Principles of Eyeglass Prescriptions

Douglas A. Kerr

Issue 3
December 5, 2010

ABSTRACT

The refractive parameters of eyeglass lenses are specified (in a "prescription") under a model that considers the refractive effect of the lens as a combination of the effect of both a spherical and a cylindrical lens. In this article we describe this model and its underlying optical principles, and describe how the parameters are usually stated in a prescription. We also look into the fact that a given lens can have its properties stated in two different, but equivalent ways, each used in separate branches of the eye care profession.

We also learn about the "optical cross", a graphical presentation of the overall refractive behavior of an eyeglass lens.

CAVEAT

I am not an eye care professional, nor do I have any formal training in the practice in that field nor in its own unique branch of optical science. The information in this article is my own interpretation of the results of extensive (mostly quite recent) research into the available literature, through the prism of my own scientific and engineering background and outlook.

HUMAN VISION

Focusing in the human eye

The human eye is organized like a camera, with the retina playing the role of the film or sensor. The lens is compound, comprising the *lens capsule* and the *cornea*, each of these being convex lenses. The curvature of the *lens capsule* can be varied[1], thus changing its refractive power and thus the net refractive power of the entire lens. This allows the eye to focus on objects at different distances, an ability spoken of as *accommodation*. For young people with "normal"

---

[1] Curiously enough, it is not known exactly how this is done.

vision, the range of distances typically extends from infinity to as close as perhaps 10 cm.

Deficiencies in accommodation

Typically, with advancing age, the eye's accommodation ability can become compromised (and the same may be true of young people as a result of congenital malformation of the eye or of various ailments). Several types of deficiency are common.

Hyperopia ("far-sightedness") is the deficiency in which the total range of accommodation is "offset out", such that distant objects (even at "infinity") can be focused on but the near limit is not nearly as close as is normal.

Myopia ("near-sightedness") is "offset in", such that close objects can be focused on but the far limit is not to infinity.

Presbyopia (the term means "old person's seeing") is the deficiency in which the total range of accommodation (the *accommodation amplitude*) is decreased. The remaining limited range may be in the far, intermediate, or near regimes, in the individual case.

The following deficiency may occur in connection with any of the above three, or by itself.

Astigmatism

Astigmatism (not a deficiency in accommodation) is the deficiency in which the refractive power of the eye's lens system is not the same in different directions. An illustrative result is that if we have astigmatism and look at a cross of thin lines on a card, we can focus so that the vertical line is sharp, or the horizontal line is sharp, but not both at the same time.

Astigmatism often results from one eye lens component (often the cornea) not being rotationally-symmetrical.

Vision correction

Mitigation of these deficiencies is often done with the use of corrective lenses, typically in the form of eyeglasses. Simplistically, their role is to "cancel out" the inappropriate aspect(s) of the eye's refractive power. Before we discuss this further, I'll talk a little about lenses.

LENSES

Lens refractive power

The *refractive power* ("power") of a lens is the degree to which it will converge (or diverge) rays of light emanating from the same point on an object and entering the lens at different points on its face.

Quantitatively, the power of a lens is the reciprocal of its *focal length*. The traditional unit of power is the diopter (symbol D)[2]. A lens with a focal length of one meter has a power of one diopter.

Spherical lenses

In optometric work, a *spherical lens* is any lens that has rotational symmetry, whether or not its surface is actually a portion of a sphere. Thus we can (and often do) have aspherical spherical lenses.

A spherical lens exhibits the same power along any direction.

A converging lens (which has a positive focal length) has a positive power. A diverging lens (which has a negative focal length) has a negative power.

We can present the variation (if any) in the refractive power of a lens with direction on a polar chart. In figure 1, panel A, we see a plot of a spherical lens with refractive power +1.0 D (a converging lens). This is a trivial case, and hardly requires a chart to explain. But we show the plot here to establish the format and notation.

