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Depth of Field in Film and Digital Cameras

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ABSTRACT

Although the image of an object created by a camera is only "perfectly focused" when the object is at the precise distance to which the camera has been focused, objects at other distances (over a certain range) will have images of what we consider "acceptable sharpness", an honor for which we must adopt some quantitative, if arbitrary, definition. The range of object distances for which this occurs is spoken of as the *depth of field* of the camera. This article discusses the traditional concept by which depth of field is defined, quantified, and calculated, and describes the rationales of two outlooks often used to develop a criterion of "acceptable sharpness". It also discusses the way in which the film frame or format size of a camera influences depth of field. The related topics of *depth of focus* and *out of focus blur performance* are also discussed.

INTRODUCTION

When a camera lens is focused on an object at a certain distance from the camera, the light from each point on the object is brought to convergence at a unique point on the film plane (if we ignore lens aberrations and diffraction effects).

For objects not precisely at this "focus distance", the convergence of the light at the film plane is imperfect. Instead, the light from each point on such an object forms a small circular blur figure at the film plane, called a "circle of confusion".[1] The overall result is that the outlines and features of such objects are "blurred" on the resulting image.

Of course, a certain amount of such blurring is not even visible to the human eye (depending on the scale of the final "print" and the distance from which it is viewed), and even an amount which could be discerned under close

[1] There is however a widespread current use of that term to refer to a different, but related, thing, the quantity we will define shortly as the "circle of confusion diameter limit", or COCDL. To avoid any confusion on the part of readers accustomed to that usage, I will here only use the phrase "circle of confusion" in the term "circle of confusion diameter limit", in which there is no chance of misunderstanding, and I will call the circle of confusion itself the "blur figure".

scrutiny may have no practical adverse effect on the image as most-commonly viewed.

The “tolerance” of some degree of imperfect focus is the basis for the concept of *depth of field*.

DEPTH OF FIELD

The concept

By *depth of field* we mean the range of object distances (with the camera focused at a certain distance) for which the objects will be imaged with what is thought of as “acceptable sharpness”.

For this concept to be meaningful, we must adopt some quantifiable definition of what we will consider acceptable sharpness. But since “sharpness” can have many meanings in photography, we will work in terms of the complementary concept of “acceptable blurring”. We do this by adopting a maximum acceptable diameter of the blur figure. Objects whose images have a blur figure whose diameter is within this limit will be considered to have acceptable blurring. I call this limit the *circle of confusion diameter limit* (abbreviated COCDL).

Having adopted a value for the COCDL, (and for a certain distance at which the camera is actually focused, a certain lens focal length, and a certain f/number), we can calculate the range of object distances over which objects will be imaged within that acceptable “blurring” criterion—the *depth of field* for that situation.

I will discuss the matter of choice of a COCDL later in this article.

Describing the depth of field

We can describe the depth of field several different ways:

- We can give the near and far limits of the field of “acceptable” focus: “The near limit of the field is 6.98 m, the far limit is 17.60 m.”

- We can describe the actual depth of the field in the near and far directions from the distance of perfect focus, perhaps like this: “The depth of field is -3.02 m, +7.60 m.” Note that the field is not in this case (nor really in any case) symmetrical.

- We can describe only the total extent of the field (not really very useful, but the most often cited!): “The total depth of field is 10.62 m.”

Speaking of asymmetry

Just above we mentioned that the field is not symmetrically disposed about the plane (or distance) of perfect focus. The field is always greater in the far direction than the near direction.

There is a widely circulated myth that the total field of acceptable focus is distributed approximately 1/3 on the near side and 2/3 on the far side. This is just not so. The proportions vary with the focal distance. For small focus distances, the two directions may be almost equal. For large focus distances, the far distance may be many, many times the near distance (we will in fact next hear of a situation in which the far distance is infinite).

Hyperfocal distance

For any given aperture (as an f/number), lens focal length, and COCDL, there is a distance to which the focus may be set such that the depth of field just extends, at the far end, to an unlimited distance ("infinity"). This means that any object at any distance at or beyond the associated near limit distance will be in focus within the degree we have adopted as acceptable. That focus distance is called the *hyperfocal distance*.

The associated near limit of acceptable focus is always very nearly one half that distance.

It also turns out that, if we focus the camera at infinity, the near limit of the depth of field becomes very nearly equal to the hyperfocal distance[2] (and the far limit is "beyond infinity", a concept which has no physical meaning).

Simple cameras having no focusing capability are often set at the factory to the hyperfocal distance for their lens' focal length and some arbitrarily chosen aperture. The intent is to give the user the largest practical range of object distances for which acceptable focus will be achieved.

Selecting a COCDL

How might an appropriate acceptable maximum diameter of the blur figure (COCDL) be chosen? The overall issue is complex. It involves, at the least, considerations in these two areas:

- How will different degrees of blurring be perceived by the viewer of the image? For example, if the misfocus on a certain object results in a blur

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[2] One focal length less, in fact. (Thanks to Doug Pardee for reminding the author of that.) Sometimes this near distance for focus at infinity is in fact also called the "hyperfocal distance", and some believe that this is actually the rigorous meaning of the term, not the meaning discussed above.

figure whose diameter is 1/500 the width of the image, this could be very prominent to the viewer if the image is presented as a 12 x 8 inch print viewed from a distance of 24 inches, but unnoticeable on a 3 x 2 inch print viewed at that same distance. And it would be hard to tell what the impact would be on an 8-foot high billboard image.

