

Equivalent f-number

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ABSTRACT

In digital photography, a convention called “equivalent focal length” (more precisely, “35-mm equivalent focal length”) is widely used to allow comparison, across cameras of different format sizes, of the field of view implications of the use of a lens of a certain focal length on a particular camera. It works by describing the focal length of the lens that, if used on a 35-mm film camera, would give the same angular field of view as the lens of interest gives on “this” camera.

In recent times, we hear, again in connection with a lens being used on a camera whose format size is not that of a 35-mm film camera, of its “effective f-number”.

What might that mean? In this article, I describe one thing that is often meant.

1 THE F-NUMBER OF A LENS

1.1 Effect on exposure

The f-number of a lens (expressed, for example, in the form “f/8.0”) is the ratio of the focal length of the lens (in mm, for example) to the diameter of the lens’ *entrance pupil* (then also in mm). It is often, but imprecisely, described as “the ratio of the focal length of the lens to the diameter of the lens’ *aperture*”. But the distinction is one we need not fret over here.

The significance of the f-number is that it consistently tells us the ratio between the *luminance* of a point in the scene and the *illuminance* of the image of that point on the film or digital sensor.¹ Thus it is a key parameter in determining various matters pertaining to photographic exposure. The larger the f-number, then for a given illuminance of that point on the object the less is the illuminance of its image (we can say, “the less is the exposure”).

Note that that this photometric property of a lens (characterized in terms of the f-number) is based in its optical properties, and is not

¹ Only exactly so for focus at infinity, and assuming the lens has 100% transmission.

affected by, for example, the format size of the camera on which it is being used.

1.2 Effect on depth of field

But we also may be concerned with the f-number of a “shooting setup” when we try and determine what *depth of field* it will yield.

Because it is vital, if this story is to mean anything, to understand the concept of depth of field, I will take the time to give a brief review of that topic.

2 DEPTH OF FIELD

2.1 Misfocus blurring

If a camera is focused at a certain distance, then (ignoring any lens aberrations), any point on an object at that distance will lead to a point (which has zero diameter) on the image, on the film or digital sensor, of that object. The overall result, for an object or part of an object at that distance, is that the image will not be “blurred” from “misfocus”.

2.2 The circle of confusion

But any point on an object at a different distance will lead to a “spot” of finite diameter in the image (called a “blur figure”, or sometimes a “circle of confusion”, and I will use that latter term from here on) on the film or digital sensor. The result is that, for an object or part of an object at such a distance, the image will be blurred.

2.3 The meaning of “depth of field”

By depth of field, we mean: With the camera focused at a certain distance, the *depth of field* is the range of distances over which any part of an object will have an image that is not blurred from misfocus by more than some amount that we choose.

2.4 Quantifying blurring

If we are to work with this matter on a mathematical basis, we need to first decide how we will quantify the “degree of blurring” that the image of an object at some distance (other than the distance at which the camera is focused) will suffer. We usually do that simply in terms of the diameter of the blur figure that would occur. Of course, the greater the diameter of the circle of confusion, the “worse” is the blurring.

2.5 The circle of confusion diameter limit (COCDL)

We then need to consider what diameter of the circle of confusion will we consider to be the greatest that can occur for us to say that the

object involved is within the “depth of field” of this shooting setup. I call this the circle of confusion diameter limit (COCDL).²

One outlook on this is to choose this COCDL to be a certain value, perhaps the size we believe will just make the blurring “significant”.

A complication is that the mathematics predicts the diameter of the blur figure on the film or digital sensor (I will from now on say “focal plane” for conciseness), whereas in most cases we look at the “deliverable image” as a print or screen image much larger than the camera’s format size. And our ability to discern blurring has an angular basis, so the distance at which we view the deliverable image figures into the reckoning.

To take all this into account, it is common then to choose a “maximum acceptable blur figure diameter” that is a certain fraction of the size (usually the diagonal dimension) of the camera format. Often a fraction of 1/1400 is used.

Note that the smaller the COCDL (**which we choose**), the smaller will be the calculated depth of field.

2.6 What depth of field is not

Keep in mind that the depth of field for a certain shooting setup is not a measurement of an intrinsic physical phenomenon, like the luminance of some spot on an object. It is a value that results from our adapting a criterion for how much misfocus blurring **we** will consider “negligible”.

2.7 Inputs to the calculation

When we calculate the depth of field for a certain proposed shooting “setup”, the inputs to the calculation are:

- The distance at which the camera is focused.
- The focal length of the lens.
- The f-number of the lens aperture.
- The diameter (on the focal plane) we have chosen as the “maximum acceptable diameter of the circle of confusion”. I call that the “circle of confusion diameter limit”, or COCDL.

² Sadly, it is extremely common in this field for the *circle of confusion diameter limit* to be called the “circle of confusion”.

2.8 Outputs of the calculation

The outputs of the calculation are the *near object distance* (less than the distance at which the camera is focused) and the *far object distance* (greater than the distance at which the camera is focused) between which an object can be so that its image is not blurred by misfocus by greater than the amount we have prescribed by choosing a COCDL.

If we want a single value for depth of field, it is the difference between those two distances.

Note that, for all other parameters being equal, the greater the f-number, the greater the depth of field.