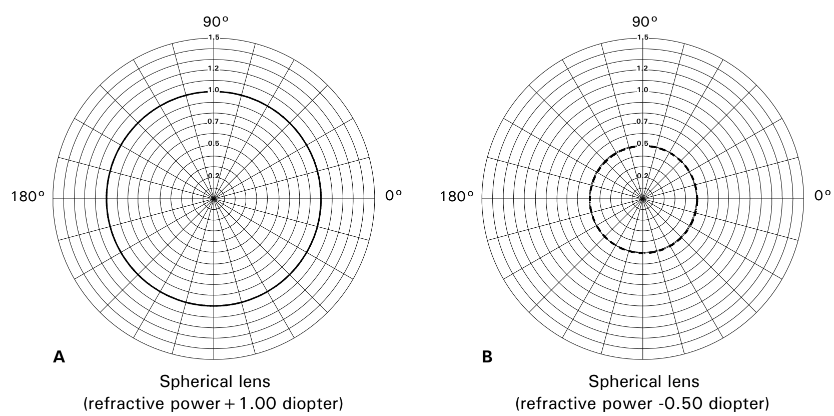

Figure 1. Spherical lens—power plot

The radius to the curve in a certain direction indicates the refractive power (in diopters) for that direction. Recall that a "direction" here

[2] The unit preferred in modern scientific work is the inverse meter (m^{-1}). However, we do not find this in optometric practice.

means both ways: either way along the line at a certain angle (called a *meridian* in optometric practice). Because of that symmetry, we only need to plot half the curve. But I show the curve for a full 360° for aesthetic completeness.

The usual scientific convention for angle is followed, with the angle reference (0°) being to the right (but there is a wrinkle, about which more shortly) and the angle measured counterclockwise.

It is difficult to express negative values on a chart in polar coordinates—a "negative" radius would put the point on the opposite side of the chart, where it would just look like the (positive) value for an angle 180° from the actual angle.

To escape this difficulty, here I will plot negative values of the refractive power as a dotted line. And we see that in figure 1, panel B, the plot for a spherical lens with a refractive power of –0.5 D (a diverging lens).

Cylindrical lenses

A cylindrical lens has a surface that is a portion of a cylinder (which may or may not be exactly a circular cylinder). A cylindrical lens exhibits a certain power (its "rated" power) in one direction (perpendicular to its axis). Along its axis, it exhibits zero power. At intermediate angles, it exhibits intermediate values of power. We see this illustrated in figure 2 for two cylindrical lenses, one with a positive power and one with a negative power.

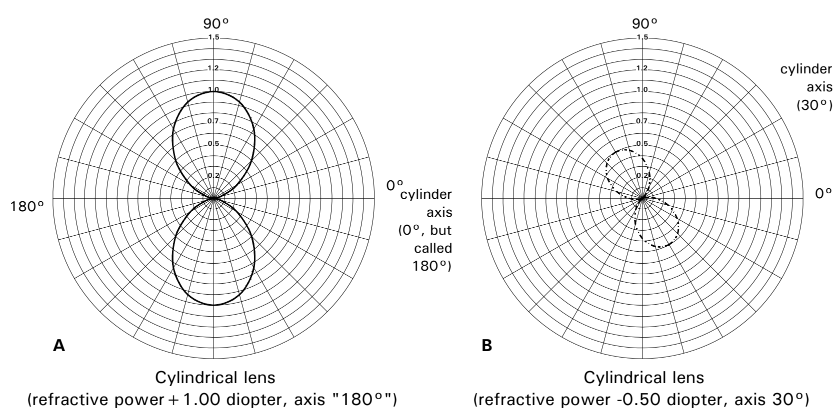

Figure 2. Cylindrical lens—power plot

Imagine that we combine a spherical lens and a cylindrical lens (and we assume here the convenience of the fanciful "thin lens" conceit, which, although impossible to have in practice, makes all the math work out in a very simple way).

In the direction of the cylinder lens axis, where the cylinder lens has zero power, there is no effect of the cylindrical lens on the overall result. In the direction at right angles to that, the power of the cylindrical lens combines with that of the spherical lens (taking into account the applicable algebraic signs) and so we have a power different from that of the spherical lens alone (perhaps even of the opposite sign).

