The Mathematics of Extension Tubes in Photography

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ABSTRACT

Extension tubes are devices that are placed between an interchangeable camera lens and the camera body to shift the focusing range of the camera to embrace shorter subject distances. The object is generally to achieve a greater image magnification than can be had with the lens in its normal situation. In this article we review the optical principles involved with the use of extension tubes, and give various equations useful in their application.

INTRODUCTION

Camera lenses (whether of the interchangeable type or not) have a range of the distance at which they allow the camera can be focused.[1] Often we are interested in focusing at a smaller distance with a certain lens than the normal minimum. This most commonly arises in connection with the desire for a greater image magnification than the camera exhibits with the lens at its closest focusing distance setting.

MAGNIFICATION

The *image magnification* of a camera[2] is defined as the ratio of a linear dimension of a feature of the image to the linear dimension of the same feature of the object itself. (Canon prefers to use the term *reproduction ratio*, a term originally derived from copy camera practice.) The magnification is in fact the ratio of (a) the distance to the image (measured from the 2nd nodal point of the lens) to (b) the distance to the subject (measured from the 1st nodal point of the lens).[3]

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[1] Note that a lens cannot be focused at a certain distance, only a camera of which the lens is a part.

[2] Strictly speaking, a lens does not (by itself) have a magnification, only a camera.

[3] We will speak here both of the *nodal points* of a lens and its *principal points*. In all cases of interest to us, the two nodal points fall at exactly the same location as the two principal points. But we will selectively use the two terms because the two pairs of points have different significance in the optical theory of camera operation.

Because of the relationships of the basic focus equation, we can also say that the magnification depends on the focal length of the lens and the distance to the subject (again measured from the 1st nodal point of the lens).

The greatest magnification is ordinarily attained at the closest focus distance of the camera.

THE EXTENSION TUBE

A direct way to increase the maximum magnification of the camera with a certain lens is to physically move the entire lens outward from the camera body. If the lens is of the interchangeable type (as is common with cameras of the single lens reflex—SLR—type, but uncommon with cameras of other types), we can do this by inserting a special device, called an *extension tube*, between the lens and its mount on the camera body. The tube of course must be designed to cater for the specific type of lens mount in use, emulating the lens mount of the camera on its forward end and the fitting on the rear of the lens on its rear end. In addition, some extension tubes have provisions for carrying through them the electrical and/or mechanical connections between the lens and the camera body, allowing retention of various automatic features of the camera system.

The most common type of extension tube (and the only type we will discuss here) has no optical elements in it—it is just an empty tube. It does all its work by shifting the available range of distance from the 2nd nodal point of the lens to the film or digital sensor.

MAGNIFICATION WITH AN EXTENSION TUBE IN PLACE

If we know the magnification the lens (without an extension tube) provides when at some focus setting (including, but not necessarily, closest focus), we can readily calculate the magnification it will give, at that same focus setting, with an extension tube in place:

$$m' = m + \frac{L}{f} \qquad (1)$$ [4]

where *m′* (read "m prime") is the magnification with the extension tube in place, *m* is the magnification with no extension tube (but the lens at the same focus setting), *L* is the "length" of the extension tube

[4] This equation is derived in Appendix A.

(the amount by which it moves the lens forward), and f is the focal length of the lens (L and f in consistent units).

Note that this is equally workable whether the lens focus setting involved is its "closest" one or not.

EFFECT ON THE RANGE OF FOCUS DISTANCE

With an extension tube in place, both the "shortest" and "farthest" focus distances are reduced from those provided by the lens in its normal use. The former is of course usually the object of the use of the extension tube; the latter is just a side effect (sometimes troublesome).

Calculation of new minimum and maximum focus distance

Often we wish to know the actual minimum and maximum distance to which a camera can be focused with an extension tube in place. These calculations involve the classical "Gaussian" focus equation, which in its fundamental form is:

$$\frac{1}{P}+\frac{1}{Q}=\frac{1}{f} \qquad (2)$$

where P is the distance to the object, measured from the 1st principal point of the lens; Q is the distance to the image (at the film or sensor, if the camera has been "focused" on the object) measured from the 2nd principal point of the lens; and f is the focal length of the lens, all in consistent units.

We can solve this for P in terms of Q and f:

$$\frac{1}{P}=\frac{1}{f}-\frac{1}{Q} \qquad (3)$$

$$\frac{1}{P}=\frac{Q-f}{Qf} \qquad (4)$$

$$P=\frac{Qf}{Q-f} \qquad (5)$$

Similarly, we can solve for Q in terms of P and f, and get:

$$Q=\frac{Pf}{P-f} \qquad (6)$$

Now, with those in hand, let's examine the situation in which the lens itself is set to its normal minimum focus distance, P_1. Then the distance to the image, Q_1, will be given by:

$$Q_1 = \frac{P_1 f}{P_1 - f} \qquad (7)$$

If we then insert an extension tube of length L, its effect is to increase Q, such that:

$$Q_1' = Q_1 + L \qquad (8)$$

where Q_1' is the new value of Q.