- How does the blurring resulting from imperfect focus interact with blurring from other phenomena, such as lens aberrations and the limited resolution of the film or digital sensor? For example, if our camera is only able to resolve points spaced by at least 0.01 mm, it would not be sensible to define acceptable blurring in terms of a COCDL of 0.003 mm—the blurring caused by that degree of misfocus would be "swamped" by the camera's finite resolution.

Thus, a truly appropriate assessment of the expected depth of field performance of a particular camera setup in a particular situation must take into account many considerations. In demanding work, such as product photography and professional cinematography, the calculations and criteria are indeed often particularized with respect to these considerations.

However, most photographers are anxious to have a "general-use" basis for predicting depth of field performance ("without going into all that complicated stuff"). To accommodate this, various rationales have come into wide use for choosing a COCDL that will lead to hopefully-meaningful calculated depth of field results in a wide range of practical situations. Most of these rationales are based on one of two distinct "outlooks".

Appendix A discusses these two outlooks and their rationales, along with how they are commonly applied, finally indicating typical COCDL values that have been adopted through their use.

Calculation of depth of field distances

The various distances of interest can be calculated by the equations in Appendix B. They are presented in two versions, a "precise" version, and an "approximate" version which yields quite accurate results for focus distances substantially greater than the focal length involved.

Effect of format size on depth of field performance

By *format size* we mean the size of the film frame or digital sensor used in a camera. As digital cameras emerge with a wide range of format sizes, there is often interest in the effect that this difference has on depth of field performance. Format size in fact does influence the calculated depth of field, although in a surprising way, involving the choice of a COCDL and the focal length lens we might use in a particular photographic situation. (We'll see later how that happens.)

But making a meaningful comparison of depth of field performance between two cameras with different format sizes is not as simple as might at first be thought. We will probably wish to adopt an "all other things being equal" approach. But what would **that** mean?

One reasonable approach would be to make the comparison under the following provisos:

- The focal lengths of the lenses involved in the comparison produce consistent fields of view in both cases. (This is sometimes thought of in the digital camera world as the lenses having the same "full-frame 35-mm camera equivalent focal length".)

- The aperture (as an f/number) is the same for both cases.

- Focus is at the same distance for both cases.

- We use a consistent COCDL when expressed as a fraction of the size of the format. (This follows a common application of the first "outlook" discussed in Appendix A.)

It will turn out that, under those conditions, the camera with the smaller format size will exhibit greater depth of field.

Looking at it from the other direction, to achieve a certain depth of field on a camera with a larger format requires the use of a smaller aperture than would be needed on a camera with a smaller format. In particular, it would require an aperture whose f/number is *j* times the f/number of the aperture used on the smaller-format camera, where *j*, the *format size factor*, is the ratio of a linear dimension of the format of the larger-format camera to that of the smaller-format camera[3] (and the other provisos above are held).

A mathematical demonstration of this relationship is given in Appendix C, which also discusses related matters pertaining to format size.

Depth of field "calculators"

There have been made available many "calculators" intended to facilitate the determination of depth of field performance. Some use spreadsheets, others are "on-line", others are sets of tables, and yet others are essentially "circular slide rules".

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[3] This is the same factor we often use in determining the "35-mm equivalent focal length" of a lens of a certain focal length when used on a camera with a format size different from that of a 35 mm film camera.

Some of these "calculators" are wholly generic, but others are in some way "intended" for use with a certain type of camera (in terms of format size), often a full-frame 35-mm camera, but sometimes another specific camera having a different format size.

Often the question arises, "How can I adapt the use of such a calculator to my particular camera, which has a different format size?" The answer depends on the way the "calculator" is set up.

This matter is discussed thoroughly in Appendix D.

Quasi-reciprocity

A very interesting relationship is almost true in depth of field calculations. It is perhaps most easy described with an example. Suppose that, with a certain focal length, f/number, and COCDL, we assume focus at a distance of 10.0 meters and calculate the near limit of the depth of field as 6.98 meters.

Now, holding the basic parameters constant, we assume that we focus the camera at a distance of 6.98 meters. Then the far limit of the depth of field will be very close to 10.0 meters.

The same situation occurs in the other direction.

Recognizing this can be handy when looking into various depth-of-field related matters.[4]

DEPTH OF FOCUS

The term "depth of focus" is often, but incorrectly, used as a synonym for *depth of field*. It describes a different, although related, concept arising from the same optical principles.

Assume that we have an object at a certain distance and the camera has been focused to bring the object to a perfectly-focused image at the film plane. Now suppose that we move the film plane in and out, spoiling the focus, but we only go so far in each direction that the diameter of the blur figure reaches the limit we have adopted—the COCDL. That range of motion of the film plane is the *depth of focus*. It is primarily of concern to camera designers, helping to assess the effects of such things as film curvature. It also figures into the analysis of focusing accuracy of cameras as it is affected by tolerances on the location of the various optical components.

[4] The term "reciprocity" to describe this was suggested by Leon Wittwer.

Even though depth of focus is (numerically) quite different from depth of field, the two are intimately related. For example, the accuracy tolerance on a camera's automatic focus mechanism may be described in terms of the depth of focus (of course having adopted a certain COCDL): "accurate within the depth of focus". Sometimes an enthusiast will misquote that in an online forum as "accurate within the depth of field", and will then be chastised by "better informed" colleagues for not understanding the distinction.

