Astigmatism in Camera Lenses—Meridional and Sagittal MTF Response

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ABSTRACT

When presenting lens performance data in the form of a modulation transfer function (MTF), we often see separate curves for *meridional* and *sagittal* response. This primarily relates to a lens aberration called *astigmatism*. In this article, we discuss astigmatism and the significance of the terms meridional and sagittal.

SUMMARY

Astigmatism is a lens aberration that results in the cone of light from an object point not being converged to a "point image" at any place behind the lens. Rather, the cone is converged in one direction (for example, horizontal) at a certain location, and in the other direction (in that example, vertical) at a different location.

This phenomenon can be caused by asymmetry in the lens. However, even in a perfectly symmetrical lens, the phenomenon will still occur for object points not on the optical axis.

If we focus the lens so that the point imaged of an on-axis object point will lie exactly on the film plane, then the two locations of convergence for an off-axis plane will both lie forward of the film plane. At the film plane, the cone of light from the off-axis point will form a "blur figure", whose width is greater in one direction than the other—somewhat elliptical in shape. The orientation of this figure is in a predictable orientation with respect to the location of the object point and the optical axis. Its long axis is always "points to" where the optical axis strikes the film plane.

The phenomenon that causes the resolution of a lens (perhaps as reflected in the plot of its modulation transfer function, or MTF) to be limited is that the image of an object point is not a point, but rather a blur figure. The degree to which the resolution is limited (or the MTF becomes less than 1.0) depends on the width of the blur figure. In our case, with astigmatism present, the width that matters is the width in the direction of the "track" along which we determine the response of the lens to the detail in the test target.

Accordingly, in the presentation of the MTF plot for a lens, we have two sets of curves, one describing the response along a track along a line from the object point of interest through the optical axis, and the other for a track at right angles to that line.

The direction along the line through the optical axis is called the *meriodional*, or *tangential*, direction, and the one at right angles to that line is called the *sagittal* direction.

THE MODULATION TRANSFER FUNCTION (MTF)

The modulation transfer function (MTF) plot of a camera lens describes its ability to transfer to the film or digital sensor the patterns of contrast that constitute detail in the scene.

The MTF value varies with a number of parameters, including:

- Model of lens
- Focal length (for a zoom)
- Focus distance (but usually tested at infinity)
- Aperture in use
- The spatial frequency of the detail (how "fine" it is)
- Location in the overall frame (distance from the center)
- The wavelength of the light involved
- The orientation of the detail

It is the last of these that will figure most prominently in the topic of this article.

The dependence of the MTF on the orientation of the detail is usually recognized by the presence of two sets of curves in the MTF presentation, often said to be for *meridional* and *sagittal* response.

Although the underlying concept is straightforward, the terminology used is rather curious. Explanations of the terminology are often confusing at best and erroneous at worst.

The term *meridional* is sometimes replaced with the term *tangential*, especially in scientific work, and (less frequently) *sagittal* is replaced by *radial (*and in even rarer cases by *equatorial*). We'll explain the origin of each term at the appropriate point, but for consistency will use the terms *meridional* and *sagittal* throughout the main text of this article.

ASTIGMATISM IN CAMERA LENSES

Introduction

Astigmatism is a lens aberration in which the lens does not converge all the light coming from a point on the object at a single point behind the lens. the same distance behind the lens, for two different directions of convergence. Rather, the "cone of light" is converged in one direction (for example, horizontal) at a certain distance, and in the other direction (in that example, vertical) at a different distance.

In the human eye, astigmatism results from the eye's lens system (cornea plus lens proper) not being a true "figure of revolution"; that is, not having the same cross section at different planes through the lens axis. This cause of astigmatism is almost absent from camera lenses.

In a camera lens, one form of astigmatism can result from improper alignment ("decentering") of the individual lens elements. This form affects the focusing of object points whether they are on the lens axis or off.

Oblique astigmatism in a camera lens does not result from any defect in manufacture or assembly but rather is an inherent phenomenon of basic lens behavior. As its name suggests, it only affects object points not lying on the lens axis (and, generally speaking, is more severe the greater the distance of the point from the axis). It can be reduced (corrected) by taking various steps in the design of complex lenses, but it is never practical to completely eliminate it (especially while at the same time acceptably mitigating other types of aberration).

Our concern in this article is wholly with oblique astigmatism and its effect on the response of a lens to detail having different orientations.

We can understand the source and significance of oblique astigmatism with the aid of Figure 1.

In the figure, we assume a simple camera lens exhibiting uncorrected oblique astigmatism. Note that we have shown a relatively-unlikely ratio of object distance to image distance merely to make the figure more manageable.

