A Metric for Chromaticity Difference

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ABSTRACT

In many areas of photographic practice, we are concerned with the difference between two chromaticity values, especially in connection with "white balance color correction" matters. An example would be the departure of the recorded image of a "white" object from the reference white chromaticity of the color space in use, or the departure form "neutrality" of the reflective chromaticity of a "neutral target" (gray card).

In this article, the author suggests the use of the metric "du′v′" as a single-valued measure of the degree of a chromaticity difference. The metric is defined and a rationale given for its use. An appendix defines the way this metric can be calculated from the sRGB coordinates R, G, and B of the two colors whose chromaticity we wish to compare. An available spreadsheet is also described that can be used to perform this determination.

BACKGROUND

The need

In many areas of photographic practice, we are concerned with the difference between two chromaticity values, especially in connection with "white balance color correction" matters. An example would be the departure of the recorded image of a "white" object from the reference white chromaticity of the color space in use, or the departure form "neutrality" of the reflective chromaticity of a "neutral target" (gray card). For many purposes, it is valuable to have a single-valued metric that describes the "magnitude" of the chromaticity difference ("how far off is it?"). Ideally, this would have a consistent implication as to the human perception of this difference.

One common practice

Many metrics of chromaticity difference are found today. Often. the manufacturer of a "neutral measurement target" (a "gray card") may say, "We carefully control the reflective color of our Gray Eminence Target Card so that, stated in the L*a*b* color model, it does not show a value of either a* or b* that is beyond ±1.0 unit".

There are several problems with this approach. The most serious is that the a*b* plane is not a *chromaticity* plane but rather a *chrominance* plane. In brief, this means that, for any given chromaticity, the values of a* and b* scale with the value of the "lightness" coordinate, L*.[1]

By way of example, two colors with the following L*a*b* coordinates:

1. L*=80, a* = +10, b*=-8

and

2. L*=40, a* = +5, b*=-4

have exactly the same chromaticity.

Thus we can see that describing a chromaticity difference (or a discrepancy from some desired value) in terms of a* and b* alone is not meaningful.

Other approaches

Chromaticity can be stated in "absolute" terms by citing the x and y coordinates under the CIE xyY color model. A graph whose axes are x and y is called the "CIE (1931) x-y chromaticity diagram". It is the familiar diagram in which we see the region of visible chromaticities bounded by a horseshoe-shaped curve (the "spectral locus").

We could thus describe the difference between two chromaticities by citing the distance (on the x-y plane between the two points representing their chromaticities.

But there are two problems with this:

- The perceived difference in chromaticity to a human observer for a certain distance is not the same over all portions of the chart.

- The perceived difference in chromaticity to a human observer for a certain difference in x is not the same as for the same difference in y.

We can, to a great degree, overcome these problems by instead "plotting" our two chromaticities on the CIE (1976) u'v' chromaticity chart. On that chart, the human perception of the chromaticity

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[1] The asterisks in the formal names of the coordinates is meant to remind us that they are nonlinear values. Often, for editorial convenience a* and b* are just referred to as "a" and "b". This does not imply any change in their definition.

difference is very much the same for a variation along either axis (or in any "direction" in between) and very much the same in different parts of the chart.

A PROPOSED METRIC

Accordingly, I suggest that a useful single-valued metric for chromaticity difference (or discrepancy) is the distance on the CIE u′v′ "chromaticity plane", expressed in the same units as u′ and v′. I suggest calling this metric "du′v′" (evocative of "delta u′v′, or perhaps Δu′v′, but easier to render).

Do not confuse this with "duv"

The user may have already see the term "duv" in a similar connection. The difference is:

- The metric duv is based on distance on the CIE (1960) uv chromaticity diagram. This is a predecessor of the u′v′ diagram, which does not exhibit the same degree of "perceptual uniformity" of the u′v′ diagram.

- The metric duv is primarily used in connection with the expression of a "generally white" chromaticity in terms of *correlated color temperature* and *Planckian locus offset*. Correlated color temperature is (by definition) the actual color temperature of a point on the Planckian (blackbody) locus that (when plotted on the CIE uv diagram) is nearest to the point representing the chromaticity of interest. Planckian offset is the distance (on the uv diagram) between the point representing the chromaticity of interest and that nearest point on the Planckian locus.[2]

HOW CAN WE APPRECIATE THE SCALE?