Before we continue, let me mention the small wrinkle about stating the angle of the cylinder axis. As stated in ordinary scientific work, the angle of the cylinder axis can only vary over the range 0° through infinitesimally less than 180°. In optometric practice, the range of the angle is considered to be from infinitesimally greater than 0° through 180°(that is, we never write "0°").

In figure 3, we see one example of the combination of a spherical and a cylindrical component:

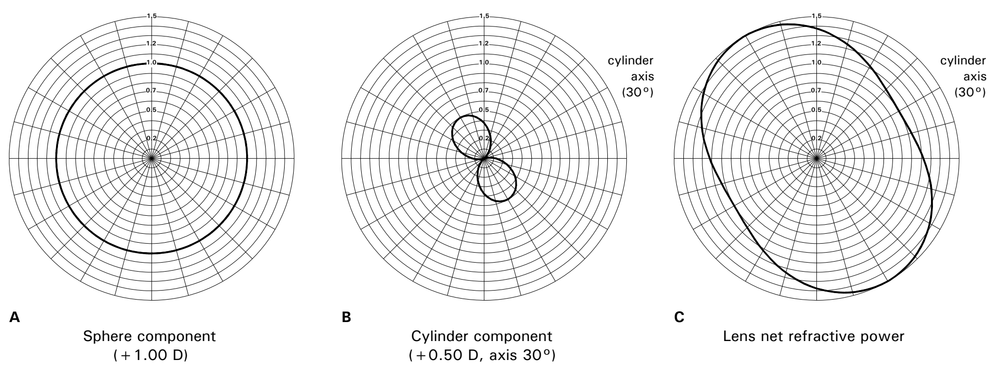

Figure 3. Composite lens—power plot

In figure 4 we see a different example:

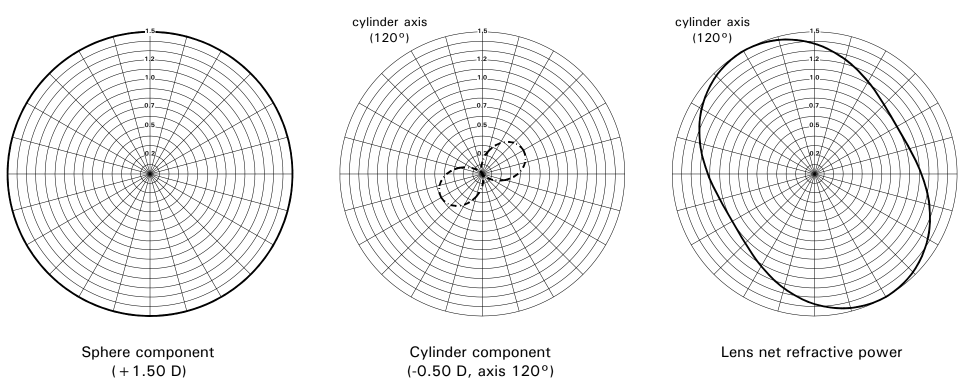

Figure 4. Composite lens—power plot

Note that the result here is identical to the previous case.[3] This is reminiscent of the ways we might make an ellipse. We might start with a circle of small diameter, and stretch it in the direction of the ellipse's major axis. Or we might start with a circle of large diameter, and shrink it in the direction of the ellipse's minor axis.[4]

We can of course make a single lens that will exhibit this overall behavior. A simple version (although not often used in modern ophthalmic practice) would have a front surface that is a portion of a sphere (implementing the "spherical" component of the overall power) and a rear surface that is a portion of a cylinder (implementing the "cylindrical" component of the overall power). This might be done with the cylinder surface either convex or concave, following the composition intimated by figures 3 or 4, respectively. The behaviors of the two will be identical (if we put aside some small wrinkles).

LENSES FOR VISION CORRECTION

The basic concept

In the application of eyeglass lenses, a role is played by both "spherical" (rotationally-symmetric) and "cylindrical" refractive behavior.

To correct for hyperopia (farsightedness), we provide a converging lens (net convex, with a positive power) to shift the range of focusing ability "closer". (Photographers do the very same thing with an *auxiliary closeup lens* to allow their cameras to focus at a closer distance than they would otherwise be able to.)

