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Issue 12 September 27, 2023

#### ABSTRACT

Although the image of an object created by a camera is only "perfectly focused" when the object is at the precise distance to which the camera has been focused, objects at other distances (over a certain range) will have images of what we still consider "acceptable sharpness", an honor for which we must adopt some objective, if arbitrary, criterion. The range of object distances for which this occurs is spoken of as the *depth of field* of the camera. It is not a physical property, merely a man-made construct that can help the planning of photographic setups.

This article discusses the traditional concept by which depth of field is defined, quantified, and calculated, and describes an alternate philosophy sometimes used to develop a criterion of "acceptable sharpness". It also discusses the way in which the film frame or digital sensor size of a camera influences depth of field. The related topics of *depth of focus* and *out of focus blur performance* are also discussed.

An appendix gives the equations by which depth of field can be reckoned. Another appendix discusses some of the ramifications of the use of the "depth of field calculators" widely available on-line.

#### 1 INTRODUCTION

When a camera lens is focused on an object at a certain distance from the camera, the light from each point on the object is brought to convergence at a unique point on the film plane (if we ignore lens aberrations and diffraction effects).

For objects not precisely at this "focus distance", the convergence of the light at the film plane is imperfect, and an image point is not created from the object points. Instead, the light from each point on such an object forms a small circular *blur figure* at the film plane, often called a "circle of confusion". The overall result is that the outlines and features of such objects are "blurred" on the resulting image.

Of course, a certain amount of such blurring is not even visible to the human eye (depending of course on the scale of the final "print" and the distance from which it is viewed).

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The "tolerance" of some degree of blurring from imperfect focus is the basis for the concept of *depth of field*.

#### 2 DEPTH OF FIELD

#### 2.1 The concept

By *depth of field* we mean the range of object distances (with the camera focused at a certain distance) at which the objects will be imaged with what is thought of as "acceptable sharpness".

For this concept to be meaningful, we must adopt some objective, quantifiable definition of what we will consider acceptable sharpness. But since "sharpness" can have many meanings in photography, we will work in terms of the complementary concept of "acceptable blurring".

We do this by adopting a maximum acceptable diameter of the blur figure. Objects whose image points have a blur figure whose diameter is within this limit will be considered to have acceptable blurring. I call this limit the *circle of confusion diameter limit* (abbreviated COCDL).

Having adopted a value for the COCDL, (and for a certain distance at which the camera is actually focused, a certain lens focal length, and a certain f/number), we can calculate the range of object distances over which objects will be imaged within that acceptable "blurring" criterion—the *depth of field* for that situation.

I will discuss the matter of choice of a COCDL later in this article.

## 2.2 Not a physical property

It is important to recognize that depth of field is not a physical property of a given photographic setup.

We could not take a photographic optical setup on an optical bench and in any way by measurement, or from an analysis based on all its optical parameters, determine with the depth of field of that setup is.

Rather, depth of field is a man-made construct, based on an essentially-arbitrary criterion, that can help us plan the photographic setup for a particular photograph task.

#### 2.3 Describing the depth of field

We can describe the depth of field of as given photographic setup in several different ways:

- We can give the near and far limits of the field of "acceptable" focus: "The near limit of the field is 6.98 m, the far limit is 17.60 m."
- We can describe the actual depth of the field in the near and far directions from the distance of perfect focus, perhaps like this: "The depth of field is -3.02 m, +7.60 m." Note that the field is not in this case (nor really in any case) symmetrical.
- We can describe only the total extent of the field (not really very useful, but the most often cited!): "The total depth of field is 10.62 m."

# 2.4 Speaking of asymmetry

Just above we mentioned that the field is not symmetrically disposed about the plane (or distance) of perfect focus. The field is always greater in the far direction than the near direction.

There is a widely circulated myth that the total field of acceptable focus is distributed approximately 1/3 on the near side and 2/3 on the far side. This is just not so. The proportions vary with the focal distance. For small focus distances, the two directions may be almost equal. For large focus distances, the far distance may be many, many times the near distance (we will in fact next hear of a situation in which the far distance is infinite).

## 2.5 Speaking of confusion

There is widespread current use of the term "circle of confusion" to refer to a different, but related, thing, the quantity we will define shortly as the "circle of confusion diameter limit", or COCDL. To avoid any confusion on the part of readers accustomed to that usage, I will here use the phrase "circle of confusion" only in the term "circle of confusion diameter limit", in which there is no chance of misunderstanding, and I will call the circle of confusion itself the "blur figure".