But in fact an automatic focusing error that shifts the plane of focus in the camera from where it should fall by exactly what we consider the depth of focus will result in the plane of ideal focus in subject space being displaced from the subject on which we tried to focus by exactly the depth of field in the pertinent direction. And it is of course that effect—not the one in image space—that is of importance to the photographer.

THE CONCEPT OF "OUT OF FOCUS BLUR PERFORMANCE"

Often we are interested in comparing the behavior of two different "situations" with respect to depth of field in a qualitative, rather than quantitative, way. A common such question is, "[All other factors being equal[5]], if I have two digital cameras with different format (sensor) sizes, which will give me the greater depth of field?" A useful outlook is one that does not require us to deal with the matter of the choice of COCDL criterion for the two different camera types—actually, to really deal with "depth of field". The property of interest in that outlook is what I call "out of focus blur performance"[6]

Here is the concept, as applied to the particular comparison mentioned just above. Imagine that we have two cameras, with different sensor sizes. We equip them with lenses whose focal lengths give equivalent fields of view on the two cameras. We use the same aperture on each camera. We shoot the same scene from the same point, having focused at the same distance in each case.

We then examine the two images at comparable display or print sizes, from the same viewing distances (perhaps 12" x 8" glossy prints, laid side-by side

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[5] Usually the "all other factors being equal" clause isn't really stated by the questioner, but of course it has to be dealt with, and, as we saw earlier, some thought needs to be given to just what we might mean by that.

[6] Thanks to Michael Schaefer, a member of one of the Digital Photography Review forums, for recommending this term. I had at first introduced this concept in the forum under the term "depth of field performance", but Michael pointed out that it might be better to limit the phrase "depth of field" for situations in which we were actually quantifying a "depth of field".

on our coffee table). What will we see with regard to the matter of blurring caused by imperfect focus for scene objects not at the focus distance?

For one thing, for either camera's image the blurring will be greater the further that the distance to the specific object departs from the "distance of perfect focus".

But, for any given object at a specific distance, not the perfect focus distance, the degree of blurring (diameter of the blur figure) will be greater in the image from the camera with the larger sensor. Thus, we can say that the larger-sensor camera exhibits worse "out of focus blur performance".

Note that, since we have not established (in this exercise) a criterion for what diameter of the blur figure constitutes a limit of "acceptable blurring", we can't say, for either camera, over what range of object distances is blurring from misfocus acceptable—what depth of field we ascribe to each. But we can nevertheless clearly see that, for objects not at the proper focus distance, the blurring is worse for the larger-sensor camera.

The most valuable property of this outlook is that it does not require us to adopt any particular outlook on establishing a COCDL for either camera (as we would need to do to calculate a numerical "depth of field" in either case).

But will the camera with "better out of focus blur performance" exhibit greater depth of field than the other camera, "all other factors being equal"?

If included in that stipulation of "all other factors being equal" is that the COCDL is, for both cameras, chosen as a fixed fraction of the diagonal sensor size of the camera, then "yes". If however, the COCDL is set in terms of the pixel size of the sensor, then "not necessarily".

These two outlooks on choosing a COCDL are described in Appendix A, which includes a discussion of the perhaps-surprising result mentioned in the preceding paragraph.

Another aspect of blur performance is the situation in which we cultivate a "tasteful" out of blur situation for objects in front of or behind our "main subject" as a matter of artistic style.[7]

Often photographers attempt to estimate or describe the degree of blurring of an out-of focus object in terms of depth of field, but of course here it is

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[7] Blur intentionally exploited for this purpose is sometimes called "bokeh", a word coined from a Japanese word (spelled, in Latin characters, "boke") meaning, roughly, "blurred". The spelling "bokeh" is intended to prevent readers from thinking the word should be pronounced like "spoke".

the complementary matter of out-of focus blur performance that is the indicator of the effect to be expected.

WHEN WORSE IS BETTER

Photographers whose interest at the moment is in *boke* will sometimes speak of a camera setup that produces greater blurring as having “better depth of field”. Of course they mean that the limited depth of field that setup produces goes hand-in-glove with the substantial blurring that they seek for that work.

But this often produces confusion to those readers who, recognizing that the basic concept of photography is to render an accurate image of the subject, think that “better” depth of field means “greater” depth of field.

So I discourage the simplistic use of terms “better” or “worse” in connection with depth of field performance, or out of focus blur performance, unless the context of the discussion has first been articulated.

A CLOSING CAUTION

It is easy to be seduced by the intricate trains of thought involved in the calculation of depth of field behavior and believe that the results of these calculations will tell us whether or not the results of a particular setup will yield an “acceptable” result. They can’t. For one thing, the calculation process, as I describe it in this article, depends on an arbitrary measure of the degree of blurring, to which we assign an arbitrary “bogey”, and all this is done within a framework of numerous assumptions and arbitrary predicates about how the image will be viewed and other matters. The concept of depth of field is a wholly “man-made” construct.

The process of course can take no account of such matters as the nature of the different scene elements whose degree of focus is of interest, the purpose of the image, how it will actually be viewed, or the perception of the ultimate “client”.

Nevertheless, so long as we remember this, the results of depth of field calculations, thoughtfully considered, can be very useful in guiding our photographic technique.

ACKNOWLEDGEMENTS

I would like to acknowledge the contributions of the numerous colleagues, both those named here and otherwise, that have contributed to my outlook on these matters.