To correct for myopia (nearsightedness), we provide a diverging lens (net concave, with a negative power) to shift the range of focusing ability "farther".

In astigmatism, the eye's lens has a different refracting power in different directions, the maximum in a certain direction and the minimum in the direction at right angles to that. (The specific direction varies from person to person, from eye to eye.)

We could compensate for that by using a cylindrical lens with a positive power equal to the difference between the eye lens' maximum and minimum power (in a given state of focus), with its

---

[3] The specific mathematical variation of the power of a cylindrical lens with angle makes this equivalence exact.

[4] Notwithstanding this metaphor, the plot of the power of such a composite lens is not an ellipse.

cylinder axis aligned with the eye lens' direction of maximum power (so the power of the cylindrical lens adds to the lesser power of the eye's lens).

Or we would use a cylindrical lens with a negative power equal to the difference between the eye's maximum and minimum power (in a given state of focus), with its cylinder axis aligned with the eye lens' direction of minimum power (so the power of the cylindrical lens subtracts from the greater power of the eye's lens).[5]

Now, for a person having, for example, both hyperopia and astigmatism, we can visualize a sandwich of two lenses, a spherical lens to shift the focusing range (to overcome the hyperopia), as was just discussed, and a cylindrical lens "trimming out" the difference in the eye lens' refractive power in different directions (to overcome the astigmatism).

Of course, as we discussed earlier in a more abstract context (figures 3 and 4), we can make a single lens that does the same thing as that sandwich. A simplistic view would be a lens with a spherical surface on one face and a cylindrical surface on the other—two different ways.

Or we could visualize a lens with one face planar ("plano") and the other face having a compound curve, with one curvature along a certain direction and a different curvature along the direction at right angles to that.

In fact such a compound curve is found at the surface of a recognized three-dimensional figure, the *torus*. (A doughnut is nominally a torus in shape, as is a "rootbeer barrel".)

As a result, a lens having both spherical and cylindrical aspects to its refractive power, especially when conferred on one surface, is often referred to as a *toric* lens[6].

Interaction between spherical and cylindrical components

Assume that astigmatism is present, and suppose that the spherical lens we spoke of as correcting the person's range of accommodation was the best compromise for focus in the two critical directions—the one where the eye's refractive power was greatest, and the one where it was least (said to be the two *meridians* of the eye).

---

[5] Note that the staring spherical power would have to be different in these two cases.

[6] Pretty fancy for guys afraid of zero!

Now, if we chose to add a cylinder lens (or its equivalent as a component of the lens' overall refractive effect) with a positive power along the meridian in which the eye had the least power, the magnitude of its power being the difference between the eye's power along that meridian and its power along the other meridian, the overall "optical system" (eye plus corrective lens) would now have equal power along both meridians. Thus the astigmatism would have been overcome. But that power would not be ideal in either direction—it would be too high in both directions.

To get the ideal result, we would also need to decrease the spherical lens power by half the magnitude of the cylindrical component.

If, on the other hand, we chose to add a cylinder lens (or its equivalent as a component of the lens' overall refractive effect) with a negative power along the meridian in which the eye had the most power, the magnitude of its power being the difference between the eye's power along that meridian and its power along the other meridian, the overall "optical system" (eye plus corrective lens) would now have equal power along both meridians. Thus the astigmatism would have been overcome. But that power would not be ideal in either direction—it would be too low in both directions.

To get the ideal result, we would also need to increase the spherical lens power by half the magnitude of the cylindrical component.

In summary, if we had determined the ideal spherical correction as a compromise for focus along the two meridians, then when we introduce a cylindrical component to overcome astigmatism, we must change the spherical power by the negative of half the cylindrical power (observing the algebraic signs).

THE PRESCRIPTION

General

An eyeglass prescription is a specification for the lenses in the glasses. It is done in terms of the model we saw above, in which the overall refractive pattern of the lens is described in terms of the joint effect of two hypothetical lenses, one spherical and one (only present if there is a correction for astigmatism) cylindrical.