# **3 HYPERFOCAL DISTANCE**

## 3.1 The concept

For any given aperture (as an f/number), lens focal length, and COCDL, there is a distance to which the focus may be set such that the depth of field just extends, at the far end, to an unlimited distance ("infinity"). This means that any object at any distance at or beyond the associated near limit distance will be in focus within the degree we have adopted as acceptable. That focus distance is called the *hyperfocal distance*.

The associated near limit of acceptable focus is always very nearly one half that distance.

# 3.2 A related relationship

It also turns out that, if we focus the camera at infinity, the near limit of the depth of field becomes very nearly equal to the hyperfocal distance<sup>1</sup> (and the far limit is "beyond infinity", a concept which has no physical meaning).

Sometimes this near distance for focus at infinity is inappropriately called the "hyperfocal distance", and some (erroneously) believe that this is actually the meaning of the term, not the proper meaning discussed above.

# 3.3 Fixed-focus cameras

Simple cameras having no focusing capability are often designed to focus at the hyperfocal distance for their lens' focal length and some arbitrarily chosen aperture. The intent is to give the user the largest practical range of object distances for which acceptable focus will be achieved.

# 4 TWO PHILOSOPHIES OF "ACCEPTABLE BLURRING"

There are two general philosophies of what we should consider "acceptable blurring" held by different camps in the photographic community (or perhaps, for a given worker, used in two different situations.

# 4.1 Visually negligible blurring

This is the classical philosophy as to this matter. Under this philosophy, we treat as "acceptable" any blurring that would not be noticed by a hypothetical "average observer". (Of course specific conditions of viewing must be adopted in this regard.)

## 4.2 Blurring that does not degrade the camera resolution

This philosophy has largely come into use with the advent of digital cameras. It says that we treat as acceptable blurring only blurring that does not noticeably degrade the camera's (potential) resolution (that suggested by its pixel pitch).

# 5 SELECTING A COCDL

How might an appropriate acceptable maximum diameter of the blur figure (COCDL) be chosen? The overall issue is complex, and differs between the two philosophies distinguished above. The matter is discussed in detail in Appendix A.

<sup>&</sup>lt;sup>1</sup> One focal length less, in fact. (Thanks to Doug Pardee for reminding the author of that.)

## 6 CALCULATION OF DEPTH OF FIELD DISTANCES

The various distances of interest can be calculated by the equations in Appendix B. They are presented in two versions, a "precise" version, and an "approximate" version which yields quite accurate results for focus distances substantially greater than the focal length involved.

## 7 EFFECT OF FORMAT SIZE ON DEPTH OF FIELD PERFORMANCE

By *format size* we mean the size of the film frame or digital sensor used in a camera. As digital cameras emerge with a wide range of format sizes, there is often interest in the effect that this difference has on depth of field performance. Format size in fact does influence the calculated depth of field, although in a surprising way, involving the choice of a COCDL and the focal length lens we might use in a particular photographic situation. (We'll see later how that happens.)

But making a meaningful comparison of depth of field performance between two cameras with different format sizes is not as simple as might at first be thought. We will probably wish to adopt an "all other things being equal" approach. But what would **that** mean?

One reasonable approach would be to make the comparison under the following provisos:

- The focal lengths of the lenses involved in the comparison produce consistent fields of view in both cases. (This is sometimes thought of in the digital camera world as the lenses having the same "full-frame 35-mm camera equivalent focal length".)
- The aperture (as an f/number) is the same for both cases.
- Focus is at the same distance for both cases.
- We use a consistent COCDL when expressed as a fraction of the diagonal size of the format.

It will turn out that, under those conditions, the camera with the smaller format size will exhibit greater depth of field.

Looking at it from the other direction, to achieve a certain depth of field on a camera with a larger format requires the use of a smaller aperture than would be needed on a camera with a smaller format. In particular, it would require an aperture whose f/number is j times the f/number of the aperture used on the smaller-format camera, where j, the format size factor, is the

ratio of a linear dimension of the format of the larger-format camera to that of the smaller-format camera<sup>2</sup> (and the other provisos above are held).

A mathematical demonstration of this relationship is given in Appendix C, which also discusses related matters pertaining to format size.

#### 8 DEPTH OF FIELD "CALCULATORS"

There have been made available many "calculators" intended to facilitate the determination of depth of field performance. Some use spreadsheets, others are "on-line", others are sets of tables, and yet others are essentially "circular slide rules".