I would also like to express my appreciation to Carla Kerr for her meticulous and insightful copy editing of this difficult manuscript. She points out, however, that I am on my own so far as the equations are concerned!

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APPENDIX A

Choice of a circle of confusion diameter limit (COCDL)

INTRODUCTION

In making quantitative determinations of the depth of field to be expected from a particular photographic setup, we draw upon the concept of "acceptable sharpness", or its counterpart, "acceptable blurring". We quantify this by adopting a maximum acceptable diameter of the *blur figure* created in the image, as a result of imperfect focus, from each point in the subject. In this article, I call this diameter the "circle of confusion diameter limit", or COCDL.

For many years, a particular outlook, based on human visual acuity, was widely used as the basis for choosing an appropriate "general use" value for this criterion.

In modern times, especially when digital cameras are involved, an alternative outlook, based on camera resolution, is often followed.

In this appendix, I will, for each of these outlooks, reconstruct their rationales, and explain the numerous assumptions embraced by the common application of the outlook, finally arriving at some specific COCDL values that are widely used today under each outlook.

Note that it is not our purpose here to either endorse nor deprecate the use of either of these outlooks as a basis for the adoption of a COCDL value.

THE 'VISUAL ACUITY" OUTLOOK

This outlook begins with the recognition that the human eye has a finite *angular resolution*—its ability to distinguish two points separated by a certain angle. Clearly, setting a criterion for "acceptable blurring" that is finer than the eye's resolution is naïve if the image is intended for human viewing.

It is generally considered that the resolution of the human eye is approximately one-third of a milliradian—that is, the eye can distinguish two points separated by an angle of 1/3000 radian, very roughly one minute of arc (1/60 degree). This would mean, for example, the ability to distinguish two points one inch apart at a distance of 3000 inches (about 250 feet).

How can we relate this to an appropriate COCDL on our camera's film or digital sensor, where the blurring actually occurs? Here is where the assumptions in the rationale start to appear.

Imagine that we have taken a photo of a scene with a full-frame 35-mm camera using a lens of focal length 50 mm (often, quite arbitrarily, said to be the "normal" focal length for such a camera) and have produced from the negative a print of size 12 x 8 inches, which we then view from a distance of 16.7 inches. This combination of circumstances produces what we may call "actual-size viewing". That means that the angular distance between any two points on the print, as seen by the viewer on that print at the stated viewing distance, is exactly the same as the angular distance between the two corresponding points of the scene as viewed with the "naked eye" from the camera location.

Based on a visual resolution of 1/3000 radian, the viewer of this print should be able to resolve two points on the print separated by 0.0056 inch (about 1/180 inch). (I had that ability once!)

Tracing this back to the negative (which, for the camera described, would have a size of 36 x 24 mm), this would correspond to a separation of 0.00066 inch, or 0.017 millimeter.[8] Perhaps we should thus adopt this as our COCDL.

But (continues the rationale) a degree of blurring defined by this criterion is unrealistic, since blurring from such phenomena as lens aberrations, as well as the inherent resolution limit of the film, would "swamp" that amount of blurring from imperfect focus. Thus, perhaps we should adopt a more coarse COCDL, maybe (**very** arbitrarily) twice as great a diameter, or 0.034 mm.

Accordingly, a COCDL in the range 0.030-0.035 mm has been widely used by camera manufacturers in preparing general-purpose depth of field tables for their lenses basically intended for use with full-frame 35 mm cameras. Canon, for example, seems to use 0.035 mm; Leica, on the other hand, has traditionally used 0.024 mm, a more stringent criterion.

Use of lenses of other focal lengths

It might seem that, when we calculate the depth of field for a particular focal length lens, we should first choose a COCDL using a model based on actual-size viewing of an image taken with a lens of that focal length, not arbitrarily based on actual-size viewing of an image taken with the "normal" lens focal length (as we did just above).

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[8] This result can be attained in a much more direct way, without resorting to the scenario of a print of a certain size viewed at a certain distance: it is just the assumed visual angular resolution times the focal length of the reference lens. Nevertheless, I find that people can often grasp the significance of the result best by getting at it in this "more tangible" way.

This would indeed be appropriate if we were to stipulate that, regardless of the focal length used to take the picture, we would wish the objects in it to have the same apparent size in the viewed print as they appear in real life.

But (goes the rationale) when we use a greater focal length to "zoom in" on a subject, it is usually because we want the objects to appear correspondingly larger in the print than they would have to the naked eye from the camera position—that's the whole object of using a larger focal length. Based on that concept, a single COCDL—based on a model involving the "normal" lens focal length—is probably most appropriate for depth-of-field calculations made for any focal length lens.

Application to other format sizes

Consider the use of a COCDL of 0.031 mm for a full-frame 35 mm camera. That diameter is about 1/1400 the diagonal size of the film frame of such a camera. If we wish to apply the same outlook to a camera having a different "format size" (film frame or digital sensor size), we can do so by keeping that same fraction of the sensor diagonal size.

Thus for a Canon EOS-20D, with a sensor 22.5 x 15.0 mm in size (a diagonal size of 27.0 mm), the corresponding COCDL value would be 0.019 mm.