Recall that, as we saw in figures 3 and 4, the identical lens behavior could be conceptually implemented with either of two "recipes"; for that particular example, we could combine:

- A spherical lens with power +1.00 D
- A cylindrical lens with power +0.50 D and axis 30°

or

- A spherical lens with power +1.50 D
- A cylindrical lens with power −0.50 D and axis 120°

And either of those two "recipes" could be viewed as specifications of the identical lens behavior (which would then apply to lenses made with either recipe).

Two systems of notation

Either model we saw above could be used (at our choice) as the premise for specifying a certain lens **behavior** in a prescription. It turns out that when the prescription is written by an ophthalmologist (a physician and surgeon specializing in the eyes), it would be in the first form (the cylinder component always having a positive power), called the "positive cylinder" form.

When the prescription is written by an optometrist (a Doctor of Optometry, qualified and certified to examine eyes and issue eyeglass prescriptions), it would be in the second form, (the cylinder component always having a negative power), called the "negative cylinder" form.

This is not only the result of accidental historical "diversity". It is claimed that, when determining the optimal prescription for corrective lenses using combinations of "trial" lenses, or later using the variable power lenses in a *refractor* (the instrument used to determine the optimal lens prescription), the pragmatic strategy used in searching for the best combination plays out better under one scheme for stating the final result than for the other.

Format of the prescription

In a prescription for eyeglasses, there is a line (or section) for each eye. In each, there are three parameters stated:

- The power of the spherical component (could be zero, often written as "plano" rather than "0"). This is normally in increments of 0.25 D.

If there is a cylindrical component:

- Its power (normally in increments of 0.25 D)
- The angle of its cylinder axis (normally in increments of 5°, from 5° through 180°.

Often, the indicators OD (from the Latin, *oculus dexter*) and OS (*oculus sinister*) are used for the right and left eyes, respectively. (OU—*oculus uterque*—indicates both eyes.)

We will give our first complete example in the "positive cylinder" system.

There a complete prescription might look like this:

OD +1.25/+0.50 X 130

OS +1.50/+0.75 X 25

Sometimes the decimal points are omitted (and all powers stated to two decimal places), so it would look like this:

OD +125/+050 X 130

OS +150/+075 X 25

There are many other variations in style.

For the very same pair of lenses, under the "negative cylinder" system, the prescription might look like this:

OD +1.75/–0.50 X 40

OS +2.25/–0.75 X 115

BIFOCALS

Especially when presbyopia is present, there may be no single corrective lens prescription that will allow the eye to focus on both near and far objects. We address this with *bifocal glasses*, which have two areas of different refractive power, one (covering most of the lens) used to see distant objects and the other (called the near-vision segment, or just the "segment"), usually at the bottom of the overall lens, used to see near objects.

Normally, the properties of the segment are the same as for the base lens except that the spherical power is greater. This is specified in the prescription in an incremental way: an "add" value that tells how much greater is the sphere power in the segment than in the rest of the lens for that eye:

OD +1.25/+0.50 X 120 add 1.75

OS +1.50/+0.75 X 22 add 1.75

Very commonly the add value is the same for both eyes (being, to a great extent, determined only by the distance at which the person wants best "near-vision" results). In that case, the add value may be stated only once in the prescription:

OD +1.25 +0.50 X 120

OS +1.50 +0.75 X 25

add +1.75

In either case, the implication of this is that the actual "prescription" for the segment for the right eye (OD) is:

+3.00 +0.50 X 120

That is, the sphere power in the segment (+3.00) is the sum of the base sphere power (+1.25) and the "add" sphere power (+1.75).

PRISM CORRECTION

In some cases, the "pointing directions" of the two eyes are not consistent (at least not happily). This can cause a problem with the brain "fusing" the images from the two eyes (in some cases causing "double vision").

This can be mitigated by including in one or both lenses a "prism" component. Its purpose is just to deflect the line of sight, in a certain direction, by a certain angle. Further discussion of this aspect of eyeglass optics and the prescription notation for it is beyond the scope of this article.

Conversion between equivalent forms

If we have a prescription in either "positive cylinder" or "negative cylinder" form, we can easily convert to the other form, this way:

- Add the present sphere and cylinder powers (observing the signs). The result is the new sphere power.
- Reverse the sign of the cylinder power; the result is the new cylinder power.
- Add or subtract 90° to/from the current cylinder axis angle (so that the result will be non-zero and positive but not over 180°). The result is the new cylinder axis angle.