In view A of the figure, for reference, we see a stylized representation of the ideal operation of the lens in forming a "point image" of a point on the object—in particular for a point on the optical axis. Oblique astigmatism does not impact this situation.

Ideally, the entire cone of light from the object point is converged to a point on the focal plane where we will find the film or digital sensor array.

If we examine the situation in front of the image plane, or behind it, we see that the cone of light has a finite circular cross section. We describe that figure as a *blur circle* or *circle of confusion*. (Of course, these portrayals have been rotated into the plane of the paper so we can see them.) If, through incorrect focusing, convergence does not occur precisely at the focal plane, such a blur circle results on the image for each point of the object, resulting in a blurred image overall.

Views B1 and B2 of the figure show a different situation, one in which the object point of interest lies off the optical axis, in this case at the "6 o'clock" position. The behavior of the lens is not now symmetrical rotationally, so we must look at the field of battle both from above (view B1) and from the side (view B2) to see what is going on.

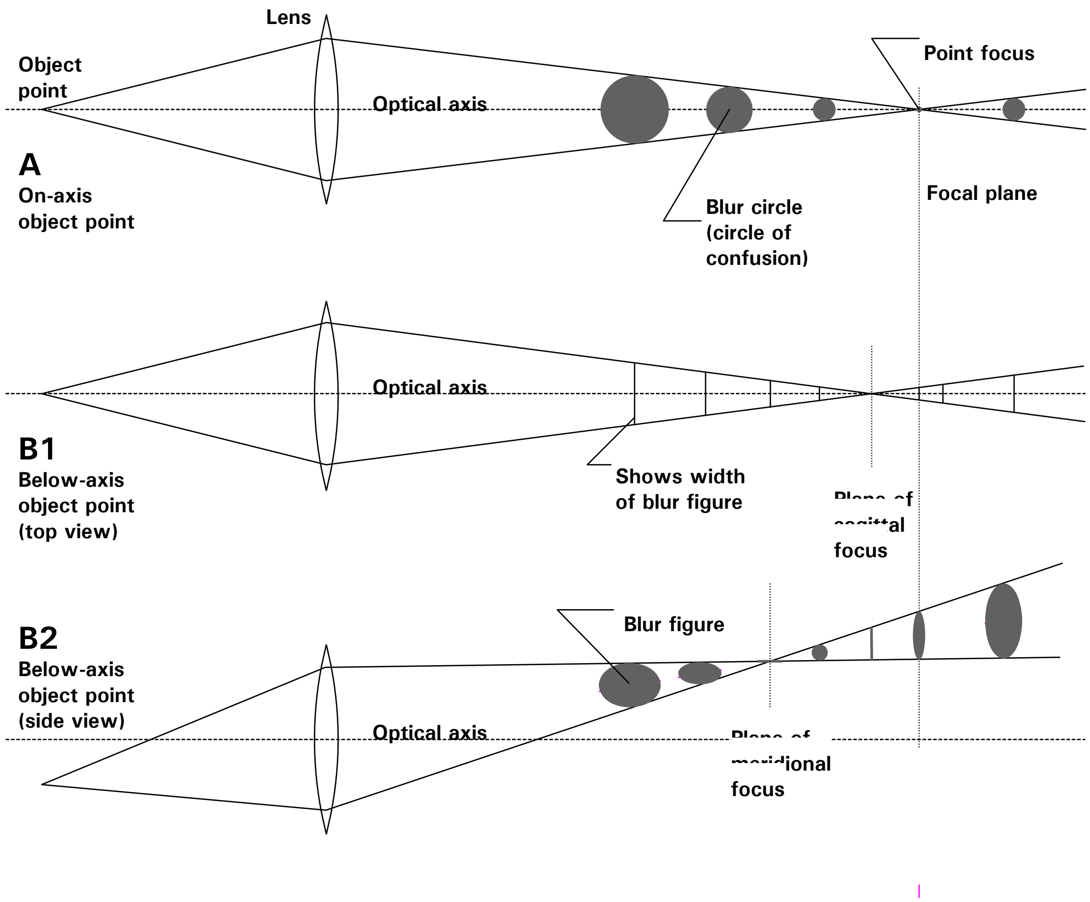

Figure 1. Oblique astigmatism

Observing from above (view B1), we note that the width of the "cone" of light decreases to zero at a certain distance behind the lens, a location called the *plane of sagittal focus*. (Don't try and figure out why it is called that—this will ooze to the surface later in our discussion.) Note that this is nearer the lens than the focal plane. We might just think that the lens has a shorter focal length for off-axis points, but the situation more complicated than that.