Often we may not have the actual sensor dimensions in hand for such a camera, but we may well know the "format size factor", the ratio of the format size of the full-frame 35 mm camera (in this case) to the format size for the camera of interest.[9] Thus, if we start with a COCDL we consider appropriate on a full-frame 35-mm camera, then to determine the corresponding COCDL value for another camera we must divide the 35-mm COCDL value by the applicable format size factor. (For the Canon EOS-20D, that factor is 1.6.) Thus, starting with a basic COCDL value of 0.031 mm for a full-frame 35 mm camera, we would get 0.019 mm for this camera.

Effect of format size on calculated depth of field

In Appendix C, I discuss at some length the effect of differences in the format sizes of two cameras on their respective depth of field performance. Let us just note here that the only ways format size enters into the actual

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[9] This factor is widely used, for example, to determine what focal length lens on a 35 mm camera would exhibit the same field of view as would a lens of a certain focal length on the camera of interest. It is often (but inappropriately) spoken of as the "field of view crop factor" ("FOV crop factor").

calculation of expected depth of field performance are that a different format size will, as I have just discussed, suggest a different COCDL, and will also suggest a different focal length for any particular photographic composition objective (part of the "all other factors being equal" stipulation).

THE CAMERA RESOLUTION OUTLOOK

Another outlook on the adoption of an appropriate COCDL is based on the premise that any misfocus blurring that substantially compromises the resolution of the camera is "unacceptable". Under this outlook, we choose a COCDL that is a fixed multiple of the "resolvable line spacing" of the camera.

Suppose for example that it is considered that a certain digital camera can successfully resolve 1000 lines per picture height (to make it independent of format size). A stringent view is that any misfocus that gives a blur figure of diameter greater then 1/1000 of the picture height (the resolution line spacing) degrades the image, and should thus be considered unacceptable.

What might this lead to in an illustrative case? Imagine a digital camera with a format of 22.5 x 15.0 mm, and an image layout of 3000 x 2000 pixels. It would not be surprising to find, by image testing, that its resolution was 1500 lines per picture height. (It will always be less than the "geometric" resolution, based on pixels per picture height, 2000 in the example.)

Thus, following the camera resolution outlook, we might adopt a COCDL of 1/1500 of the picture height, or 0.01 mm.

To compare it to the criterion discussed earlier, this would be 1/2700 of the format diagonal, a criterion twice about as stringent as the one typically adopted for this camera under the first outlook (visual acuity based).

Others might apply this outlook but suggest that for practical reasons, a COCDL of twice the resolution line spacing be used. That would lead, in our example, to a COCDL of 0.02 mm, essentially the same value resulting from the visual acuity outlook.

Some workers prefer to think in terms not of the measured resolution of the camera but only its pixel pitch (the "geometric resolution"). Here, it is common to adopt a COCDL of twice the pixel pitch, or 0.015 mm for our example camera. Again for comparison, this would be 1/1800 of the format diagonal, about 1.3 times as stringent as under the visual acuity outlook.

Application of this outlook can result in some disturbing results. For example, suppose I have a camera such as I postulated for the example above. I consider purchasing an improved newer model, with the same format size but a greater pixel density, with an image layout of 3750 x 2500 pixels, and accordingly a higher resolution to match (perhaps 1875 lines per picture height.

I am now horrified to find, after a colleague does some calculations for me, that this expensive, higher-resolution new camera (with the same format size) would give me less depth of field than my present camera for any combination of focal length, aperture, and subject distance.

Where does this paradox in calculated depth of field come from? When we utilize the “camera resolution” outlook on choice of a COCDL, we hold “acceptable” misfocus blurring to a limit proportional to the camera’s resolution. Thus with a higher resolution, we automatically position ourselves to tolerate less misfocus blurring. The result is that the range of object distances for which this stricter blurring criterion is maintained is less. We have no choice but to describe this as “less depth of field”.

Of course, images of the same scene, taken with both cameras with lenses of identical focal length and aperture and focused on the same object, preferably observed as prints of the same size seen from the same distance, will exhibit the identical decline of focus for objects at distances varying from the ideal focus distance.[10] The difference is that the photograph taken with the “new” camera will likely be, overall, “sharper”, at least if looked at sufficiently closely. If viewed from a distance such that the difference in resolution is not perceptible, the two photos will seem identical. If looked at closely, the photograph from the new camera may look sharper for objects at the perfect focal distance, but not elsewhere.

This paradox, incidentally, is one of my reasons for being suspicious of the camera-resolution outlook as a basis for the choice of a COCDL. (Ooh—I said I wasn’t going to do that.)

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[10] This is the property discussed in the body of the article under the heading “The Concept of Out of Focus Blur Performance”

APPENDIX B

Depth of field equations

In this appendix, I present the "classical" equations used to determine depth-of-field performance, based on the rationale and general assumptions described elsewhere in this article. They are derived from fundamental considerations of geometric optics.

These equations come from those presented in *Photographic Optics* by Allen R. Greenleaf, The MacMillan Company, New York, 1950 (pp 25-27). Thanks to Don Fleming (developer of the DOFMaster line of depth of field calculating products) for making them conveniently available on his Web site. They have been independently verified by this author, working from fundamental optical principles.

These equations are only strictly valid for a lens design in which the entrance and exit pupils of the lens are located at the 1st and 2nd principal points of the lens, respectively (a design for which it is said that the *pupil magnification* is 1). Since we rarely know the pupil magnification for any particular lens design, we rarely could take that into account anyway. So we have no choice but to accept any error resulting from ignoring that parameter. Fortunately, for most cases of interest, the error is very slight.