The matter of "how much chromaticity difference can a human perceive", in absolute terms, is a complex issue.

As a very rough rule of thumb in a typical photographic situation, different color-corrected versions of the same taken image in which the chromaticity of some item (perhaps a "white" item included in the image for calibration purposes) differ by a du′v′ of from 0.001 to

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[2] When a color correction value is expressed in Photoshop in terms of "color temperature" and "tint", "color temperature" means *correlated color temperature*, and "tint" is 3000 times the *Planckian locus offset*, with a positive value meaning that the point is above the Planckian locus (the "greener" direction).

0.002 can be discerned, in "A-B" testing in a "casual" viewing environment, to be different.

CALCULATING THE METRIC

A common situation of interest is one in which we examine an image recorded in the sRGB color space. Our image editor will usually report for us (at a chosen point in the image, or an average over some rectangular area we nominate) the R, G, and B coordinates of the recorded color there.

We may wish to compare the chromaticity of two regions on the image. Or (perhaps more commonly), we may wish to compare the chromaticity of some area with the "reference white chromaticity" for the color space in use, which in any RGB color space is one for which R = G = B.[3]

Appendix A gives the entire chain by which an RGB value can be turned into a chromaticity expressed as u'v', and then by which a value of du'v' can be calculated for two different RGB values. This is of course not a process we would like to do with our hand-held calculator when contemplating "how accurate" is a particular color correction result.

However, there is available (from the author) an Excel spreadsheet that performs this entire calculation chain in a convenient way. Instructions for its use, and a link to where it is (at this writing) available on the Internet) are given in Appendix B.

ACKNOWLEDGEMENT

Thanks to Bruce Lindbloom, on whose Web site are presented, in easy-to-follow form, many of the equations involved in the calculations described in Appendix A and the values of the transformation matrix, [M], for many RGB-family color spaces.

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[3] In one outlook, the theoretically-Ideal result of a color correction (to overcome the perceptual impact of a non-ideal incident light chromaticity on a photographic scene) is when a "neutral" item in the scene has, in the corrected image, a representation whose chromaticity is the reference white chromaticity for the color space in use.

APPENDIX A

Calculating du'v' from the sRGB coordinates R, G, and B

Here we give the entire chain of mathematical calculations required to:

- Determine, from the sRGB color space coordinates (R, G, and B) of a color observed in a digital photographic image, the u' and v' coordinates that describe the chromaticity of that color in absolute terms.

- Determine the value of the chromaticity difference metric du'v' for the sRGB coordinates for two colors being compared (or between one color of interest and the reference white chromaticity of the sRGB color space).

Notation

In certain steps, we will give a single equation that is used to convert some form of the coordinates R, G, or B to some other form of the same coordinates. To allow the equation to be stated only once, we will use this notation: the symbol "C" (in one form or another) stands for either R, G, or B (in the corresponding form):

This if we have an equation that allows us to determine r from R, g from G, or b from B, we will show it as determining c from C.

The color space

Note that the particulars of the calculations shown here apply to the sRGB color space. A similar chain of course will exist for other "RGB-family) color spaces, but the details are different.

THE CALCULATIONS

Normalizing R, G, and G

We normally have R, G, and B stated on a scale of 0-255. The standard equations assume R, G, and B to be on a scale of 0-1. Thus we must first normalize R, G, and B:

$$C = \frac{C_{255}}{255} \qquad (1)$$

We use the unsubscripted symbol for the normalized form to simplify the equations to follow.

Linearizing R, G, and B

The values R, G, and B are nonlinear indicators of the “potency” of the respective tristimulus values, as a result of the application of what is often called “gamma precompensation”. For our further work, we need linear tristimulus values, which we denote r, g, and b (again, shown in terms of our “proxy”, C):

$$c = \frac{C}{12.92} \qquad C \leq 0.04045 \tag{2}$$

$$c = \left(\frac{C + 0.055}{1.055}\right) \qquad C > 0.04045$$

Determining X, Y, and Y

We must now transform the color expressed as r, g, and b to the CIE tristimulus color coefficients, X, Y, and Z Matrix multiplication is involved here.

$$\left[X\ Y\ Z\right] = \left[r\ g\ b\right]\left[M\right] \tag{3}$$

[*M*] is a transformation matrix whose coefficients reflect the coordinates of the three tristimulus primaries defined for the specific RGB color space and the “luminance weighting” function.