If we now look at the situation from the side (view B2), we see that the height of the cone of light decreases to zero at a **different** distance behind the lens[1], a location called the *plane of meridional focus*. (same warning as before—don't try and figure out why it is called that.) Here, it is as if the lens had an even shorter focal length than it exhibited when the action was viewed from above.

Thus there is no place on the emerging "cone" of light where all the rays converge to a point—no point at which the film could be placed to receive a proper "point image" of any off-axis point of the object.

[1] That being the case, "cone" is not really apt, but we will continue to use it as a handy metaphor.

In view B2 of the figure, we see the two-dimensional cross section of this "cone" at several different locations. Again, these portrayals have been rotated into the plane of the paper so we can see them.

Near the lens (we don't really show this), the cross section is nearly circular. As we proceed farther to the rear, the shape becomes essentially elliptical (with the major axis of the ellipse horizontal), and, at the plane of meridional focus, becomes just a horizontal line (called the *meridional line image*). (We have arbitrarily shown the line with some thickness just so we can see it.) We can think of this as a place where the image of the object point is converged vertically but still spread horizontally. (The formation of the meridional line image is discussed in further detail in Appendix A.)

As we continue farther to the rear, the line opens into nearly a circle, and then changes to an ellipse with its axis vertical (not shown). Later, at the plane of sagittal focus, the figure again becomes a vertical line (called the *sagittal line image*). We can think of this as a place where the image of the object point is converged horizontally but still spread vertically.

By the time we reach the focal plane, the cross-section of the cone has again become essentially an ellipse, this time with its major axis vertical.

In fact, if the camera is still focused as it was in view A, it is this elliptical spot that is the "blur figure" which falls on the film for the off-axis point.

It is this behavior that is described as the aberration of *oblique astigmatism*—the aberration that is commonly referred to as just *astigmatism*.

Impact on MTF

A camera lens does not exhibit an ideal MTF plot (MTF = 1 for all situations) if, for any situation, a point on the object is not imaged as a true point on the film, but rather as a "blur figure". Normally, the MTF declines as the spatial frequency (fineness of detail) increases. The larger the blur figure, the earlier this decline begins.

Note that the blur figure of which we speak need not result only from misfocus. (If it did, then the lens would exhibit perfect MTF so long as it was focused properly.) The blur figure, even at best focus, results from various lens aberrations (only one of which is astigmatism, by the way).

Suppose the blur figure isn't circular but, as in our example of astigmatism, nearly elliptical (in the example, with the long axis vertical).

If we are talking about the reproduction of detail as we move along a horizontal path across the object (perhaps across a test pattern of vertical lines), the effective diameter of the blur figure is small (the "width" of the ellipse), and the decline in MTF with spatial frequency which that causes has a certain modest degree. But if we instead consider moving along a vertical path across the object (perhaps across a pattern of horizontal lines), the effective diameter of the blur figure is larger (the "length" of the ellipse), and the decline in MTF with spatial frequency which that causes is greater.

It is for this reason that a difference in MTF for different directions of traverse across the object is an indication of the presence of astigmatism.

Note that the two directions of traverse across the object are not always vertical and horizontal. That is only true in our example because, for convenience, we chose an off-axis point the was directly below the lens axis.

In the more general outlook, the two directions of traverse of interest are:

- Along a line passing through the point of interest and the optical axis
- Along a line at right angles to the first line.

It is these two directions of traverse across the image that are spoken of as the *meridional* and *sagittal* directions, respectively. (We'll see why shortly.)

For astigmatism of the type we saw in the figure (the most common type), we can see that the MTF in the meriodional direction (vertical in the example) will decline faster than the MTF in the sagittal direction (horizontal in the example).

NOW, THE TERMS

In our work so far, we have used two terms, *meridional* and *sagittal*, to refer to two different directions. We have urged the reader not to struggle to understand exactly what those words mean. We are now ready to actually look into their significance.

These terms actually have synonyms, including:

Meridional = **tangential** = circumferential

Sagittal = radial = equatorial

The first listed term of each group is the one most often used in practical technical information about camera lenses; the ones shown in bold are the most customary in formal technical writing about this topic.

We can actually best follow the logic of the terminology by first considering the terms *tangential* and *radial*. They have a direct and obvious meaning in geometry, as shown on figure 2.

We see on the figure a circle with a point of interest on its periphery, a radius of the circle through that point, and a tangent to the circle through that point (a line through the point perpendicular to the radius). To avoid the implication that "horizontal" and "vertical" directions are involved here, we have intentionally chosen a point not lying at a cardinal direction from the axis.