I will give two versions of the equations, first the "precise" version, and then an "approximate" version, which gives reasonable results for situations in which the hyperfocal distance is perhaps 10 times the focal length of the lens. For each version, I will give the equations in three forms: consistent units for all linear dimensions (meters, for example); focal length and COCDL in millimeters and all distances in meters; focal length and COCDL in millimeters and all distances in feet.

All distances in the following equations are reckoned from the 1st principal point of the lens (which is also the location of the entrance pupil since the equations assume a pupil magnification of 1). If the distances involved are substantial, little error will occur with the use of any handy point on the camera as the reference.

SYMBOLS

The following symbols are used in all the sets of equations:

D represents a distance in front of the camera, in particular:

- D_h represents the hyperfocal distance
- D_n represents the near subject distance limit for acceptable blurring
- D_f represents the far subject distance limit for acceptable blurring

P represents the distance to which the camera is focused

f represents the focal length of the lens (actual, not "equivalent")

n represents the lens aperture, as an f/number

c represents the chosen maximum acceptable diameter of the blur figure (circle of confusion diameter limit, COCDL)

THE "PRECISE" EQUATIONS

For *D*'s, *P*, *f*, *c* in any consistent units

Hyperfocal distance

$$D_h = \frac{f^2}{nc} + f \qquad (1)$$

Near limit of depth of field

$$D_n = \frac{P(D_h - f)}{D_h + P - 2f} \quad \text{or} \quad D_n = \frac{Pf}{f + nc\left(\frac{P}{f} - 1\right)} \qquad (2a,b)$$

(The second form avoids the need for use of the intermediate result D_h, hyperfocal distance.)

Far limit of depth of field

$$D_f = \frac{P(D_h - f)}{D_h - P} \quad \text{or} \quad D_n = \frac{Pf}{f - nc\left(\frac{P}{f} - 1\right)} \qquad (3a,b)$$

For *D*'s and *P* in meters, *f* and *c* in millimeters

Hyperfocal distance

$$D_h = \frac{f^2}{1000nc} + \frac{f}{1000} \qquad (4)$$

<u>Near limit of depth of field</u>

$$D_n = \frac{P\left(D_h - \dfrac{f}{1000}\right)}{D_h + P - \dfrac{2f}{1000}} \quad \text{or} \quad D_n = \frac{Pf}{f + nc\left(\dfrac{1000\,P}{f} - 1\right)} \qquad (5a,b)$$

<u>Far limit of depth of field</u>

$$D_f = \frac{P\left(D_h - \dfrac{f}{1000}\right)}{D_h - P} \quad \text{or} \quad D_n = \frac{Pf}{f - nc\left(\dfrac{1000\,P}{f} - 1\right)} \qquad (6a,b)$$

For *D*'s and *S* in feet, *f* and *c* in millimeters

<u>Hyperfocal distance</u>

$$D_h = \frac{f^2}{304.8nc} + f \qquad (7)$$

<u>Near limit of depth of field</u>

$$D_n = \frac{P\left(D_h - \dfrac{f}{304.8}\right)}{D_h + P - \dfrac{f}{152.4}} \quad \text{or} \quad D_n = \frac{Pf}{f + nc\left(\dfrac{304.8\,P}{f} - 1\right)} \qquad (8a,b)$$

<u>Far limit of depth of field</u>

$$D_f = \frac{P\left(D_h - \dfrac{f}{304.8}\right)}{D_h - P} \quad \text{or} \quad D_n = \frac{Pf}{f - nc\left(\dfrac{304.8\,P}{f} - 1\right)} \qquad (9a,b)$$

Cousins of the hyperfocal distance

<u>Near limit for focus at the hyperfocal distance</u>

The near limit of the depth of field, for focus at the hyperfocal distance, is approximately equal to half the hyperfocal distance.

The precise equation is:

$$D_{nh} = \frac{\left(\frac{p^2}{nc} - f\right)}{2\left(1 - \frac{nc}{f}\right)} \qquad (10)$$

where D_{nh} is the near limit of the depth of field for focus at the hyperfocal distance and f, n, and c have their usual significance.

This can be rewritten in terms of the hyperfocal distance, D_h, thus:

$$D_{nh} = \frac{D_h}{2} \frac{1}{\left(1 - \frac{nc}{f}\right)} \qquad (11)$$

Thus we see that D_{nh} is very nearly half the hyperfocal distance if nc/f is small, as it is for most cases of interest.

<u>Near limit for focus at infinity</u>

The near limit of the depth of field when focus is at infinity, D_{ni}, is approximately equal to the hyperfocal distance. [11] The precise equation is:

$$D_{ni} = \frac{f^2}{nc} \qquad (12)$$

Note that this is always less than the hyperfocal distance by exactly the focal length, and so for a hyperfocal distance many times the focal length the two distances can be considered essentially the same.

Fun with algebra

If we look at the second forms of equations 2 and 3, we see that we consolidate them both into this form:

$$D_{n,f} = \frac{Pf}{f \pm nc\left(\frac{P}{f} - 1\right)} \qquad (13)$$

But we don’t seem to see the same symmetry in the first forms. What gives? (It baffled me at first!)

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[11] This is often stated as if the equivalence were exact, which is interpreted by some as meaning that this distance is an alternate definition of hyperfocal distance.

In fact, they are symmetrical, and we can expose that symmetry if we rewrite them thus:

$$D_{n,f} = \frac{P\left(D_h - f\right)}{D_h + P - f \mp f} \qquad (14)$$

Of course, the forms originally given are individual simplifications of this, handier for practical use.

