or transcriptions.

In the case of a city of particular interest (in the past, and to a great extent still today, known as "Mecca"), my understanding is that its "official" English-language name is now "Makkah", and I will use that. I will not here join in the argument as to whether or not the use of "Mecca" is erroneous or disrespectful.

The Kaaba

The Kaaba[3] is a rectangular stone structure located in the Sacred Mosque[4] in Makkah[5], Saudi Arabia (also often called the Grand Mosque). It is considered to be the most holy place in Islam.

The exhortation to "face The Sacred Mosque"

The exhortation to "face the Sacred Mosque" (during formal prayer) appears many places in the Holy Quran[6]. One citation often given is this:

"Turn then Thy face in the direction of the Sacred Mosque" [Quran, Surah 2: Verse 144 (in part); I quote from the Yusuf Ali translation.]

This exhortation is today generally interpreted to mean specifically in the direction of The Kaaba.

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[2] American Library Association/Library of Congress

[3] الكعبـــــة, *al-Ka'bah,* The Kabaa, The Kabaah

[4] الحـرام المسـجد, *Al-Masjid al-Ḥarām*

[5] مكة, *Makkah* (to give its short Arabic name), Mecca

[6] القـــرآن, *al-qur'ān*, The Koran, The Qur'an, many other transliterations

Qibla

The direction "of the Holy Mosque" is referred to as the *Qibla*[7], an Arabic word meaning roughly "direction".

GEODESY AND SPHERICAL GEOMETRY

The dilemma

The "instruction" to "face the Holy Mosque" (for precision, we will generally characterize it from here on as to "face The Kaaba") seems at first simple. If we are at a location "two miles southwest" of the The Kaaba, then clearly the Qibla there is to the northeast.

If we wished to be wholly precise (and extreme precision is of course not necessarily needed in the matter of the direction of prayer), "two miles to the southwest" does not even have a simple obvious meaning. The reason is that the surface of the Earth is not flat, but rather is (approximately) a sphere.[8] The complication comes to the fore in matters of land surveying, even when relatively small distances are involved.

If we now consider a point at a substantial distance from Makkah, the issue becomes much more important.

Islamic scholars early realized the issue, and for many years there have abounded innumerable learned writings on how to deal with it.

Modern practice

Today, many instructions to Muslims as to how to reckon the Qibla for their prayer location revolve around two basic geodesic premises, which we can describe thus:

a. The "great circle" premise: the direction toward The Kaaba at any place on the Earth is considered to be the departure azimuth of the shorter limb of the *great circle* passing that location and the location of The Kaaba.

b. The "rhumb line" premise: the direction toward The Kaaba at any place on the Earth is considered to be the heading of the *rhumb line* from that location to the location of The Kaaba.

The pivotal terms *great circle* and *rhumb line* will be defined shortly.

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[7] قبلــــة, Qiblah, Kiblah, Kibla

[8] More precisely, approximately a *spheroid*; often assumed to be one of several precisely defined spheroids for formal geodesic technical work.

Note that these two premises are wholly incompatible. For most locations they produce gigantically different directions as "toward The Kaaba". (I will philosophize about this paradox later.)

The great circle premise

I will sneak up on the working of the great circle premise through an example for an actual site at a substantial distance from The Kaaba.

Here in my office in Weatherford, Texas, USA, to direct my gaze directly toward The Kaaba (futile from a visual standpoint, of course, since it cannot be seen from here) I would need to:

- First face a compass heading of approximately 39.5° (that is, 39.5° east of north).

- Then direct my gaze approximately 56.2° below horizontal.

That is, the line joining my location with that of The Kaaba has an *azimuth*[9] of 39.5° and an *elevation* of −56.2°.

These are, for example, the azimuth and elevation coordinates that would be given by a surveyor's transit, properly leveled and aligned with true north, with its telescope pointing directly at The Kaaba.

Under this premise, the azimuth of that line (39.48°) is considered to be the direction "toward The Kaaba".

Why the technical name of the premise? This compass direction turns out to be the "shorter limb departure azimuth" of a *great circle* of the Earth passing thorough my location and that of The Kaaba. (A proof is given in Appendix A.)