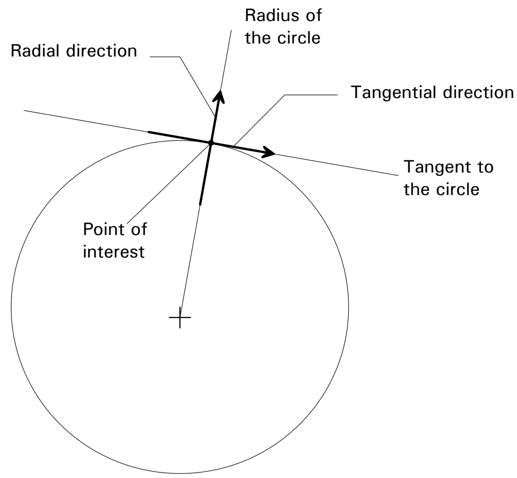

Figure 2. Radial and tangential directions in geometry

The *radial* direction is the direction lying along the radius; the *tangential* direction is the direction lying along the tangent. (Duh!)

As we mentioned earlier, the radial direction is also called the *sagittal* direction. Sagittal comes from the Latin, and means "as the arrow flies"[2]. In this case, the metaphor is an arrow shot from the optical axis toward the point of interest. From here on we will continue to use the term sagittal rather than radial, since it is what we find in most optical and photographic writing.

(For those of you who already know the ultimate punch line of this topic, you may think that I have this backwards, considering the way these two terms are used in discussions of MTF and astigmatism. Stand easy—I'm not done yet. Figure 2 merely gives the geometric meaning of the terms. Some curious things will happen by the time we get to their application to our subject.)

Figure 3 shows how this notation relates to the orientation of the two line images we saw generated in figure 1, views B1 and B2.

It shows the two line images as they would be seen in their respective focal planes. Note that one of them falls in the sagittal (radial) direction, and the other in the meridional direction. The images get their names from this.

[2] It has the same root as the name of the mythological figure Sagittarius, "The Archer".

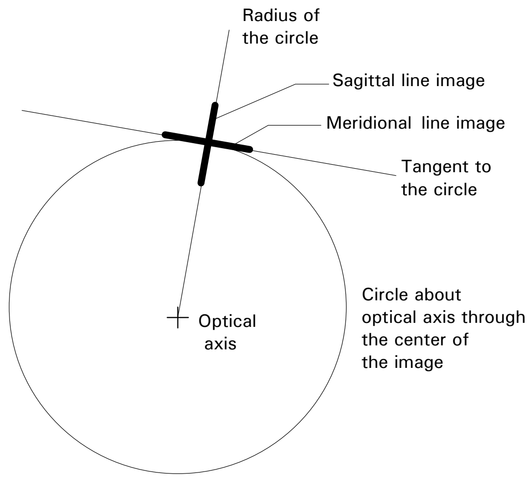

Figure 3. Orientation of line images

THE PLANES, THE PLANES!

Often, in dealing with the matter of astigmatism in lenses, it is convenient to segregate the rays in the cone emerging from an object point and captured by the lens into groups having consistent behavior. One way this is done is to identify two different planes in the space traversed by the rays (figure 4).

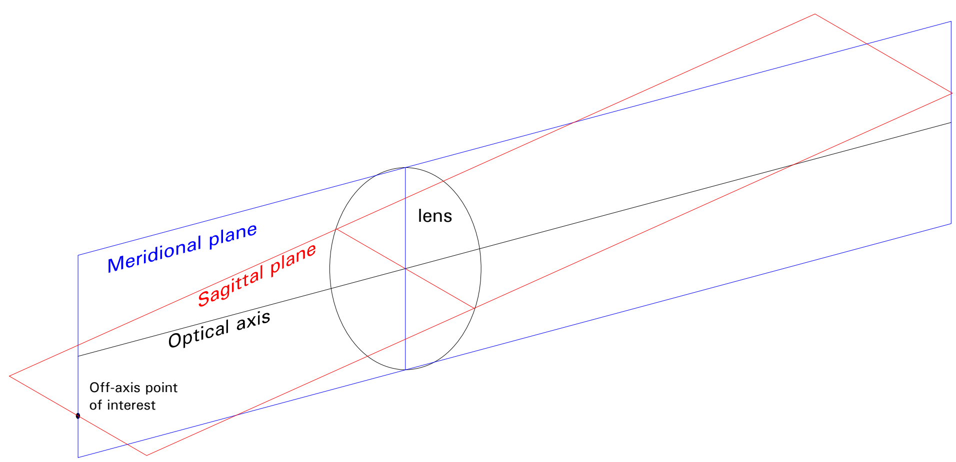

Figure 4. Meridional and sagittal planes

The *meridional plane* is a plane that includes both the optical axis and the off-axis point of interest. In our example, it is a vertical plane, but would not always be, depending on the direction in which the point is off-axis. The *sagittal plane* is a plane that includes the off-axis point of interest and the center of the lens and is perpendicular to the meridional plane. (No, we still can’t see yet why they have those names!)