Some of these "calculators" are wholly generic, but others are in some way "intended" for use with a certain type of camera (in terms of format size), often a full-frame 35-mm camera, but sometimes another specific camera having a different format size.

Often the question arises, "How can I adapt the use of such a calculator to my particular camera, which has a different format size?" The answer depends on the way the "calculator" is set up.

This matter is discussed thoroughly in Appendix D.

## 9 QUASI-RECIPROCITY

A very interesting relationship is almost true in depth of field calculations. It is perhaps most easy described with an example. Suppose that, with a certain focal length, f/number, and COCDL, we assume focus at a distance of 10.0 meters and calculate the near limit of the depth of field as 6.98 meters.

Now, holding the basic parameters constant, we assume that we focus the camera at a distance of 6.98 meters. Then the far limit of the depth of field will be very close to 10.0 meters.

The same situation occurs in the other direction.

Recognizing this can be handy when looking into various depth-of-field related matters.<sup>3</sup>

 $<sup>^2</sup>$  This is the same factor we often use in determining the "35-mm equivalent focal length" of a lens of a certain focal length when used on a camera with a format size different from that of a 35 mm film camera.

<sup>&</sup>lt;sup>3</sup> The term "reciprocity" to describe this was suggested by Leon Wittwer.

#### DEPTH OF FOCUS

The term "depth of focus" is often, but incorrectly, used as a synonym for *depth of field*. It describes a different, although related, concept arising from the same optical principles.

Assume that we have an object at a certain distance and the camera has been focused to bring the object to a perfectly-focused image at the film plane. Now suppose that we move the film plane in and out, spoiling the focus, but we only go so far in each direction that the diameter of the blur figure reaches the limit we have adopted—the COCDL. That range of motion of the film plane is the *depth of focus*. It is primarily of concern to camera designers, helping to assess the effects of such things as film curvature. It also figures into the analysis of focusing accuracy of cameras as it is affected by tolerances on the location of the various optical components.

Even though depth of focus is (numerically) quite different from depth of field, the two are intimately related. For example, the accuracy tolerance on a camera's automatic focus mechanism may be described in terms of the depth of focus (of course having adopted a certain COCDL): "accurate within the depth of focus". Sometimes an enthusiast will misquote that in an online forum as "accurate within the depth of field", and will then be chastised by "better informed" colleagues for not understanding the distinction.

But in fact an automatic focusing error that shifts the plane of focus in the camera from where it should fall by exactly what we consider the depth of focus will result in the plane of ideal focus in subject space being displaced from the subject on which we tried to focus by exactly the depth of field in the pertinent direction. And it is of course that effect—not the one in image space—that is of importance to the photographer.

## 10 THE CONCEPT OF "OUT OF FOCUS BLUR PERFORMANCE"

#### 10.1 General

Often we are interested in comparing the behavior of two different "situations" with respect to depth of field in a qualitative, rather than quantitative, way. A common such question is, "[All other factors being equal<sup>4</sup>], if I have two digital cameras with different format (sensor) sizes, which will give me the greater depth of field?" A useful outlook is one that does not require us to deal with the matter of the choice of COCDL criterion for the two different camera types—actually, to really deal with "depth of

<sup>&</sup>lt;sup>4</sup> Usually the "all other factors being equal" clause isn't really stated by the questioner, but of course it has to be dealt with, and, as we saw earlier, some thought needs to be given to just what we might mean by that.

field". The property of interest in that outlook is what I call "out of focus blur performance"<sup>5</sup>

Here is the concept, as applied to the particular comparison mentioned just above. Imagine that we have two cameras, with different sensor sizes. We equip them with lenses whose focal lengths give equivalent fields of view on the two cameras. We use the same aperture on each camera. We shoot the same scene from the same point, having focused at the same distance in each case.

We then examine the two images at comparable display or print sizes, from the same viewing distances (perhaps  $12'' \times 8''$  glossy prints, laid side-by side on our coffee table). What will we see with regard to the matter of blurring caused by imperfect focus for scene objects not at the focus distance?

For one thing, for either camera's image the blurring will be greater the further that the distance to the specific object departs from the "distance of perfect focus".

But, for any given object at a specific distance, not the perfect focus distance, the degree of blurring (diameter of the blur figure) will be greater in the image from the camera with the larger sensor. Thus, we can say that the larger-sensor camera exhibits worse "out of focus blur performance".

Note that, since we have not established (in this exercise) a criterion for what diameter of the blur figure constitutes a limit of "acceptable blurring", we can't say, for either camera, over what range of object distances is blurring from misfocus acceptable—what depth of field we ascribe to each. But we can nevertheless clearly see that, for objects not at the proper focus distance, the blurring is worse for the larger-sensor camera.