We can then calculate the new actual minimum focusing distance (again measured to the 1st principal point[5]) thus:

$$P_1' = \frac{Q_1' f}{Q_1' - f} \qquad (9)$$

There is of course a consolidated solution for P_1' in terms of P_1, L, and f, but it is very cumbersome, and in practice it is easier to calculate the result in steps, as shown above.

Calculation of the new maximum focus distance proceeds along similar lines. We assume that the maximum focus distance with the lens alone is infinite ($P_2 = \infty$). Then:

$$Q_2 = f \qquad (10)$$

With the extension tube in place:

$$Q_2' = f + L \qquad (11)$$

We can then calculate the corresponding focus distance (the new maximum focus distance) thus:

$$P_2' = \frac{Q_2' f}{Q_2' - f} \qquad (12)$$

[5] Note that when "minimum focusing distance" is stated for a lens on a certain camera, it is usually stated from the object to the focal plane. We cannot work with that here since it depends on a parameter of the lens we do not normally know.

EFFECT ON EXPOSURE

We normally treat the f-number of a lens as the indicator of its effect on exposure. In fact, it is only strictly that for the situation in which the lens is focused at infinity (or, in a practical sense, in which it is focused at a distance that is very large compared to the focal length).

Thus, when we focus at a shorter distance, the exposure impact of the lens will not be that suggested by the f-number—in fact, it will be a lesser exposure impact.

This effect often becomes of importance when we use an extension tube, since we will then in fact be focusing at a shorter distance than we could have before.

It is common to attribute this decline in exposure performance to the extension tube, but in fact it is a creature of focus at a short distance—however we bring that about. (Many lenses can focus at rather short distances without an extension tube, and this phenomenon occurs significantly there just the same.)

Often the term “loss of light” is used to describe this effect, but that is not technically apt. No light is lost as we focus at closer and closer distances—the same amount of luminous flux is collected from any “patch” of the scene (for a given luminance of the patch, of course). But when we focus at a closer distance, the image of any given subject patch is larger (this is why we focus at a close distance for “macro” work), and thus the collected luminous flux from a particular subject patch is spread over a larger area on the film or sensor. This constitutes a decline in the *illuminance* on the film or sensor, which is (along with exposure time) the factor to which the film or sensor responds.

Because we are used to thinking of the exposure impact of a lens in terms of f-number, it is often handiest for us to reckon the exposure impact of a lens in a close focus situation in terms of an “effective f-number”. This is the f-number that, for a lens focused at infinity, would produce the same exposure impact that our lens exhibits when focused at our actual subject distance. We will see how to reckon effective f-number after we dispose of a little complication.

Pupil magnification

The *entrance pupil* of a lens is the “port” through which it collects light. It is the virtual image of the actual physical aperture stop (iris or diaphragm) as seen from in front of the lens. If we look at a typical lens from the front, we may think we see the aperture stop, but in fact we see its virtual image. It is not in the same place that the stop actually is, and its diameter is not that of the physical stop (all this because of the refractive effect of the lens elements in front of the aperture stop).

It is in fact the diameter of the entrance pupil—not the diameter of the physical aperture stop—that is used in the reckoning of the f/number of the lens.

The *exit pupil* is the “port” through which the lens discharges light. It is the virtual image of the actual physical aperture stop as seen from behind the lens.

In a symmetrical lens design, the entrance pupil is located at the 1st principal point of the lens, and the exit pupil is located at the 2nd principal point. The two pupils have the same diameter (not generally the same as the diameter of the physical aperture stop).

But often in real lens designs, the two pupils are not located at the two principal points. If the entrance pupil is displaced a certain distance in a certain direction from the 1st principal point, then the exit pupil must be displaced a certain (different) distance, in the same direction, from the 2nd principal point. And for any given situation of displacement, the diameters of the exit and entrance pupils have a certain ratio. That ratio is called the *pupil magnification*. By stating it, we state the whole story of the locations of the pupils.

When we predict the actual effect of focus at a close distance on the exposure impact of the lens (perhaps by calculating an “effective f-number), we must take the locations of the pupils into account. We can do that by including the pupil magnification in the formula. But very rarely do we know the pupil magnification of the lens (or the actual locations of the pupils). Thus we most often just have to ignore the effect of pupil magnification, and calculate the effective f-number as though the pupil magnification were 1 (that is, as if the two pupils were actually located at the two principal points of the lens).

The formula for effective f-number we will develop shortly is based on the assumption that there is no displacement of the pupils.

Calculating the effective f-number

If (as discussed just above) we ignore the possibility of a pupil magnification different from 1, the effective f-number of a lens is given by:

$$n' = n(1+m) \qquad (13)$$

where n' is the effective f-number, n is the actual f-number, and m is the magnification of the lens for an object at the current focus distance (whether focus at that distance is attained with the help of an extension tube or not).