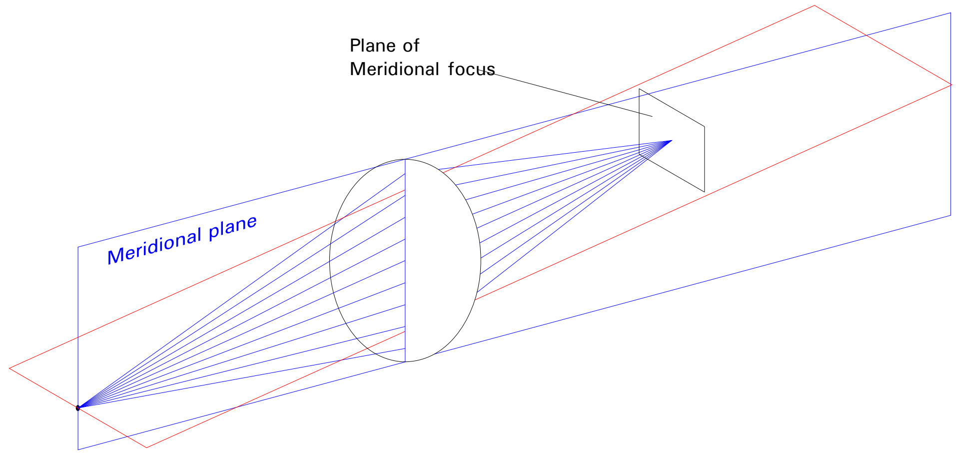

Figure 5. The meridional fan of rays

In figure 5, we see only those rays emanating from our object point that lie in the meridional plane. They of course do not form a cone, but only a flat fan-like arrangement, and in fact this collection of rays is often called the *meridional fan*.

Note that the rays of the meridional fan converge at a point lying in the *plane of meridional focus*. (This point is in fact one point of the *meridional line image* we saw in figure 1, views B1 and B2.)

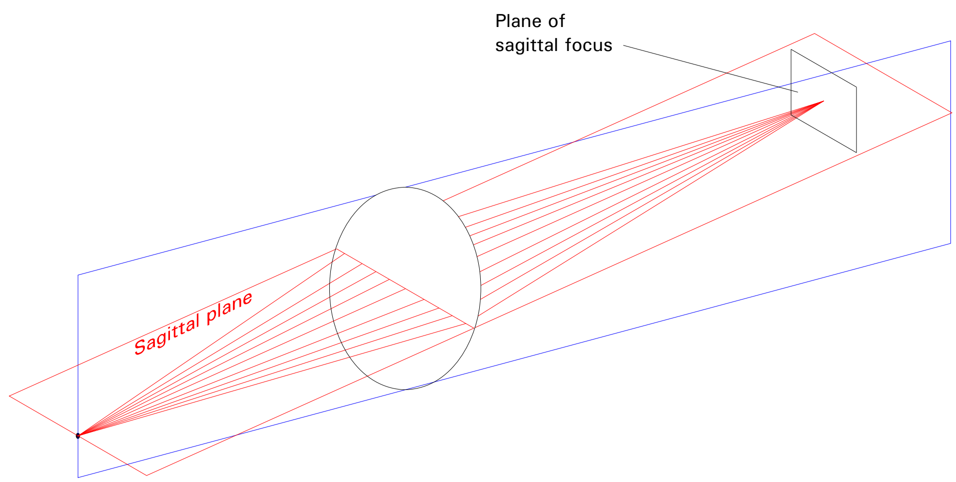

Figure 6. The sagittal fan of rays

In figure 6, we see only those rays emanating from our object point that lie in the sagittal plane. Again, these form a flat fan-like arrangement, called the *sagittal fan*.

Note that the rays of the sagittal fan converge at a point lying in the *plane of sagittal focus*. (This point is in fact one point of the *sagittal line image*.)

In these two figures, we have exaggerated the difference in the distance from the lens to the meridional and sagittal focus planes (compared to the relationship shown in figure 1) in order to make the difference more obvious.