The most valuable property of this outlook is that it does not require us to adopt any particular outlook on establishing a COCDL for either camera (as we would need to do to calculate a numerical "depth of field" in either case).

But will the camera with "better out of focus blur performance" exhibit greater depth of field than the other camera, "all other factors being equal"?

If included in that stipulation of "all other factors being equal" is that the COCDL is, for both cameras, chosen as a fixed fraction of the diagonal

<sup>&</sup>lt;sup>5</sup> Thanks to Michael Schaefer, a member of one of the Digital Photography Review forums, for recommending this term. I had at first introduced this concept in the forum under the term "depth of field performance", but Michael pointed out that it might be better to limit the phrase "depth of field" for situations in which we were actually quantifying a "depth of field".

sensor size of the camera, then "yes". If however, the COCDL is set in terms of the pixel size of the sensor, then "not necessarily".

Another aspect of blur performance is the situation in which we cultivate a "tasteful" out of blur situation for objects in front of or behind our "main subject" as a matter of artistic style.<sup>6</sup>

Often photographers attempt to estimate or describe the degree of blurring of an out-of focus object in terms of depth of field, but of course here it is the complementary matter of out-of focus blur performance that is the indicator of the effect to be expected.

## 10.2 WHEN WORSE IS BETTER

Photographers whose interest at the moment is in the artistic role of blurred representation of background or foreground objects will sometimes speak of a camera setup that produces greater blurring as having "better depth of field". Of course they mean that the limited depth of field that setup produces goes hand-in-glove with the substantial blurring that they seek for objects far from the focus distance.

But this often produces confusion to those readers who, recognizing that the basic concept of photography is to render an accurate image of the subject, think that "better" depth of field means "greater" depth of field.

So I discourage the simplistic use of terms "better" or "worse" in connection with depth of field performance, or out of focus blur performance, unless the context of the discussion has first been clearly articulated.

## 11 A CLOSING CAUTION

It is easy to be seduced by the intricate trains of thought involved in the calculation of depth of field behavior and believe that the results of these calculations will tell us whether or not the results of a particular setup will yield an "acceptable" result. They can't.

For one thing, the calculation process, as I describe it in this article, depends on an arbitrary measure of the degree of blurring, to which we assign an arbitrary "bogey", and all this is done within a framework of numerous assumptions and arbitrary predicates about how the image will be viewed

<sup>&</sup>lt;sup>6</sup> Blur intentionally exploited for this purpose is sometimes called "bokeh", a word coined from a Japanese word (spelled, in Latin characters, "boke") meaning, roughly, "blurred". The spelling "bokeh" is intended to prevent readers from thinking the word should be pronounced like "spoke".

and other matters. The concept of depth of field is a wholly "man-made" construct.

The process of course can take no account of such matters as the nature of the different scene elements whose degree of focus is of interest, the purpose of the image, how it will actually be viewed, or the perception of the ultimate "client".

Nevertheless, so long as we remember this, the results of depth of field calculations, thoughtfully considered, can be very useful in guiding our photographic technique.

## 12 ACKNOWLEDGEMENTS

I would like to acknowledge the contributions of the numerous colleagues, both those named here and otherwise, that have contributed to my outlook on these matters.

I would also like to express my appreciation to Carla Kerr for her meticulous and insightful copy editing of the first issue of this difficult manuscript. She points out, however, that I am on my own so far as the equations are concerned! And any editorial flaws in the current issue are wholly my responsibility.

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#### Appendix A Choice of a circle of confusion diameter limit (COCDL)

#### A.1 INTRODUCTION

In making quantitative determinations of the depth of field to be expected from a particular photographic setup, we draw upon the concept of "acceptable sharpness", or its counterpart, "acceptable blurring". We quantify this by adopting a maximum acceptable diameter of the *blur figure* created in the image, as a result of imperfect focus, from each point in the subject. In this article, I call this diameter the "circle of confusion diameter limit", or COCDL.

I will discuss here this matter as it applies to the "visually negligible blurring" philosophy of depth of field reckoning.

Conceptually, this is predicated on the matter of human visual acuity. There are long-well-recognized values for the acuity of the typical human visual system, and so this process should be straightforward. But it isn't.

I will review the various "complications" in this process, starting with the camera and working back to the human viewer of the image.