Note, as a credibility check, if we consider focus at a large distance, m becomes very small, and thus n' becomes nearly n (essentially, our usual assumption as to the meaning of the f-number of a lens).

Note also that for the focus distance at which the magnification become 1 (1:1), for some reason an important "landmark" in macrophotography, the effective f-number becomes twice the real f-number. This represents a decline of exposure impact of 2 stops from that suggested by the f-number.

The bellows factor

Suppose we define a constant, b, this way:

$$b = 1 + m \tag{14}$$

Then equation 13 can be rewritten as:

$$n' = bn \tag{15}$$

The factor b, which capsulizes the effect of a short focus distance upon effective f-number, is often called the "bellows factor". The name comes from cameras with a bellows between the lens and the body proper. The focus distance in such a camera varies with the position of the lens, and thus with the extension of the bellows. Thus the factor b, which varies with focus distance, was attributed to bellows extension, and was named accordingly.

Note, as a credibility check, that for focus at an infinite distance (for which m is zero), b will become 1, again indicating that the effective f/number in that situation is just the actual f-number.

Effect on other uses of the f-number

The f-number of a lens appears in many photographic calculations (for example, depth of field calculations). In most cases, it is in fact the actual f-number, n, that is to be used, even in cases of close focus, not the "effective f-number", n'. (Of course, focus distance plays a role in those calculations, but that is taken into account by other parts of the equations involved.)

A CAUTION

Classical camera lenses were "focused" by moving the entire lens (often on a bellows). Some more modern lenses of are focused by moving all the lens elements together (essentially moving the entire "lens" within the lens housing).

However, in most modern lenses, focusing is attained by moving only one or more of the groups of lens elements. This is done to meet various design objectives.

For certain of those focusing schemes, the actual focal length changes as the focusing setting is changed. In the case of zoom lenses, where we want to be able to change the focal length, for a given setting of the focal length control the actual focal length may vary with changes in the focusing setting.

As a matter of industry practice, the focal length of a lens (focal length range, for a zoom lens), is stated for focus at infinity (but this is not always mentioned in the lens specifications).

Commonly, for the "closest" focusing setting, the focal length is less than the rated focal length.

Of course, moving the entire lens (housing and all) by the insertion of an extension tube has no effect on the focal length of the lens. But since we commonly use an extension tube in a situation in which the lens is also set to its closest (or almost closest) focusing setting, we may in fact be working with a focal length not that marked on the lens (or the lens zoom control).

Since focal length appears in equation 1, a consequence can be that the attainable magnification with an extension tube in place may not be as great as that expected.

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Appendix A

Derivation of the equation for the effect of an extension tube

Here we derive equation 1, restated here for reference:

$$m' = m + \frac{L}{f} \qquad [1]$$

The image magnification of a lens is given by:

$$m = \frac{Q}{P} \qquad (16)$$

where P is the distance to the object, measured from the 1st principal point of the lens, and Q is the distance to the image (at the film or sensor, if the camera has been "focused" on the object) measured from the 2nd principal point of the lens.

P and Q are related by the basic focus equation, which we here see in its "Gaussian" form:

$$\frac{1}{P} + \frac{1}{Q} = \frac{1}{f} \qquad [2]$$

Or, solved for P:

$$P = \frac{Qf}{Q-f} \qquad [5]$$

Substituting into equation 16, we get:

$$m = \frac{Q(Q-f)}{Qf} \qquad (17)$$

And simplifying we get:

$$m = \frac{(Q-f)}{f} \qquad (18)$$

Solving for Q, we get:

$$Q = mf + f \qquad (19)$$

which will be useful shortly (as we will not want Q in our final result).

If we insert an extension tube of "length" L, then we have a new value of *Q*, we will call *Q*', thus:

$$Q' = Q + L \qquad (20)$$

We can then substitute that in equation 18 and calculate the new magnification, m':

$$m' = \frac{Q + L - f}{f} \qquad (21)$$

Then substituting for Q from equation 19, we get

$$m' = \frac{mf + f + L - f}{f} \qquad (22)$$

or

$$m' = \frac{mf + L}{f} \qquad (23)$$

or

$$m' = m + \frac{L}{f} \qquad (24)$$

Quod erat demonstrandum.

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If we know *Q*, we can calculate *P*. We do that here for our new value of *Q* (that is, *Q*'), thus getting the new value of *P* (that is, *P*'), the new distance from the plane of focus to the 1st principal point of the lens:

$$P' = \frac{Q' f}{Q' - f} \qquad (25)$$

Substituting for Q' from equation ***, we get:

$$P' = \frac{(Q+L)f}{(Q+L)-f} \tag{26}$$

Substituting this value of P' into equation ***, we get:

$$m' = \frac{Q((Q+L)-f)}{(Q+L)f)} \tag{27}$$

Dividing top and bottom by (Q + L), we get:

$$m' = \frac{Q - f/(Q+L))}{f} \tag{28}$$