BACK TO THE MTF

When we test for MTF, we take account of the different behavior of the lens with respect to detail of differing orientation (the result of astigmatism) by testing along "tracks" of differing orientation. Here we see the notation associated with these tracks. [Note that the term "track" is the author's, and is not generally used in technical writing about this topic.)

Note that the synonyms of these names are just the opposite of the geometric names associated with these two directions, one source of confusion in this whole area!

These names are based on the orientation of the two line images shown in figure 1. As we travel along any track, the response of the lens to the detail is based on the narrowness of the line image that is oriented perpendicular to the track. In the case of the track running outward from the optical axis, that is the *meridional line image*. Thus, we speak of the direction of that track as the *meridional* direction. In the case of the track running perpendicular to that track, that is the *sagittal line image*. Thus, we speak of the direction of that track as the *sagittal* direction.

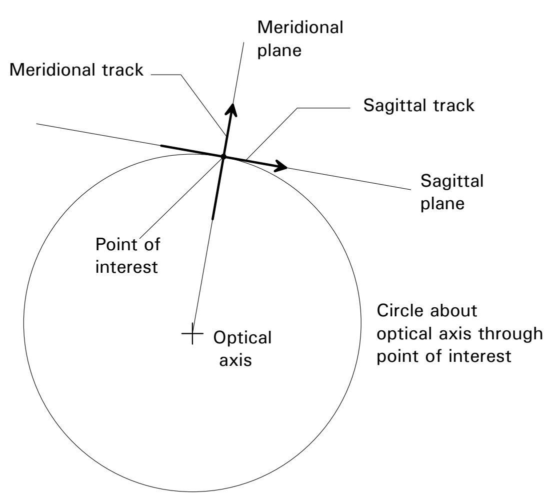

Figure 7. MTF track directions

We can see another aspect of this by examining the orientation of the test patterns used to test the response in the two directions—along the two tracks (figure 8).

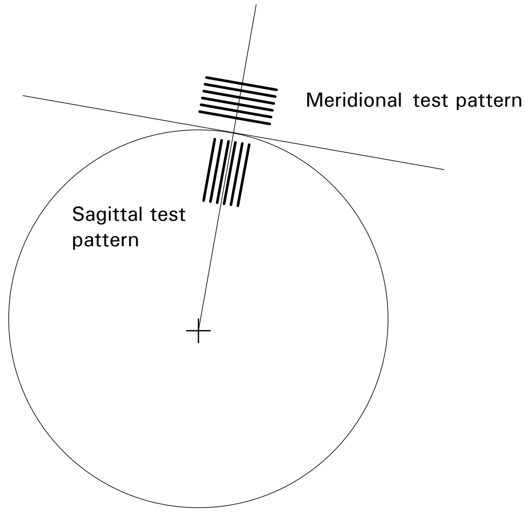

Figure 8. MTF test patterns

For testing along the meridional track, we use a pattern of lines running perpendicular to that track. For testing along the sagittal track, we use a pattern of lines running perpendicular to that track. Of course, for actual testing, we have several sets of such lines for each orientation, one for each spatial frequency for which we wish to measure the MTF. And the patterns aren't really "sharp edged" lines, but rather a pattern of sinusoidal variation in luminance.

What about "meridional"?

So far, we have consistently used the term "meridional" for one of the two directions associated with many concepts in this subject, without explaining its basis. In fact, for most lens MTF data published by lens manufacturers, the term *meridional* is used instead of the term *tangential*, used in most scientific work. You've already seen the rationale, tortured as it is, for the application of the term tangential. But where did *meridional* come from, and why?

I'm not certain what brought this about. My guess is that the term "tangential" just seemed too counter-intuitive. For example, if we compare figures 2, 4, and 7, we see that the MTF direction we call meridional (tangential), and the orientation of the ray plane we call meridional (tangential), are in fact those that correspond to the **radial** direction of geometry. (How this happened I hope that I have explained to you as we went.)

It may be that the introduction of the alternate term "meridional" was meant to overcome this apparent inconsistency.

The basis of the word is that the plane whose name it bears corresponds conceptually somewhat to a plane through the earth's axis, a plane that defines the earth's meridians.

What about “sagittal”? Isn’t it also counter-intuitive, for the same reason? Well, it is. If the wonks had stuck with “radial”, there would have been the same seeming inconsistency that there is with “tangential”. But since (trying to show off their knowledge of Latin, I suppose) they had already largely replaced “radial” with “sagittal”, and since nobody could understand what sagittal meant in this context, nothing seemed counter-intuitive (or intuitive either)!

In some fairly rare cases the term “equatorial” is used as an alternate for sagittal, I assume for consistency with “meridional”.