The great circle

A *great circle* of the Earth is a circle through the surface of the Earth (we assume for purposes of the definition that the Earth is precisely spherical) lying in a plane through the center of the Earth. Thus the Equator, and all meridians of longitude, are great circles. Parallels of latitude other then the Equator are not. There are an infinity of other great circles on a sphere.

A great circle has several important properties. It is (as its name suggests) the largest circle that can be formed on the surface of a given sphere. [This is however not of importance to the issue here.] The shortest travel distance along the surface of a sphere from one point on the surface to another is along the (shorter portion of) a great

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[9] A word of Arabic orign, from a word that also means, roughly, "direction".

circle passing through those two points. [This is however not of importance to the issue here.]

Another metaphor

This metaphor is often advanced with regard to the great circle premise for the Qibla.

Suppose we are at a point sufficiently far from The Kaaba that it is well below the horizon (the distance being enough that the effect of the curvature of the earth would have to enter into precise calculations).

Imagine now that we have a powerful beam of light directed directly upward from The Kaaba, and we can see that (owing to scattering in the atmosphere) from our point of observation. We observe the compass direction toward the beam of light at the horizon.

That direction is precisely the one that would result from the model I first described. (The proof is in Appendix A.)

Yet another metaphor

Some might suggest that a better metaphor would have the beam of light arising from The Kaaba site not vertically but rather parallel to a vertical line at the point of observation.

We then again observe the compass direction toward this beam of light at the horizon.

This turns out to be the same compass direction as in the previous model. (The proof is in Appendix A.)

The rhumb line premise

This premise for determining the compass direction "toward The Kaaba" is commonly (and aptly) called the "constant compass direction" premise.

This premise can be described as follows. Consider the compass direction such that, if I set off from the point of observation on a journey along the Earth's surface, initially in that compass direction, and continuously maintain that compass direction as my heading for the rest of the journey, I would eventually arrive at The Kaaba.

Under this premise, that compass direction is considered to be the direction "toward The Kaaba".

Why the technical name of the premise? The track of such a journey is called, in navigational theory, a *rhumb line*. Maritime travel to a

destination on such a basis is called *Mercator sailing*, since a rhumb line becomes a straight line on a map using the Mercator projection.

A rhumb line in actual space is of course technically not a line (which term implies "straight line"), since it follows a curved surface (the surface of the Earth).

Of the great circles of the Earth, the Equator and all meridians of longitude are also rhumb lines; other great circles are not. All parallels of latitude (not great circles except for the Equator) are rhumb lines.

Although this is of no consequence insofar as the matter of the Qibla is concerned, from an actual standpoint of geodesy we note that the total distance along a rhumb line from point A to point B is in general greater than the distance from point A to point B along a great circle of the Earth.

MY OUTLOOK

Introduction

We sometimes hear that we should not attempt to understand or evaluate various premises for reckoning the Qibla using logic, geometry, trigonometry, or other mathematical tools. This is, "they" say, because the Qibla is a matter of religious doctrine, which transcends (perhaps "trumps") such secular conceits, rendering them inappropriate and useless.

Fair enough. As Hamlet says[10], "There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy."

My work here, though, merely seeks to understand the technical implications of two (wholly incompatible) premises, described in "secular" terms, recommended by two camps of Islamic "experts". I could hardly suggest that one or the other is "more appropriate"—that it is the best fulfillment (from a religious standpoint) of the instruction in the Holy Quran to "face the Sacred Mosque".

That having been said, let me make some personal observations on the two premises. The reader, perhaps aware of and harking to some consideration or outlook of religious (or other) origin, may come to a quite different conclusion.

"The direction toward The Kaaba"

In my opinion, the least technically ambiguous meaning of "the direction toward The Kaaba" would be the direction of the line joining

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[10] Shakespeare, W, "Hamlet", Act 1, scene 5

the location of the person with the location of the Kaaba—yes, in most cases that would be through the Earth.