THE OPTICAL CROSS

- We almost never see polar plots of refractive power, such as we see in this article, in the literature of eyeglass optics. But there is another useful graphic convention that is often used: the *optical cross*. Its purpose is to show the overall result of the spherical and cylindrical components: what the finished lens does in terms of refractive power. It is in fact sometimes used to define the desired properties of a lens (rather than using the "prescription" form)

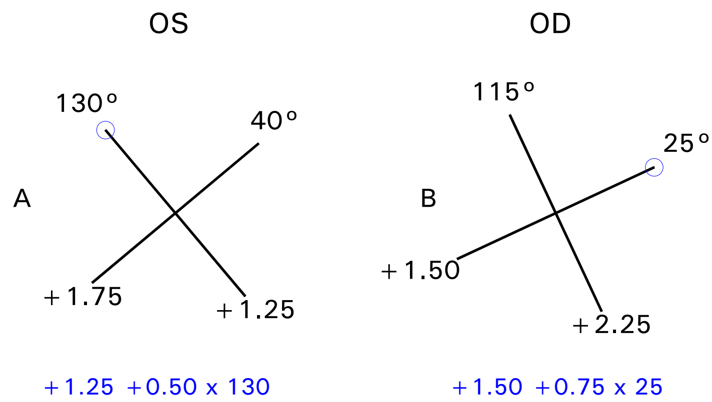

Figure 5. Optical crosses—based on plus cylinder prescription

We see it in Figure 5, displaying the characteristics of these two arbitrarily-defined lenses (expressed here in "positive cylinder" form):

OD +1.50 +0.75 X 25

OS +1.25 +0.50 X 130

The two legs of the cross are said to represent the two "principal meridians" of the lens: the directions of maximum and minimum net refractive power of the lens. One is always the cylinder axis; the other is always at right angles to that (the direction in which the cylinder lens has its maximum power, of whatever sign), but not necessarily respectively. (The cylinder axis meridian will have the minimum power under the negative cylinder form, and the maximum power under the negative cylinder form.)

Each leg is labeled at its upper end with the angle of the meridian it represents, and at its lower end with the net power in that direction.

For the benefit of the reader, I have stated the prescription for each lens under its cross representation. This would not usually be the case in practice; the cross is an alternate way to describe the lens' desired performance. Also for the convenience of the reader, I have indicated the meridian that corresponds to the cylinder axis with a small circle around its tip. Again, this would not be seen in the actual use of the optical cross. (These two "private" annotations are in blue, for the benefit of readers seeing this in color.)

We note that on the cylinder axis, along which the cylinder component has no power, the net power is just the sphere component. On the other meridian, where the cylinder component exhibits its maximum (and stated) power, we note that the net power is the sum of the sphere and cylinder powers.

Now we know that this pair of prescriptions, in "minus cylinder" form, defines exactly the same pair of lenses:

OD +2.25 -0.75 X 115

OS +1.75 -0.50 X 40

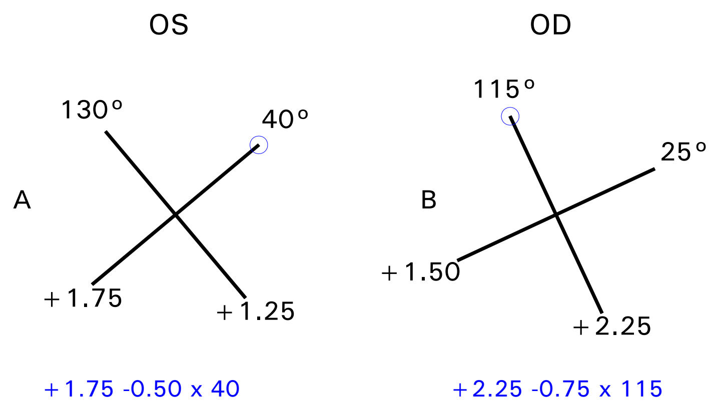

Figure 6. Optical crosses—based on minus cylinder prescription

In figure 6, we show the optical crosses for that outlook:

Notice that these are identical to the ones in figure 5 (except of course for my "private" annotations, which would not appear on a real optical cross). (Seems reasonable—they are for the same lens.)