THE TRAIL OF NAMES

Let’s summarize the trail by which the various items of interest get their “orientation” names.

- The meridional and sagittal line images get their names from their geometric orientations (see figure 3).
- The planes of meridional and sagittal focus get their names from the names of the line images they contain (see figure 1).
- The meridional and sagittal planes get their names from the fact that the rays in each one are brought to a focus at the correspondingly-named plane of focus, as part of the correspondingly-named line image (see figures 5 and 6).
- The meridional and sagittal directions of traverse across the image get their names from the planes in which they lie (see figure 7).

MISCONCEPTIONS

There are a number of misconceptions floating around that help to confuse us when trying to understand this already-confusing subject. I’ll discuss two of the most problematical ones here.

The “frame diagonal” misconception

We often read (in descriptions pertaining to the MTF testing of camera lenses) that the meridional direction is defined as the direction along the frame diagonal, and the sagittal direction is the direction perpendicular to the frame diagonal.

That turns out to be true in the case where the point of interest happens to lie on the frame diagonal. But that is not the general case, and the true definitions are not based on that presumption. (Note that in the explanations above, the frame diagonal hasn’t even been mentioned.)

How did that misconception get started? Here’s my guess.

The MTF of a lens varies with several parameters, one of which is the distance from the frame center. In fact, in the form of the MTF chart most often used in presenting lens characteristics, distance from the center of the frame is the independent variable (on the horizontal axis).

We are interested in the MTF for the full range of distances from the center that we might encounter. Of course, the largest distance from the center occurs at the corners of the frame. Thus we need to be sure to take measurements all the way out to a corner.

Having decided that, we might as well, for the sake of orderliness, take all our measurements—at different distances from the frame center—at points along a diagonal (which of course reaches the corner).

Then, for any such test point, the "meridional" direction is indeed along a line from the center of the frame, which of course the diagonal is. And the sagittal direction is perpendicular to that. But this does not constitute the definitions of the two directions.

The "fan gives a line image" misconception

Even in well-respected textbooks on optical engineering, the statement is often made that "the rays in the meriodional fan form a line image at the plane of meridional focus; the rays in the sagittal fan form a line image at the plane of sagittal focus." That's just not so. The rays of the meridional fan form a **point image** at the plane of meridional focus[3]; the rays of the sagittal fan form a **point image** at the plane of sagittal focus (as we see in figures 5 and 6.)

What does form the meriodional and sagittal line images is **all** the rays emanating from the object point of interest—that is, the entire "cone" of rays from the object point that is accepted by the lens. We actually see that in figure 1.

I can't even guess where this misconception came from.

In any case, the matter of the formation of the meridional line image is discussed in considerable detail in Appendix A.

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[3] This is not precisely true if we consider the effects of other lens aberrations, but that does not disrupt matter at issue in this section.

APPENDIX A

Formation of the meridional line image

Even in well-respected textbooks on optical engineering, the statement is often made that “the rays in the meriodional fan form a line image at the plane of meridional focus; the rays in the sagittal fan form a line image at the plane of sagittal focus.” That’s just not so. The rays of the meridional fan form a **point image** at the plane of meridional focus; the rays of the sagittal fan form a **point image** at the plane of sagittal focus. The entire ensemble of rays in the ”cone” is required to form a line image.

This can be most persuasively illustrated by considering the matter of formation of the meridional line image. Please see figure 9.

Section A of the figure shows the case where there are only meridional rays (the “meridional fan”). In view A1, we see the lens space looking at the meridional plane. We will assume the same situation as in our earlier example—the off-axis point at the 6 o’clock position—so we can conveniently speak of right and left, up and down.

In this view, we see that all the meriodional rays are vertically converged at the plane of meridional focus.[4]

In view A2, we are looking down on the lens space, essentially looking at the sagittal plane (although it is sloping up to our right).

Approaching the lens, the rays of the meridional fan are (by definition) confined to the meriodional plane (which we see edge-on). Passing through the lens, they are not deviated to either side. (An intuitive proof of this relies on symmetry: if they would be deviated, to which side would it be?) Thus the rays remain confined to the meridional plane downstream of the lens.

Thus, at the plane of meridional focus, the image can have no width—there are no rays outside the meridional plane. Accordingly, it is a point image that is formed, not a line image. We show on view A2 where the meridional line image would be, if generated. Note that there are no rays to form any part of it other than its very center—a point image. A line image is not formed by the meridional rays alone.

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[4] In fact, if the lens has uncorrected spherical aberration, the vertical convergence will not be perfect. This however does not disrupt the point being made here.