THE "APPROXIMATE" EQUATIONS

These equations provide a good approximation to the precise result so long as the focus distance, P and the hyperfocal distance, D_h, are large compared to the focal length.

For *D*'s, *S*, *f*, *c* in any consistent units

Hyperfocal distance

$$D_h = \frac{f^2}{nc} \qquad (15)$$

Near limit of depth of field

$$D_n = \frac{PD_h}{D_h + P} \quad \text{or} \quad D_n = \frac{Pf^2}{f^2 + Pnc} \qquad (16)$$

(The second form avoids the need for use of the intermediate result D_h, hyperfocal distance.)

Far limit of depth of field

$$D_f = \frac{PD_h}{D_h - P} \quad \text{or} \quad D_f = \frac{Pf^2}{f^2 - Pnc} \qquad (17)$$

For *D*'s and *P* in meters, *f* and *c* in millimeters

Hyperfocal distance

$$D_h = \frac{f^2}{1000\,nc} \qquad (18)$$

Near limit of depth of field

$$D_n = \frac{PD_h}{D_h + S} \quad \text{or} \quad D_n = \frac{Pf^2}{f^2 + 1000\,Pnc} \qquad (19a,b)$$

Far limit of depth of field

$$D_n = \frac{PD_h}{D_h - S} \quad \text{or} \quad D_f = \frac{Pf^2}{f^2 - 1000\,Pnc} \qquad (20a,b)$$

For *D*'s and *P* in feet, *f* and *c* in millimeters

Hyperfocal distance

$$D_h = \frac{f^2}{304.8\,nc} \qquad (21)$$

Near limit of depth of field

$$D_n = \frac{PD_h}{D_h + P} \quad \text{or} \quad D_n = \frac{Pf^2}{f^2 + 304.8\,Pnc} \qquad (22a,b)$$

Far limit of depth of field

$$D_n = \frac{PD_h}{D_h - P} \quad \text{or} \quad D_f = \frac{Pf^2}{f^2 - 304.8\,Pnc} \qquad (23a,b)$$

DEPTH OF FIELD IN MACROPHOTOGRAPHY

In connection with macrophotographic work, our interest in depth of field makes us encounter the following realities:

- The depth of field may be rather small, and we are more likely interested in the actual depth of the field than in its near and far limit distances.
- We probably do not know the focus distance P to the precision that is required in this case (among other things, we probably don't know where the 1st principal point of the lens is).
- We probably are conscious of the image magnification that applies to our setup.

There is an alternate form of the depth of field equations that caters to this overall set of considerations. We will see them here in their "precise" form[12].

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[12] Thanks to Helmuth Schumann, who derived these equations, thus saving me the tedious algebra that is required.

The following symbols are used these equations:

d_n represents the "near depth of field" (that is, the distance toward the camera from the plane of focus within which blurring is acceptable)

d_f represents the "far depth of field" (that is, the distance away from the camera from the plane of focus within which blurring is acceptable)

d_t represents the total depth of field (that is, the entire range of object distance within which blurring is acceptable)

f represents the focal length of the lens (actual, not "equivalent")

m represents the image magnification of the "setup"

n represents the lens aperture, as an f/number

c represents the chosen maximum allowable diameter of the blur figure (circle of confusion diameter limit, COCDL)

Here are the precise equations, set up for d_n, d_f, d_d, f, and c in consistent units (typically millimeters):

Near depth of field

$$d_n = \frac{\left(\frac{nc}{m}\right)\left(1+\frac{1}{m}\right)}{1+\frac{nc}{fm}} \tag{24}$$

Far depth of field

$$d_f = \frac{\left(\frac{nc}{m}\right)\left(1+\frac{1}{m}\right)}{1-\frac{nc}{fm}} \tag{25}$$

Total depth of field

$$d_t = \frac{\left(\frac{nc}{m}\right)\left(1+\frac{1}{m}\right)}{1-\left(\frac{nc}{fm}\right)^2} \tag{26}$$

Note that, for reasonably-large magnifications (specifically, when nc/fm is much smaller than 1), this can be well approximated by:

$$d_t = \left(\frac{nc}{m}\right)\left(1+\frac{1}{m}\right) \tag{27}$$

which shows us that, in such situations, the total depth of field is essentially affected by the f/number, *n*; the COCDL we adopt, *c*; and the magnification, *m*, of the setup in use, but not significantly by focal length. Note further that the total depth of field essentially varies directly with the f/number, *n*, so a doubling of the f/number essentially results in a doubling of the total depth of field.

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Appendix C

Effect of Format Size

Effect of format size

As digital cameras emerge with a wide range of *format size* (film frame or digital sensor size), there is often interest on the effect that format size has on depth of field performance. Consider a comparison between two cameras having different format sizes, under the following provisos:

- The focal lengths of the lenses involved in the comparison produce consistent fields of view. (This is sometimes thought of in the digital camera world as the lenses having the same "full-frame 35-mm equivalent focal length".)
- We use a consistent COCDL, expressed as a fraction of the diagonal size of the format. (This would be reasonable under the "first outlook" discussed in Appendix A.[13])
- Focus distance and aperture (as an f/number) are the same

It will turn out that, under those provisos, the camera with the smaller format size will exhibit greater depth of field for any given focus distance and aperture, or a smaller hyperfocal distance for any given aperture.