First, note that ultimately the COCDL is defined in terms of the diameter of the blur figure as it falls on the sensor. But since we are talking about determining an appropriate value of the COCDL based on the human perception of blurring, we recognize that this must be done in terms of, for example, a print viewed by the "hypothetical" viewer.

So at some stage of the process, we must translate the limiting diameter of the blur figure as seen on such a print to the corresponding diameter at the sensor. This then requires us to know the factor by which the image on the print is magnified from the sensor image. We most often do this by assuming a size of the hypothetical print (and later introducing into the calculations the actual size of the sensor).

Of course, the human visual system (I will say just "eye" for conciseness) works not on the actual linear dimensions of things in the visual object space but rather on the basis of the angle subtended by those things. In that light, the typical visual acuity of the eye is often stated in its ability to resolve to point objects separated by a certain angle. Or in terms of the maximum spatial frequency of luminance modulation it can resolve, in terms of cycles per unit angle (in scientific terms, perhaps cycles per radian; in practical work, perhaps in cycles per degree of angle.

Now perhaps the most vexing of the challenges. There are long-well-accepted values for the resolution of the "typical" eye, perhaps in terms of maximum spatial frequency (in cycles per degree, say) that can be resolved. But what is less clear is how that relates to the amount of blurring that will not be "visually negligible" to such a hypothetical typical viewer.<sup>7</sup>

The practical reality is almost certainly that researchers in the area (perhaps with lens or camera development laboratories) have done subjective tests, with test prints of some arbitrary size at some arbitrary distance, as to what degree of blurring can be "noticed" by the subjects.

From that, the result being translated into diameter of the blur figure on the film (this work was done before the advent of digital cameras), adopted a certain COCDL for use in reckoning the depth of field that would be given by, say, a certain lens, set for a certain aperture, and focused on a certain distance.

This process was generally predicated on a camera with a format size of about 36 mm  $\times$  24 mm (the so-called "full-frame 35-mm" frame size).

Different manufacturers adopted slightly different values of COCDL. Canon, for the most part, used a COCDL of 0.031 mm. To normalize that, that is approximately 1/1396 of the diagonal size of the "full-frame 35-mm" frame size.

Accordingly, it is often suggested that, under this philosophy, a COCDL os 1/1400 of the diagonal frame size would be appropriate.

## A.2 ON CAMERAS OF OTHER FORMAT SIZES

When digital cameras came into use, a popular frame size (known for a revolting reason as the "APS-C" size) was on the order of 22.5 mm  $\times$  25.6 mm. Based on the principles discussed above, a COCDL for such cameras, consistent with that adopted by Canon for the full-frame 35-mm frame size) would be about 0.019 mm.

That notwithstanding, Canon based depth of field calculations for its "APS-C" sensor size cameras on a COCDL of 0.031 mm. Go figure.

This matter is discussed in further detail in Appendix C.

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<sup>&</sup>lt;sup>7</sup> And I will not attempt here to develop analytically any relationship between the two.

#### Appendix B Depth of field equations

#### **B.1** INTRODUCTION

In this appendix, I present the widely cited equations used to determine depth-of-field performance, based on the rationale and general assumptions described elsewhere in this article. They are said to be derived from fundamental considerations of geometric optics, and "precise". This author has confirmed their provenance by a complete derivation from first principles, arriving at exactly the same equations widely cited.

These equations are only strictly valid for a lens design in which the entrance and exit pupils of the lens are located at the 1st and 2nd principal points of the lens, respectively (a design for which it is said that the *pupil magnification* is 1). Since we rarely know the pupil magnification for any particular lens design, we rarely could take that into account anyway. So we have no choice but to accept any error resulting from ignoring that parameter. Fortunately, for most cases of interest, the error is very slight.

I will give the equations in three forms: consistent units for all linear dimensions (meters, for example); focal length and COCDL in millimeters and all distances in meters; focal length and COCDL in millimeters and all distances in feet.

All distances in the following equations are reckoned from the 1st principal point of the lens (which is also the location of the entrance pupil since the equations assume a pupil magnification of 1). If the distances involved are substantial, little error will occur with the use of any handy point on the camera as the reference.