In section B of the figure, we consider **all** the rays emanating from the object point that pass through the lens. Looking ("from the side") at the meridional plane (in view B1), we see that all the rays are still vertically converged at the plane of meridional focus—that is in fact what that term means.

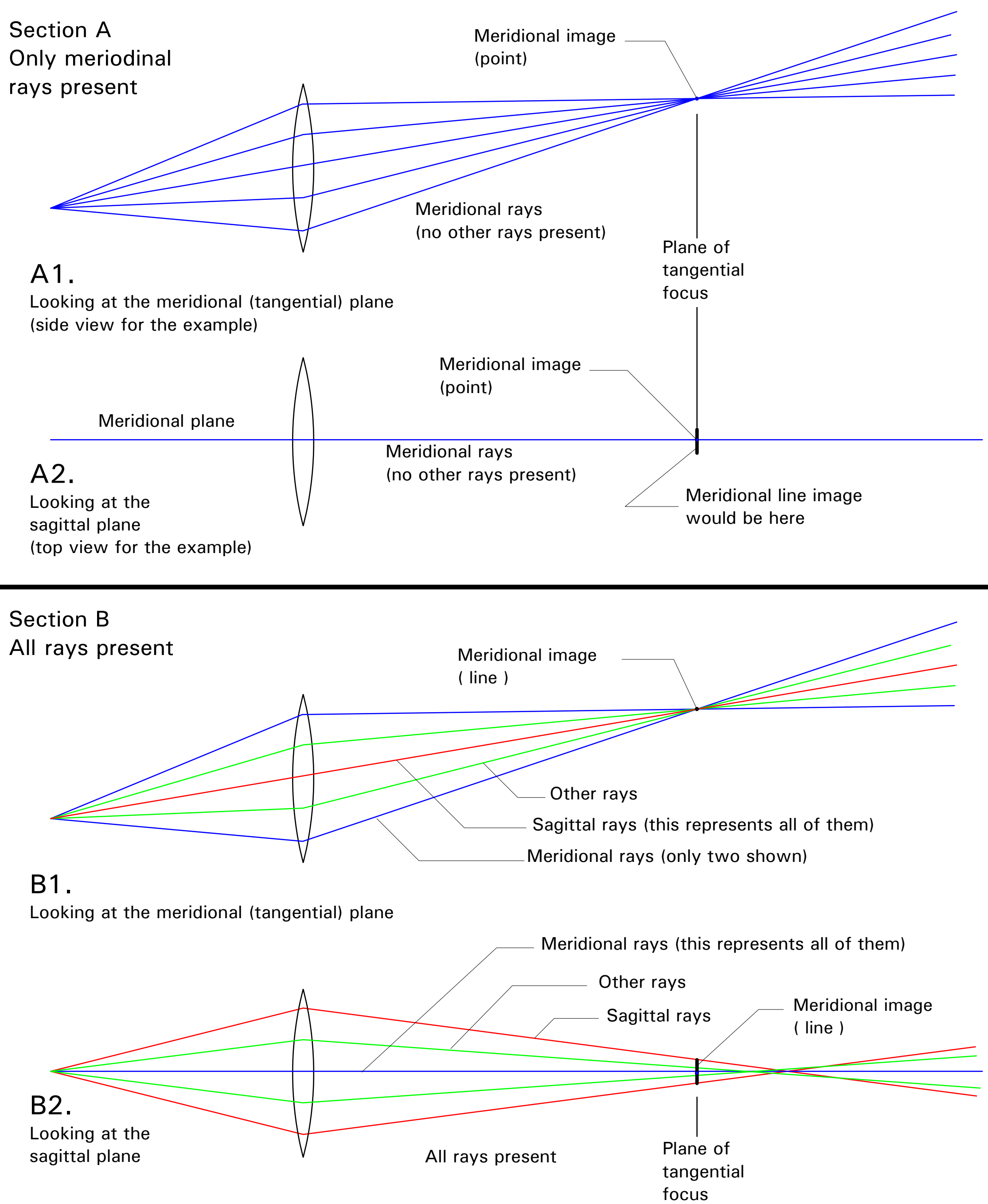

Figure 9. Formation of the meridional line image

But looking down (in view B2) we see that the rays are not converged horizontally at the plane of meridional focus—the basic symptom of oblique astigmatism. The overall result in this case is the formation by the ensemble of rays not of a point image but rather a line image—a horizontal one, for the orientation of our example, the one in any event known as the “meridional line image” (as seen in figure 1).

A similar demonstration can be made for the sagittal fan. It is slightly complicated in that we can’t rely on simple symmetry to persuade ourselves that the rays remain confined in the sagittal plane after they pass through the lens.

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