In the case of the sRGB color space, that matrix is:

$$[M] = \begin{matrix} 0.412424 & 0.212656 & 0.0193324 \\ 0.357579 & 0.715158 & 0.119193 \\ 0.180464 & 0.0721856 & 0.950444 \end{matrix} \tag{4}$$

Determining u′ and v′

Now, we must determine the chromaticity of the color of interest in terms of the CIE chromaticity coordinates, u′ and v′.

$$u' = \frac{4X}{x + 15Y + 3Z} \tag{5}$$

$$v' = \frac{9Y}{x + 15Y + 3Z} \tag{6}$$

Now, for the differences

We first determine the two-dimensional chromaticity difference (on the u′v′ plane) between the two colors of interest this way:

$$du' = u'_2 - u'_1 \qquad (7)$$

$$dv' = v'_2 - v'_1$$

Finally, we determine the single metric du’v’, the distance (on the u’=v’ plane) between the points representing the two chromaticities being compared:

$$du'v' = \sqrt{u'^2 + v'^2} \qquad (8)$$

Thus we have our result.

Adobe RGB

If we are working in the Adobe RGB color space, the same calculations are valid if we make the following changes:

Equation 2 becomes:

$$c = C^{2.2} \qquad (2a)$$

The transformation matrix (equation 4) becomes:

$$[M] = \begin{matrix} 0.576700 & 0.297361 & 0.0270328 \\ 0.185556 & 0.627355 & 0.0706879 \\ 0.188212 & 0.0752847 & 0.991248 \end{matrix} \qquad (4a)$$

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APPENDIX B
An Excel spreadsheet to calculate du′v′ from R, G, and B

For those of us who can't perform matrix multiplication without taking off our shoes and socks, I have prepared an Excel spreadsheet that calculates, for a color expressed in the sRGB coordinates R, G, and B, the u′ and v′ coordinates of the chromaticity of the color, and the difference values du′, dv′, and du′v′ that compare the chromaticity of two colors.

This spreadsheet can (as of this writing) be obtained here:

http://Pumpkin.Annex.home.att.net/XLS/RGB-upvp.xls

Attached is a screen shot of the spreadsheet. The inputs (R,G, and B) and outputs (du′, dv′, and du′v′) are in a concise panel on the left. On the right are the results of the intermediate calculation and the constant coefficient values of the transformation matrix, [*M*].

There are rows for up to 20 different colors. Space is provided for the user to enter a description of each color for reference on a printout (or copy) of the spreadsheet.

The values of the chromaticity differences, du′, dv′, and du′v′ are all reckoned with respect to the color in the topmost row ("Ref"). In that row I have prefilled the RGB values 255,255,255. This color by definition has the reference white chromaticity for the color space (sRGB).

If you wish to compare the chromaticity of some color with that of another color (the "reference color"), just enter the RGB coordinates of the reference color into the first ("Ref") row.

Enjoy!

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Determination of the chromaticity difference metric *du'v'*
from color expressed in terms of the sRGB coordinates, R, G, B

		Inputs (sRGB)			Results (uV' scale units)				
					Absolute		Relative		
Item	Description	R	G	B	u'	v'	du'	dv'	duV'
Ref	Reference white	255	255	255	0.197835	-0.468326	0.000000	0.000000	0.000000
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									

Intermediate results

Item	r	g	b	X	Y	Z	x	y	Y
Ref	1.000000	1.000000	1.000000	0.950467	1.000000	1.088969	0.312712	0.329008	1.000000
1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
2	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
3	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
4	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
5	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
6	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
7	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
8	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
9	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
10	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
11	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
12	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
13	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
14	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
15	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
16	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
17	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
18	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
19	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000
20	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000			0.000000

rgb to XYZ matrix

0.412424	0.212656	0.019332
0.357579	0.715158	0.119193
0.180464	0.072186	0.950444

NOTES

1. The metric **du'v'** is generally indicative of perceived difference in chromaticity (for modest differences), and is generally consistent across different portions of the chromatcity chart and for differences "in any direction".

2. The "prefilled" values of RGB for the "reference" item, 255,255,255, describe a color having the reference white chromaticity for the sRGB color space. If we leave it in place, then for other items, the duV' value will represent departure from the reference white chromaticity.

RGB-uV'.xls dak Issue 1 2008.02.27