#### B.2 SYMBOLS

The following symbols are used in all the sets of equations:

- *D* represents a distance in front of the camera for this setup, in particular:
  - $D_h$  represents the hyperfocal distance
  - $D_n$  represents the near subject distance limit for acceptable blurring
  - $D_f$  represents the far subject distance limit for acceptable blurring
- P represents the distance to which the camera is focused
- f represents the focal length of the lens (actual, not "equivalent")
- *n* represents the lens aperture, as an f/number
- *c* represents the chosen maximum acceptable diameter of the blur figure (circle of confusion diameter limit, COCDL)

#### **B.3 THE EQUATIONS**

#### For D's, P, f, c in any consistent units

Hyperfocal distance

$$D_h = \frac{f^2}{nc} + f \tag{1}$$

Near limit of depth of field

$$D_{N} = \frac{P(D_{h} - f)}{D_{h} + P - 2f} \quad \text{or} \quad D_{N} = \frac{Pf}{f + nc\left(\frac{P}{f} - 1\right)}$$
(2a,b)

(The second form avoids the need for use of the intermediate result  $D_h$ , hyperfocal distance.)

## Far limit of depth of field

$$D_{F} = \frac{P(D_{h} - f)}{D_{h} - P + f} \quad \text{or} \quad D_{N} = \frac{Pf}{f - nc\left(\frac{P}{f} - 1\right)}$$
(3a,b)

#### For *D*'s and *P* in meters, *f* and *c* in millimeters

## Hyperfocal distance

$$D_h = \frac{f^2}{1000nc} + \frac{f}{1000} \tag{4}$$

Near limit of depth of field

$$D_{N} = \frac{P\left(D_{h} - \frac{f}{1000}\right)}{D_{h} + P - \frac{2f}{1000}} \quad \text{or} \quad D_{N} = \frac{Pf}{f + nc\left(\frac{1000\ P}{f} - 1\right)} \quad (5a,b)$$

Far limit of depth of field

$$D_{F} = \frac{P\left(D_{h} - \frac{f}{1000}\right)}{D_{h} - P} \quad \text{or} \quad D_{F} = \frac{Pf}{f - nc\left(\frac{1000\ P}{f} - 1\right)} \tag{6a,b}$$

#### For *D*'s and *S* in feet, *f* and *c* in millimeters

Hyperfocal distance

$$D_h = \frac{f^2}{304.8nc} + f$$
(7)

Near limit of depth of field

$$D_{N} = \frac{P\left(D_{h} - \frac{f}{304.8}\right)}{D_{h} + P - \frac{f}{152.4}} \quad \text{or} \quad D_{N} = \frac{Pf}{f + nc\left(\frac{304.8P}{f} - 1\right)} \quad (8a,b)$$

Far limit of depth of field

$$D_{F} = \frac{P\left(D_{h} - \frac{f}{304.8}\right)}{D_{h} - P} \quad \text{or} \quad D_{F} = \frac{Pf}{f - nc\left(\frac{304.8P}{f} - 1\right)} \quad (9a,b)$$

#### B.4 COUSINS OF THE HYPERFOCAL DISTANCE

#### **B.4.1** Near limit for focus at the hyperfocal distance

The near limit of the depth of field, for focus at the hyperfocal distance  $(D_{Nh})$ , is approximately equal to half the hyperfocal distance.

The equation is:

$$D_{Nh} = \frac{\left(\frac{p^2}{nc} - f\right)}{2\left(1 - \frac{nc}{f}\right)}$$
(10)

where  $D_{Nh}$  is the near limit of the depth of field for focus at the hyperfocal distance and f, n, and c have their usual significance.

This can be rewritten in terms of the hyperfocal distance,  $D_h$ , thus:

$$D_{Nh} = \frac{D_h}{2} \frac{1}{\left(1 - \frac{nc}{f}\right)} \tag{11}$$

Thus we see that  $D_{nh}$  is very nearly half the hyperfocal distance if nc/f is small, as it is for most cases of interest.

#### B.4.2 Near limit for focus at infinity

The near limit of the depth of field when focus is at infinity,  $D_{Ni}$ , is approximately equal to the hyperfocal distance.<sup>8</sup> The precise equation is:

$$D_{Ni} = \frac{f^2}{nc} \tag{12}$$

Note that this is always less than the hyperfocal distance by exactly the focal length, and so for a hyperfocal distance many times the focal length the two distances can be considered essentially the same.

## B.5 DEPTH OF FIELD IN MACROPHOTOGRAPHY

In connection with macrophotographic work, our interest in depth of field makes us encounter the following realities:

- The depth of field may be rather small, and we are more likely interested in the actual depth of the field than in its near and far limit distances.
- We probably do not know the focus distance P to the precision that is required in this case (among other things, we probably don't know where the 1st principal point of the lens is).

<sup>&</sup>lt;sup>8</sup> This is often stated as if the equivalence were exact, which is interpreted by some as meaning that this distance is an alternate definition of hyperfocal distance.