But that clearly is not an outlook that is useful to the praying Muslim. Would he adopt a posture in which his chest was perpendicular to that line? This notion could not fit in with the fact that different postures (from the upright to the prostrate) are in fact taken during the different phases of the formal, obligatory prayer. The following figure, from the Wikipedia article on *Salah*, will serve to illustrate my point.
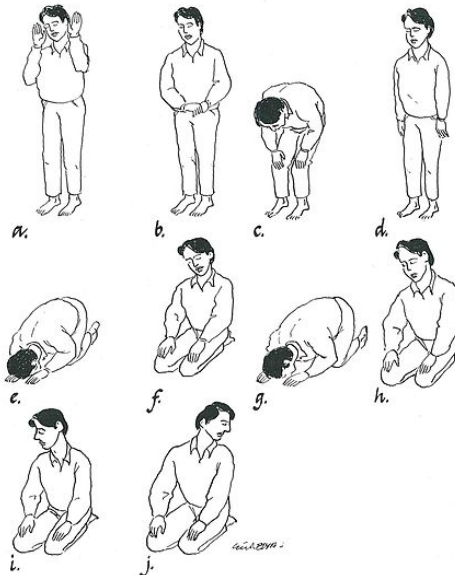
[I make no suggestion that this is a definitive description of any appropriate practice.]

So the "direction" in which the person is exhorted to face must have a practical interpretation as a direction in the horizontal plane (which I often speak of as a "compass direction"). And determining what that is becomes the dilemma central to the issue discussed in this article.

On the great circle premise

The compass direction determined in accordance with the great circle premise seems to me to most directly flow from the concept of "the direction toward The Kaaba". I find it hard to believe that, at my office, any compass direction other than (approximately) 39.5° is better considered as the compass direction "facing The Kaaba". It is the compass direction in which I must face my body to most directly "face The Kaaba" (which of course also requires that my gaze be below the horizontal).

On the rhumb line premise

I personally find no basis for embracing the rhumb line premise on "the direction facing The Kaaba", which is based on a hypothetical journey over the face of the earth from the person's location at prayer to The Kaaba. Recall that we are concerned with a static concept for a person now at a particular location, not with traveling to The Kaaba.

For reference, the direction toward The Kaaba from my office is reckoned under the rhumb line premise as approximately 95.3°. This differs from the direction determined under the "great circle" premise by about 55.8°.

Qualitative descriptions

We can see from the discussion above why we often see, in different articles discussing the general matter of the Qibla, these two statements:

a. For most points in North America, the Qibla will be to the north of east.

b. For most points in North America, the Qibla will be to the south of east.

Well! That's clear enough.

A "straw man" argument

Those arguing against the great circle premise sometimes proffer this argument:

"The great circle contains the shortest path from the point of interest to The Kaaba. However, the distance to The Kaaba is not a criterion here. Thus we should not adopt a great circle as our premise."

The first two parts are true. But consideration of the distance to The Kaaba indeed played no part in the development of the "great circle" premise I gave above, nor did I even begin by assuming that a great circle was involved. It turns out at the end that the direction of interest is in fact along a great circle, and the matter of shortest distance is a byproduct of the properties of a great circle.

QIBLA CALCULATORS

There are available many tools to assist the practicing Muslim (perhaps while "on the road") to determine the Qibla for his location, that he might observe it in connection with his obligatory prayers. Some are instruments that include a magnetic compass to allow the result to be easily utilized (there typically not being a compass rose in the typical motel room), and their workings contain provisions for taking magnetic

deviation into effect (since the basic "calculations" return a result in terms of true, not magnetic, compass direction). These tools often include a "directory" of their input parameter in terms of the name of the city.

Many of these tools are of course today available as "online calculators". Most essentially follow the "great circle" premise; a few follow the "rhumb line" premise; and some offer the user a choice of either (not normally identified by those names).

While this ambiguity might seem worrisome, we must keep in mind that this is a matter of religious doctrine, and what might seem worrisome in a context of surveying, engineering, farming, or real estate litigation may be comfortably embraced by a more spiritual outlook.

In any case, as I understand it, the ultimate Islamic doctrine on this matter is that each Muslim should determine the Qibla for his use as he is best able to do so.

Well said.