2.9 How does format size enter into this?

The optical equations we would use to calculate the depth of field for any shooting setup do not (directly) involve the format size of the camera.

But the practical reality goes beyond that. I will discuss that in section 4.3, after some other relevant matters have been covered.

3 THE CONVERSE OF DEPTH OF FIELD

For many photographic tasks, we wish foreground and background objects to be blurred so as to allow the main subject (which we assume will be located at the distance at which the camera is focused) to “stand out”. This consideration is sort of “the converse of depth of field”.

And, for the same lens focal length and the same distance at which the camera is focused, the greater the f-number, the less will be the blurring of a near or far object at a certain distance.³

As a consequence, for shots where we wish that distinction between the main subject and the incidental near and far objects to be “more”, we must use a lens with a smaller f-number. This is one reason that, even if exposure considerations do not themselves suggest it, a photographer may invest in a “large aperture” (small f-number) lens for certain work.

But note that the calculated depth of field for a certain setup does not tell us how blurred will be an object at a certain near or far distance.

³ There is, however, no generally-accepted way to quantify this behavior

4 THE "EFFECTIVE F-NUMBER"

4.1 The question

Suppose that we have a lens of a certain focal length intended for use on a camera with a format 1/1.6 times the size (in both directions) of the "35-mm" format (the latter being $36\text{ mm} \times 24\text{ mm}$ ⁴, so our camera's format is $22.5\text{ mm} \times 15\text{ mm}$ in size), and assume its maximum aperture f-number (smallest f-number) is f/2.0.

For any given distance at which the camera is focused, and for any given choice of the COCDL, then if we set the lens to its maximum aperture, the calculations will show a certain depth of field (the least the camera with that lens aboard will exhibit for that focus distance).

Now, one of the things that can be meant by "effective f-number" in this situation is the answer to this question:

For a lens on a 35-mm format size camera, what would its f-number need to be for the setup to exhibit that same depth of field as "this" camera does with "this" lens (with a certain f-number) aboard?

Why do we want to know that? Well, just as for field of view, photographers have (supposedly) often gotten used to recognizing the depth of field behavior of lenses of a certain f-number as used on a 35-mm camera. And the same is true for the "converse of depth of field" behavior.

4.2 That old "all other thing being equal" dilemma

But of course there are three parameters, beyond the f-number, that affect the reckoned depth of field (or the converse behavior). So we must say (or presume) what those parameters would be for this hypothetical 35-mm format size setup.

Now what might be a reasonable set of those parameters? Well, we **might** choose these:

⁴ That frame size is specifically what is fully called the "full-frame 35-mm" format, there being another one less widely used, the "half-frame 35-mm" format ($18\text{ mm} \times 24\text{ mm}$). But unless we hear otherwise, "35-mm" normally means " $36\text{ mm} \times 24\text{ mm}$ ". Note that this use of "full frame" is not the same as the use of "full frame" to mean, by itself, the "full-frame 35-mm format", the premise there being that this is the "entire" size of a camera format and anything smaller is somehow partial.

- The “35-mm” camera is focused at the same distance as for “this” setup.
- The lens on the “35-mm” camera has the focal length that will give the same field of view on that camera as “this” lens gives on “this” camera.
- We choose a COCDL that is the same fraction of the format size of the 35-mm camera as the fraction of the format size of “this” camera we had chosen as the COCDL for use with “this” camera for depth of field calculation.

4.3 How does format size enter into this (again)?

As I noted earlier, the optical equations we would use to calculate the depth of field for any shooting setup do not (directly) involve the format size of the camera.

But the practical reality goes beyond that. If in fact we adopt the viewpoint on choosing a COCDL suggested above as part of the “all other factors being equal” situation, then as we go to a larger camera format, we are led to choose a correspondingly greater COCDL. And that leads to a greater depth of field (because we now allow “greater blurring” in terms of the diameter of the circle of confusion.

But then, if we adopt the above viewpoint of the focal length to be used, as we go to a larger camera format, we are led to choose a correspondingly greater focal length. And this leads to a lesser depth of field. And mathematically, this is greater than the change in the other direction due to change of the COCDL, so the net result (for the same f-number, of course) is that for the larger format, the calculated depth of field is less.

4.4 How might that work out?

Suppose we consider an example in which:

- The frame size of “this” camera is $1/1.6$ times the frame size of the 35-mm frame size camera (such as for the so-called “APS-C” size).
- The f-number of the lens on this camera is $f/2.0$.

If we adopt the “all other things being equal” factors suggested earlier, then it turns out that the answer to the question above is, “approximately $f/3.2$ ” (3.2 being 1.6×2.0). Fancy that! (The approximation is quite close for relatively small focus distances, less close as the focus distance increases.)

BUT . . .

Now suppose one does not wish to adopt those definitions of “all other things being equal”? Then, for whatever set of conditions are adopted, the answer will likely be different.

What if one does not want to think about that matter? Well, then there is no answer.

Suppose that is not at all what is meant by effective f-number? Nothing I can do about that.

A CAUTION

I urge those who encounter mention of “equivalent f-number” not to read into that more than is meant by the writer, which likely (but will not necessarily) be what I discuss above. Especially beware of reading into it anything having to do with the exposure implications of the f-number.

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