 We probably are conscious of the image magnification that applies to our setup.

There is an alternate form of the depth of field equations that caters to this overall set of considerations. We will see them here in their "precise" form<sup>9</sup>.

The following symbols are used these equations:

- $D_N$  represents the "near depth of field" (that is, the distance toward the camera from the plane of focus within which blurring is acceptable)
- $D_F$  represents the "far depth of field" (that is, the distance away from the camera from the plane of focus within which blurring is acceptable)
- $D_T$  represents the total depth of field (that is, the entire range of object distance within which blurring is acceptable)
- f represents the focal length of the lens (actual, not "equivalent")
- *m* represents the image magnification of the "setup"
- *n* represents the lens aperture, as an f/number
- *c* represents the chosen maximum allowable diameter of the blur figure (circle of confusion diameter limit, COCDL)

Here are the precise equations, set up for  $D_N$ ,  $D_F$ ,  $D_T$ , f, and c in consistent units (typically millimeters):

Near depth of field

$$D_{N} = \frac{\left(\frac{nc}{m}\right)\left(1 + \frac{1}{m}\right)}{1 + \frac{nc}{fm}}$$
(13)

Far depth of field

$$D_{F} = \frac{\left(\frac{nc}{m}\right)\left(1 + \frac{1}{m}\right)}{1 - \frac{nc}{fm}}$$
(14)

<sup>&</sup>lt;sup>9</sup> Thanks to Helmuth Schumann, who derived these equations, thus saving me the tedious algebra that is required.

Total depth of field

$$D_{\tau} = \frac{\left(\frac{nc}{m}\right)\left(1 + \frac{1}{m}\right)}{1 - \left(\frac{nc}{fm}\right)^2}$$
(15)

Note that, for reasonably-large magnifications (specifically, when nc/fm is much smaller than 1), this can be well approximated by:

$$D_{\tau} = \left(\frac{nc}{m}\right) \left(1 + \frac{1}{m}\right) \tag{16}$$

which shows us that, in such situations, the total depth of field is essentially affected by the f/number, n; the COCDL we adopt, c; and the magnification, m, of the setup in use, but not significantly by focal length. Note further that the total depth of field essentially varies directly with the f/number, n, so a doubling of the f/number essentially results in a doubling of the total depth of field.

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## Appendix C Effect of Format Size

## C.1 INTRODUCTION

As digital cameras emerge with a wide range of *format size* (film frame or digital sensor size), there is often interest on the effect that format size has on depth of field performance. Consider a comparison between two cameras having different format sizes, under the following provisos:

- The focal lengths of the lenses involved in the comparison produce consistent fields of view. (This is sometimes thought of in the digital camera world as the lenses having the same "full-frame 35-mm equivalent focal length".)
- We use a consistent COCDL, expressed as a fraction of the diagonal size of the format. (This would be reasonable under the "visually negible blurring philospohy" discussed in Appendix A.<sup>10</sup>)
- Focus distance and aperture (as an f/number) are the same

It will turn out that, under those provisos, the camera with the smaller format size will exhibit greater depth of field for any given focus distance and aperture, or a smaller hyperfocal distance for any given aperture.

## C.2 ANALYTICAL DEMONSTRATION

I will use the expression for hyperfocal distance as the "indicator" of change in depth-of-field quantities. Remember, when the camera is focused at the hyperfocal distance, adequate focus is achieved for objects from very nearly one-half the hyperfocal distance to infinity. Thus, the smaller the hyperfocal distance, the greater we may say is our depth of field. (Or course, the depth of field for the camera focused at other distances will vary as well, and of course in the same direction, but to get a single quantitative value for "depth of field" in that general situation we have to subtract two fairly complicated expressions!)

We start with this "simplified" equation for hyperfocal distance (already introduced):

<sup>&</sup>lt;sup>10</sup> Note that if one holds to the "second philosophy" for adoption of a COCDL (the "not degraded camera resolution" philosophy), there is no way to compare the depth of field performance of two cameras based on sensor size alone.

$$D_h = \frac{f^2}{1000 \, nc} \tag{17}$$

(This form is for  $D_h$  in meters but f and c in millimeters.)

We will then define some new quantities:

Z represents the diagonal size of the frame/sensor in mm.

V represents the "relative field of view" in arbitrary terms, defined as

V = Z/f, where f is the focal length.

*C* represents the COCDL expressed as a fraction of the frame/sensor diagonal: C = c/Z, where *c* is the actual diameter in mm.