Analytical demonstration

I will use the expression for hyperfocal distance as the "indicator" of change in depth-of-field quantities. Remember, when the camera is focused at the hyperfocal distance, adequate focus is achieved for objects from very nearly one-half the hyperfocal distance to infinity. Thus, the smaller the hyperfocal distance, the greater we may say is our depth of field. (Or course, the depth of field for the camera focused at other distances will vary as well, and of course in the same direction, but to get a single quantitative value for "depth of field" in that general situation we have to subtract two fairly complicated expressions!)

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[13] Note that if one holds to the "second outlook" for adoption of a COCDL (the "camera resolution" outlook), there is no way to compare the depth of field performance of two cameras based on sensor size alone.

We start with this "simplified" equation for hyperfocal distance (already introduced):

$$D_h = \frac{f^2}{1000\,nc} \tag{28}$$

(This form is for D_h in meters but f and c in millimeters.)

We will then define some new quantities:

Z represents the diagonal size of the frame/sensor in mm.

V represents the "relative field of view" in arbitrary terms, defined as

$V = Z/f$, where f is the focal length.

C represents the COCDL expressed as a fraction of the frame/sensor diagonal: $C = c/Z$, where c is the actual diameter in mm.

If we rearrange our expression for hyperfocal distance into terms of the above new quantities, we get:

$$D_h = \frac{Z}{1000\,nV^2C} \tag{29}$$

Thus, for the provisos adopted above, we see that the hyperfocal distance varies directly as sensor size, Z. For camera B, with a sensor size one-third that of camera A, the hyperfocal distance would be one-third that for camera A. Thus, for example, the near limit of adequate focus, when the camera is focused at the hyperfocal distance, would be only about one-third as far from the camera as for camera A (since that distance is always nearly half the hyperfocal distance).

We can also see that, to maintain a constant hyperfocal distance as sensor size varies, the aperture (as an f/number) must vary as the sensor size (so that Z/a remains constant). Thus if camera D has a sensor size twice that of camera C, then the hyperfocal distance attained on camera C with an aperture of f/4.0 will be attained on camera D with an aperture of f/8.0. (All this again assumes focal lengths giving comparable fields of view.)

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APPENDIX D

Adaptation of Depth-of-Field "Calculators"

There have been made available many "calculators" intended to facilitate the determination of depth of field issues. Some use spreadsheets, others are "on-line", others are sets of tables, and yet others are essentially "circular slide rules".

Some of these "calculators" are wholly generic, but others are in some way "intended" for use with a certain type of camera (in terms of format size), often for a full-frame 35 mm camera, but sometimes for another camera of a specific model having a different format size.

Often the question arises, "How can I adapt the use of such a calculator to my particular camera, having a different format size?"

The answer depends on how the calculator is set up. We will consider four different situations.

The "generic" calculator

A "generic" calculator allows the user to input all parameters (including the chosen *circle of confusion diameter limit*, or COCDL)[14]. Such a calculator may be used directly for a camera of any format size. All parameters should be entered "as is", no special adjustment factors being required.

Note however that the COCDL should be one appropriate to the situation involved. If we follow the "first outlook" on adopting a COCDL, it is through this that different format sizes are accommodated. The user, having no other basis for deciding on this, may wish to adopt one of the traditional values, such as 1/1400 the diagonal size of the format.

The "semi-generic" calculator

In this type of calculator, there is a "default" COCDL already entered, presumably suitable for the "intended" type of camera (perhaps full-frame 35-mm), but it can be changed by the user.

To utilize such a calculator for a camera of another format size, enter all the applicable parameters directly, again being sure to choose a value for the COCDL that is appropriate to the sensor size involved.

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[14] Which is often spoken of as the "circle of confusion".

If you wish to hold to the outlook on COCDL followed by the author of the calculator, then replace the existing value with a value that is the default value divided by the applicable "format size factor" for the camera.

The "dedicated" calculator

This type of calculator is "dedicated" to a camera of a specific format size (often, but not always, a full-frame 35 mm camera), which means that the author's outlook on an appropriate COCDL, for the "reference" camera involved, is "built in" to the calculator—it cannot be entered by the user.

Assuming that we wish to accept, sight-unseen, the author's outlook on selecting a COCDL (that is, as a fraction of the diagonal format size), we can utilize such a calculator for a camera with a different format size this way:

1. Enter for *focal length* the actual focal length to be used, multiplied by the applicable format size factor. (Assuming that the calculator was intended for a 35 mm camera, this would in fact be the "35 mm equivalent focal length" of the focal length of interest.)

2. Enter for *aperture* (as an f/number) the actual f/number of the aperture to be used, multiplied by the applicable format size factor.

3. Enter for focus distance the actual focus distance of interest, if required. (Since hyperfocal distance **is** a focus distance, no input for focus distance is needed when calculating it.)

The "adaptive" calculator

This type of calculator has "built in" a certain outlook on COCDL (usually as a certain fraction of the diagonal size of the format), but allows the user to input the format size (or perhaps the format size factor with respect to a full-frame 35 mm camera) of the camera of interest, which allows the calculator to automatically adjust the COCDL accordingly.

Again assuming that we wish to accept the author's outlook on COCDL, this type of calculator can be used directly. Enter focal length, aperture, and (if required) focus distance as is—no adjustment factors are required. Enter the appropriate format size description (in whatever terms the calculator provides for).

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