So, seeing an optical cross in normal form, how can we tell which of the two meridians corresponds to the cylinder axis? Well, we can't, and we don't need to in order to understand what lens behavior the optical cross defines.

Recall that the distinction between one or the other meridian corresponding to the cylinder axis is an artifact of the prescription scheme, with its alternative positive cylinder and negative cylinder forms. The optical cross shows the final result in terms of lens behavior, not in terms of one of two alternative "recipes" for defining it.

From an optical cross, can we determine the "regular form" prescription for the lens it defines? Yes, but we must first choose whether we want it in the positive or negative cylinder form. The math is simple, but I won't show it here.

#

APPENDIX A

Variation of refractive behavior with direction

Effect of a cylindrical component on overall spherical power

If we have a positive sphere component of, say, +1.00 D and add a positive cylinder component of, say, +0.50 D, we report this **in the prescription** just the way I just described: Sphere: +1.00 D; cylinder: +0.50 D (and whatever axis). This is a "recipe-based" outlook.

However, in fact, the "mean" spherical power has been changed by +0.25 D (half the cylinder power), to +1.75 D. This is because the total power in one meridian is increased by 0.50 D, and in the other not at all.

In other optical work, we might describe the lens in a "results-based" way. This essentially is based on the outlook that a spherical lens is "normal" and that astigmatism is an aberration. Under that outlook, we might describe that same lens this way:

- Power [meaning mean]: +1.75 D
- Astigmatism [meaning the difference in power between the two meridians]: 0.50 D. (The axis would also have to be specified, and there would be some convention for indicating along which meridian was the power the greater and along which the lesser).

We can see that graphically in figure 7:

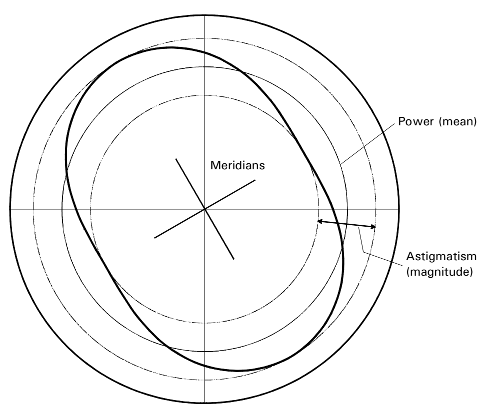

Figure 7. Lens power and astigmatism

Variation of the power of a cylindrical lens with angle

If we have a cylindrical lens, with its axis in a certain direction, and thus with its power meridian (the direction in which its power is greatest) at right angles to that, then the power exhibited in any other direction is the product of its greatest power (said to be "the power" of the lens) times the **square of the cosine** of the angle between the meridian direction of interest and the power meridian.

This situation may at first seem equivalent to the matter of resolving a vector (a force, for example), in a two-dimensional context, into two components, one lying along some direction of interest and the other in right angles to that.

In that situation, the component in the direction of interest is equal to the magnitude of the vector times the **cosine** of the angle between the direction of interest and the direction of the vector itself. We might have expected to find that in the present situation.

But the two situations are not at all equivalent.

In the case of resolving a vector, the two components **replace** the vector. If we consider them, then the vector itself is not in play anymore, and there can be no other components of the vector in play—only those two.

In the case of the variation of power of the cylinder lens, the lens exhibits power in every direction (except along the cylinder axis) all the time. Its power in some oblique direction is not a replacement for its power along its primary meridian—they both coexist, along with the powers in all other directions. [7]

So it should not be a surprise that different variations with angle are involved in the two cases.

#

---

[7] However, one can **replace** a cylindrical component by two other cylindrical components, not at right angles but rather at angles separated by 45° (which seems curious, but it in fact works that way—he proof of this is beyond the scope of this article). The concept is useful in combining the overall effect of two lenses that include one or more cylindrical components.