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APPENDIX A

Proofs pertaining to the great circle model

The surveyor's transit metaphor

Here we demonstrate that the azimuth indication of a surveyor's transit aimed directly at The Kaaba is in fact the departure azimuth of the short limb of a great circle of the Earth through the point of observation and the location of the Kaaba.

We level the transit base and orient the azimuth circle such that with the telescope pointed due north (and horizontal, if you find that comforting) the azimuth circle reads 0°.

We then (somehow) point the telescope directly at The Kaaba (along a line that in essentially every case will pass through the Earth). The "great circle" premise suggests that we should consider the reading of the azimuth circle of the transit to be the compass direction "toward The Kaaba".

Now, with the azimuth axis of the transit locked, we "plunge" the telescope until it points straight down ("to the nadir"). It now points to the center of the Earth, and the azimuth circle reads −90.0°. (Note that with the instrument leveled, its elevation axis is horizontal; that is, it is tangent to the surface of the Earth.)

Consider now a plane defined by three points: the point of observation, The Kaaba, and the center of the Earth. Because the telescope, with its azimuth axis locked, located at one of those points (the point of observation), can be swung on its elevation axis to point to the other two of those points, its pointing direction always lies in that plane (the elevation axis is perpendicular to that plane).

Consider now a circle on the surface of the Earth that lies in that plane. Since that plane passes through the center of the Earth, this circle must be a great circle of the Earth.

We now elevate the transit telescope to the horizontal. Its aiming axis intersects the great circle[11]; it lies in the plane of that circle; and it is perpendicular to the radius of the circle at the point of intersection (the line from the center of the earth to the transit). This the telescope axis is now tangent to this great circle.

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[11] It actually misses by its height above the earth, a small discrepancy in the scale involved here.

A line tangent to a circle at a point is indistinguishable from an infinitesimal segment of the circle at that point. Thus the direction of the tangent is the direction of that infinitesimal segment of the circle. We can think the direction of that segment of the circle to be the "departure azimuth" of the circle—the direction in which we would initially move to travel along the circle.

Thus the azimuth of the line that is actually directly to the Kaaba is the departure azimuth of a great circle of the Earth through both the current location and the location of The Kaaba.

Quod erat demonstrandum.

The "vertical beacon" metaphor

I mention a metaphor involving a hypothetical "beacon" of light emitted vertically upward from The Kaaba, which can be seen by a person at a point a fair distance from The Kaaba. I asserted that, looking in a horizontal direction toward that "beacon", the azimuth is again the departure azimuth of the great circle of the Earth passing through the point of observation and The Kaaba.

The line of that beacon passes through the center of the Earth. Since it also passes through The Kaaba (its point of origin), it thus lies in the same plane mentioned in the earlier proof. Thus, the telescope would "aim at it" at any elevation (since all aiming lines of the telescope would lie in that plane). Thus the azimuth reading of the telescope would be the same as before, which we earlier demonstrated was consistent with the departure azimuth of the great circle of the Earth passing through the point of observation and The Kaaba.

Quod erat demonstrandum.

The "parallel beacon" metaphor

I mention another metaphor involving a hypothetical "beacon" of light emitted upward from the Kaaba, parallel to the local vertical at the point of observation (potentially some substantial distance from The Kaaba), which can be seen by a person at that point. I assert that, looking in a horizontal direction toward that "beacon", the azimuth is again the departure azimuth of the great circle of the Earth passing through the point of observation and The Kaaba.

Now, the line of the beacon does not pass through the center of the Earth. But since it is parallel to the local vertical at the point of observation, both those lines lie in the same plane. Since a line in the plane (the local vertical at the point of observation) passes through the center of the Earth, the plane of course passes through the center of the Earth

Since that plane includes the point of observation, the center of the Earth, and the location of the Kaaba, it is the same plane referred to in the two discussions just above.

With the aiming axis of the telescope pointed at the "beacon", which lies in the plane, the azimuth of the telescope is that previously discussed.

Thus the azimuth of the telescope observing this beacon will be the same as the azimuth of departure. This metaphor is then consistent with the implications of the two previous cases.

Quod erat demonstrandum.

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