If we rearrange our expression for hyperfocal distance into terms of the above new quantities, we get:

$$D_h = \frac{Z}{1000 \, nV^2 C}$$
(18)

Thus, for the provisos adopted above, we see that the hyperfocal distance varies directly as sensor size, *Z*. For camera B, with a sensor size one-third that of camera A, the hyperfocal distance would be one-third that for camera A. Thus, for example, the near limit of adequate focus, when the camera is focused at the hyperfocal distance, would be only about one-third as far from the camera as for camera A (since that distance is always nearly half the hyperfocal distance).

We can also see that, to maintain a constant hyperfocal distance as sensor size varies, the aperture (as an f/number) must vary as the sensor size (so that Z/a remains constant). Thus if camera D has a sensor size twice that of camera C, then the hyperfocal distance attained on camera C with an aperture of f/4.0 will be attained on camera D with an aperture of f/8.0. (All this again assumes focal lengths giving comparable fields of view.)

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#### APPENDIX D

#### Adaptation of Depth-of-Field "Calculators"

#### C.3 INTRODUCTION

There have been made available many "calculators" intended to facilitate the determination of depth of field issues. Some use spreadsheets, others are "on-line", others are sets of tables, and yet others are essentially "circular slide rules".

Some of these "calculators" are wholly generic, but others are in some way "intended" for use with a certain type of camera (in terms of format size), often for a full-frame 35 mm camera, but sometimes for another camera of a specific model having a different format size.

Often the question arises, "How can I adapt the use of such a calculator to my particular camera, having a different format size?"

The answer depends on how the calculator is set up. We will consider four different situations.

#### C.4 THE "GENERIC" CALCULATOR

A "generic" calculator allows the user to input all parameters (including the chosen *circle of confusion diameter limit*, or COCDL)<sup>11</sup>. Such a calculator may be used directly for a camera of any format size. All parameters should be entered "as is", no special adjustment factors being required.

Note however that the COCDL should be one appropriate to the situation involved. If we follow the "first philosophy" on adopting a COCDL, it is through this that different format sizes are accommodated. The user, having no other basis for deciding on this, may wish to adopt one of the traditional values, such as 1/1400 the diagonal size of the format.

#### C.5 THE "SEMI-GENERIC" CALCULATOR

In this type of calculator, there is a "default" COCDL already entered, presumably suitable for the "intended" type of camera (perhaps full-frame 35-mm), but it can be changed by the user.

To utilize such a calculator for a camera of another format size, enter all the applicable parameters directly, again being sure to choose a value for the COCDL that is appropriate to the sensor size involved.

<sup>&</sup>lt;sup>11</sup> Which is often, sadly, spoken of as the "circle of confusion".

If you wish to hold to the outlook on COCDL followed by the author of the calculator, then replace the existing value with a value that is the default value divided by the applicable "format size factor" for the camera.

## C.6 THE "DEDICATED" CALCULATOR

This type of calculator is "dedicated" to a camera of a specific format size (often, but not always, a full-frame 35 mm camera), which means that the author's outlook on an appropriate COCDL, for the "reference" camera involved, is "built in" to the calculator—it cannot be entered by the user.

Assuming that we wish to accept, sight-unseen, the author's outlook on selecting a COCDL (that is, as a fraction of the diagonal format size), we can utilize such a calculator for a camera with a different format size this way:

- 1. Enter for *focal length* the actual focal length to be used, multiplied by the applicable format size factor. (Assuming that the calculator was intended for a 35 mm camera, this would in fact be the "35 mm equivalent focal length" of the focal length of interest.)
- 2. Enter for *aperture* (as an f/number) the actual f/number of the aperture to be used, multiplied by the applicable format size factor.
- 3. Enter for focus distance the actual focus distance of interest, if required. (Since hyperfocal distance **is** a focus distance, no input for focus distance is needed when calculating it.)

## C.7 THE "ADAPTIVE" CALCULATOR

This type of calculator has "built in" a certain outlook on COCDL (usually as a certain fraction of the diagonal size of the format), but allows the user to input the format size (or perhaps the format size factor with respect to a full-frame 35 mm camera) of the camera of interest, which allows the calculator to automatically adjust the COCDL accordingly.

Again assuming that we wish to accept the author's outlook on COCDL, this type of calculator can be used directly. Enter focal length, aperture, and (if required) focus distance as is—no adjustment factors are required. Enter the appropriate format size description (in whatever terms the calculator provides for).